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EVALUATION OF THERMAL-NUCLEAR EFFECTSFROM PAIR-CREATION IN THE FINAL FATE OF VERY-MASSIVE STARS

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ABSTRACT

Thermonuclear conditions found in explosive massive-stars requirethe use of not only efficient, accurate but thermodynamically consistent stellar equation of state (EoS) routines.The use of tables to describe EoS involved in stellar models is very much needed in understanding the final fate of massive stars.Many massive-low metallicity stars end their life as pair creation supernova (PCSN) through the creation of electron-positron pairs.We used thermodynamically consistentEoS tables to numerically evaluate the thermonuclear effects of the electron-positron pair creation in rotating 150 and 200 Massive starsat SMC and rotating and non-rotating 500 $M_\odot$at LMC.As expected, the effect of rotationofreducing the oxygen core masshad increasedthe thermal energy within the threshold of the pair-creation instability.Similarly, lower mass loss stars with SMC modelproduced higher thermal energies,which cancompletely explode the stars as PCSNe without remnant.On the other hand, the non-rotating 500 $M_\odot$ might have only reached the instability region due to its lower metallicity (compared to solar metallicity) that is capable of suppressing the mass loss such that the thermonuclear energy maintainscertain amount of elements into the pair creation region. At the final explosion of the stars, the helium core mass reduced the thermal energies in trying to avoid the pair-creation region. Many implications of these results for the evolution and explosion of massive stars are discussed.

Keywords:equation of state-instabilities-stars: evolution-stars: massive

INTRODUCTION

In a previous study byYusof *et al.*(2013), it was pointed out that stellar models for rotating 150 $M_\odot$ and 200 $M_\odot$at Small MagellanicCloud (SMC) metallicity androtating and non-rotating 500 $M_\odot$at Large MagellanicCloud (LMC) metallicity, are expected to explode as pair-creation supernova (PCSN).However, their work did not include, in the equation of state, the effects due to the electron-positron pair-creation. In massive stars, very energetic and pressure-supporting photonsare converted into electron-positron pairs just before ignition of any element heavier than oxygen and subsequently leads to a violent contractionthatactivates nuclear explosion(Barkat, Rakavy, & Sack, 1967; Bond, Arnett, & Carr, 1984; Carr, Bond, & Arnett, 1984; Chatzopoulos & Wheeler, 2012a; El Eid & Hilf, 1977; El Eid, Fricke, & Ober, 1983; Fraley, 1968; Ober, El Eid, &Fricke, 1983; Rakavy & Shaviv, 1967; Stringfellow & Woosley, 1988; Wheeler, 1977).The thermal concentration of these pairsoccur during the advanced burning phase of the stars'evolutionand causes dynamical instability in the star (Woosley and Heger, 2015). This instability, which leads to anexplosion of massive stars, normally results in the formation of what iscalled Pair- creation supernovae (PCSNe or pair-instability supernovaePISNe). The formation of this PCSNewas first identified by (Barkat et al., 1967), in a detailed analysis of some relevant equation of state for very massive stars at the end of their lifetimes. The pair-creation instabilityisgenerally originated when the central temperature and density of a star are relatively high,(Arnett, 1996; Barkat et al., 1967; Fraley, 1968; Phillips, 2013; Rakavy and Shaviv, 1967) andthen entered anarea (see figure 1) where the energy needed to create the rest mass of the pairs (at high entropy) softens the equation of state and reduce the adiabatic index below 4/3(Fraley, 1968).

It has been predicted, for quite long, that any massive star that entered this region will become dynamically unstable due to the pair-creationand eventually disrupt the starand produce the PCSNe. Thepair-instabilityis a vital process in the explosion and collapse of not only the massive stars but many Astrophysical objects.

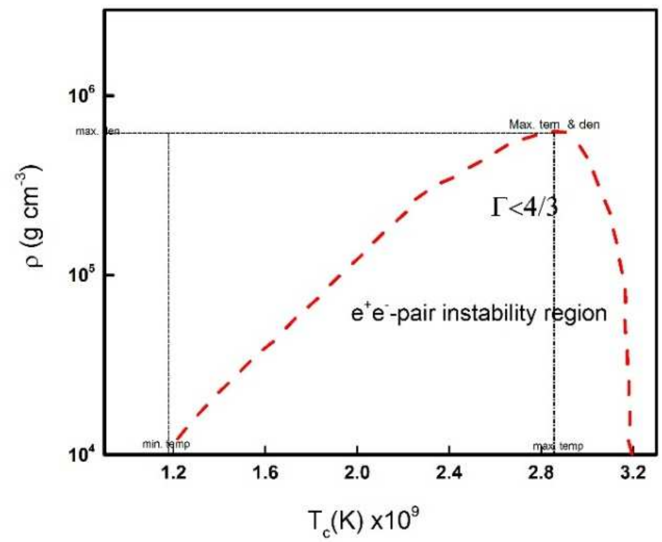

Figure 1: The electron-positron pair production regime. The adiabatic index is below 4/3 only within the unstable area (red shaded area) at maximum central density and temperature of about $7x10^5$ g cm^{-3} and $2.8x10^9$ K respectively.

The knowledge of which star may explode as PCSN has been the issue of discussion in many research works. In work by (Stanford et al., 2002), is lamented that electron-positron pair creation causeinstability in massive stars that haveinitial mass from around 120 $M_\odot$and higher(Stanford et al., 2002; Yusof et al., 2013). On the other hand, many evolutionary calculations found that stars with massive oxygen cores greater than 60$M_\odot$become dynamically unstable due to the pair-creation and the instability set in when the central temperature is high(Barkat et al., 1967).Principally, (Chatzopoulos & Wheeler, 2012b), investigated the minimum main-sequence massof a starcapable of reaching this instability regime and found that star with 65$M_\odot$will encounter full Pair-Instability Supernova(PISN) and that with 40$M_\odot$will encounter Pulsational Pair-Instability supernova (PPISN).The result predictsthe criteria for a star to enter the instability areato be controlled by themass of oxygen core; which in turnin main-sequence stars depend on metallicity, mass loss, and rotationally induced mixing as well as convective and semi-convective instability. In the case of metallicity, (Heger, et al., 2003)conclude that there is a threshold of metallicity below which PCSNe occur on account of the strong metallicity dependence on massive star winds. However, this metallicity threshold was investigated by(Langer et al., 2007).It is also found that the higher metallicity value(where the evolution of very massive stars is dominated by mass-loss) results to lower oxygen coresand therefore try to avoid the instability regime(Chatzopoulos & Wheeler, 2012a; Kozyreva, Yoon, & Langer, 2014; Vink et al., 2011), This condition is the most likely reason why the PCSNe do not exit at solar metallicity(Stanford et al., 2002; Yusof et al., 2013).Whereas, the low metallicityreduce the mass loss (see fig. 2) thereby relatively allow the lower mass main-sequence stars to encounter the instability region(Chatzopoulos & Wheeler, 2012a).However, for mass loss, stars lose mass at all evolutionary phases, and the rate of the mass loss varies depending on the initial mass of the star, thus the evolution of massive stars is strongly affected by mass loss, and there have been various mass loss prescription that is used for better understanding of the different mass loss rate in stellar evolutionary models. The mass loss prescription used in this work is the same as that used by (Yusof et al., 2013).

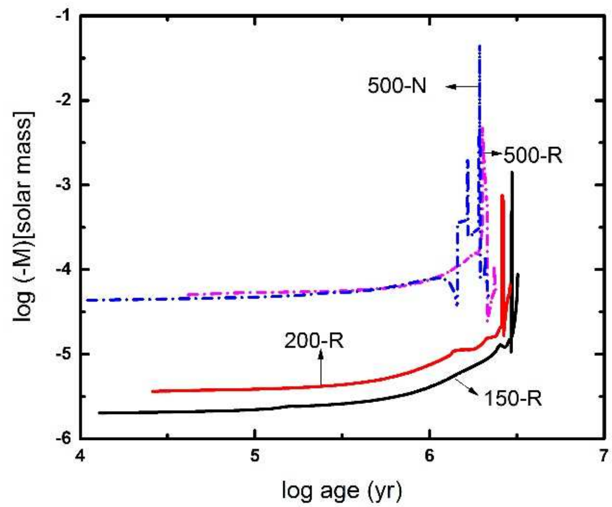

Figure 2: Mass loss rates for 150(black), 200(red) at SMC and 500 $M_{\odot}$ rotating (magenta) and non-rotating (blue) at LMC models. The maximum mass loss is proportional to an increase in the initial mass of the star, and the rotation reduces the mass loss. High metallicity results to low oxygen core and try to avoid instability, whereas, low metallicity reduce the mass loss.

In the case of rotation,the mass of the helium core for the main-sequence massincreases with respect to therotation (Figure 3)and this can affect the explosion mechanism of the stars by reducing the threshold of the pair-creation (Woosley, 2017).Another work by(Heger & Woosley, 2002), determined the range for a massive star with a helium core mass that could explode as pair-creation supernovae to beapproximate ~ 64-133 $M_{\odot}$. This mass of helium-coresignificantly affect the nucleosynthesis in pair creation supernovae.Meanwhile, stellar evolution models by (Yusof et al., 2013) indicated that massive stars progenitors expected to explode as pair-creation supernovae (PCSNe) is between about 100 and 290 $M_{\odot}$ for small MagellanicCloud(SMC) and above 450 $M_{\odot}$ rotating and non-rotating models for LargeMagellanicCloud(LMC). The advantage in this later discovery is that many effects have been put into consideration before concluding. For instance, the benchmark for helium core mass given by(Heger & Woosley, 2002) has been taken, similarly the metallicity factor, as highlighted by (Vink et al., 2011) and others, was put into consideration and finally the rotation effect which brings chemically homogenous evolution and produces higher oxygen core that is necessary for pair-creation has also been considered and compared with non-rotation.However, the progenitors also need to retain its mass very high enough, to maintain its core helium mass above ~65 $M_{\odot}$. However, this condition is not guaranteed at high metallicities where the evolution of avery massive star is dominated by stellar wind mass-loss(Vink et al., 2011).

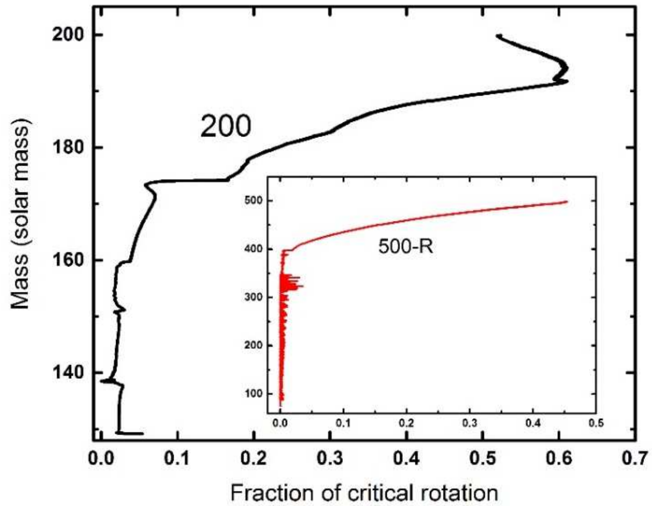

Figure 3: Plot of the mass of the star ($M_\odot$) with a fraction of critical rotation. The outer is for 200 $M_\odot$while the inner red is for 500 $M_\odot$. The helium core masses rise rapidly proportional to the critical rotation.

At theformation of the pair-creation supernova and after a sufficient portion of the star passed out from the unstable region, the pressure increases faster than the gravitational forces which reverseits collapse. And the energy released in oxygen burning again disrupt the star and eject all materials with high velocity(Fraley, 1968), this energy, which, might have to increase the temperature and provide more pressure, is wholly diverted into thecreation of electron-positron pairs. Similarly, the thermonuclear explosion disrupts the core of helium and other heavier elements, and the energy of this explosion increases with mass(Stanford et al., 2002).

On the other hand, the dynamically unstable stars at a specific temperature near the centeralso have an entropy which significantly affects its explosion(Fraley, 1968). At this extreme conditions found in massive stars; however; atoms are ionized, electrons become degenerate and relativistic (Bludman & Van Riper, 1977). Corollary,the thermonuclearenergy,relativistic electron-positron pairs isvery crucial to massive stellar cores (Blinnikov, et al., 1996) and we can understand many properties of the stellar interior by means of a thermodynamic system of properties, such that various thermodynamic properties at different macroscopic densities, chemical potential, and temperature T are vital in modeling stellar events which are characterized by thermodynamic equilibrium. This thermal equilibrium of a high-temperature plasma contains a minimum number density for electrons and positrons (Bludman & Van Riper, 1977) and therefore can be related by an equation of state (EoS). As the electron-positron pairs are created inside the massive stars at highly relativistic thermal energy (Odrzywolek, Misiaszek, & Kutschera, 2004; Phillips, 2013); some of the significant challenges are posed by the question of how these pairs affect the stability of the stars? What is the energy produced by these pairs and how does it affect the evolution and final explosion of the stars? There has been no literature that attempts to thoroughly investigate these crucial questions, except for the case of entropy production in the sun where the estimate was given by (Aoki, 1983; Kennedy & Bludman, 1997) and that of the main-sequence stars (Martyushev & Zubarev, 2015); however, these estimates were not made due to the electron-positron pairs. It is apparent that such information, when provided, is needed infullyunderstanding the explosion of massive stars, the pair-creationsupernova, and their subsequentcollapse.

In the present study,weadopt the stellar evolutionof threemodels reported by(Yusof et al., 2013) as discussed in section 2 below;specifically, we considered 150, 200, and 500 $M_\odot$.Themodels were evolved starting fromzero-aged main-sequence through atleast oxygen burning using Geneva evolution code, and then later, followed from the end of core helium burning through to explosions with the KEPLER Code. The KEPLER simulation confirms that indeedthe rotating SMC 150 and 200 $M_\odot$and the rotating and non-rotating 500$M_\odot$LMC models produce electron-positron pairs and the stars end their life as PCSNe.

We then apply the data into a modelled hydrodynamic Helmholtz equation of state (EOS) table written by (Timmes & Swesty, 2000) which relate pressure, energy, and entropy to temperature, density, and composition -and allows for electrons and positrons be relativistic and arbitrarily degenerate- to evaluate the quantitative values of the thermonuclear energy, pressure and entropy due to the electron-positron pair-creation at the final stellar masses of106.5, 129.2 and 74.8 and 94.7 $M_{\odot}$evolving from the threeselected massive starmodelsrespectively. The choice of the HelmholtzEos is due to its accuracy, speedily executable and thermodynamically consistent and it isbuild based on table interpolation of Helmholtz free energy.

STELLAR MODELS AND INPUT PHYSICAL PARAMETERS

The work byYusof et al., 2013 computed stellar modelsfor different massive stars, among which includerotating 150 and 200 $M_{\odot}$at Small Magellanic Clouds (SMC) and rotating andnon-rotating 500 $M_{\odot}$atLarge Magellanic Cloud (LMC)using Geneva stellar evolution code which has been used to solve most massive stars observed today (Crowther et al., 2010). This code in its latest developments has the prescription for both rotating, and magnetic fields included (Eggenberger et al., 2009; Ekström et al., 2012). The latterwork predicted the fate of all the models by simulating the evolution and final explosion of some selected starsusing KEPLER code and concluded that the core mass, or the carbon-oxygen (CO) core mass, is very much suitable to estimate whether the models produce PCSNe or not?(a method that has been used in various studies of very massive stars forasimilar demonstration of the fate of stars with the same CO core(Bond et al., 1984; Chatzopoulos & Wheeler, 2012a; Heger & Woosley, 2002). In another work byHeger & Woosley, 2002computed the grid models for stars with corresponding main-sequence masses of approximately 140-260 $M_{\odot}$ and found that stars having helium cores in the mass rangeM_{He} between 64 and 133 $M_{\odot}$produces electron-positron pairs and explode as PCSN, while more massive helium cores collapse to black hole (BH). However, the final results from work byYusof et al., 2013 found that; stars whose final mass is within the range$60 \leq M_{final} \leq 130M_{\odot}$will end as PCSN. This further indicates that, the KEPLER simulations confirm the rotating 150 and 200$M_{\odot}$at SMC and the rotating and non-rotating 500$M_{\odot}$at LMC indeed produces electron-positron pairs and end as PCSNe.Figure 4 shows the central temperature-density plot for the 150 and 200 $M_{\odot}$rotating SMC and 500 $M_{\odot}$rotating and non-rotating LMC, all the models fully entered the pair-creation regime except for the non-rotating 500 $M_{\odot}$where only small amount of its final mass reach the regime, this might be due to non-rotationeffect which we will explain below.At a high temperature, the thermonuclear reactions began near the centre of the star, and low mass stars stops at a specific nuclear burning phase while the massive stars continue to end at the silicon burning (Arnett, 1996; Odrzywolek et al., 2004).

This work adopted the stellar models and the final fate of very massive stars reported byYusof et al., 2013, such that we considerstars (rotating 150, 200 $M_{\odot}$, at SMC and the500 $M_{\odot}$rotating and non-rotating at LMC) that explode as PCSNe and encountered the instability regime (Fig. 4) attheirfinal stellar masses of 106.5$M_{\odot}$and 129.2$M_{\odot}$and 74.8 $M_{\odot}$ and 94.7 $M_{\odot}$respectively.Radiative line-driven winds from Vink, de Koter, & Lamers, 2001were used for the mass-loss prescription. This radiative mass lossreductionhas greatly influencedthe fate of the stars andthemaximum mass-loss rates for the initial masses under considerationwere found to be aroundlog (-2.85) $M_{\odot} yr^{-1}$,log (-3.12)$M_{\odot}$ yr^{-1}, log (-2.32) $M_{\odot}$ $yr^{-1}$and log (-1.33) $M_{\odot}$ yr^{-1}for150,200, 500$M_{\odot}$rotating and non-rotating models respectively. In Fig. 2 the evolution of the mass loss for the models under consideration is shown, we see that the increase in the initial mass is proportional to the mass loss and the rotation reduced the mass loss.Initial abundances and properties used are given in table 1, the abundances are similar to those adopted byAsplund, et al., 2009,and the isotopic ratios are byLodders, 2003.

Table 1. Initial chemical abundances of the models, isotopes, and mass fraction

Elements	Isotope	Mass fraction
H	H	7.200e-01
He	^{3}He	2.659e-01
	^{4}He	4.415e-05
C	^{12}C	2.283e-03
	^{13}C	2.771e-05
N	^{14}N	6.588e-04
	^{15}N	2.595e-06
O	^{16}O	5.718e-03
	^{17}O	2.266e-06
	^{18}O	1.290e-05
Ne	^{20}Ne	1.877e-03
	^{22}Ne	1.518e-04

Summary of the model'sfundamental properties are given in table 2, and we compare the adopted models byYusof et al., 2013with those reported by Kozyreva et al., 2014,Heger & Woosley, 2002and byLanger et al., 2007. The nuclear reaction rates are taken from the Nuclear Astrophysics Compilation of Reaction Rates (NACRE) database (Angulo et al., 1999), and the effect of these rates in stellar evolution are well explained, see Ekström et al., 2012. The models evolved with different degrees of rotation (having fraction of critical rotation to be $\Omega/\Omega_{crit} = 0.002\ and\ 0.006$) and a non-rotating degree (0%) for the ZAMS.

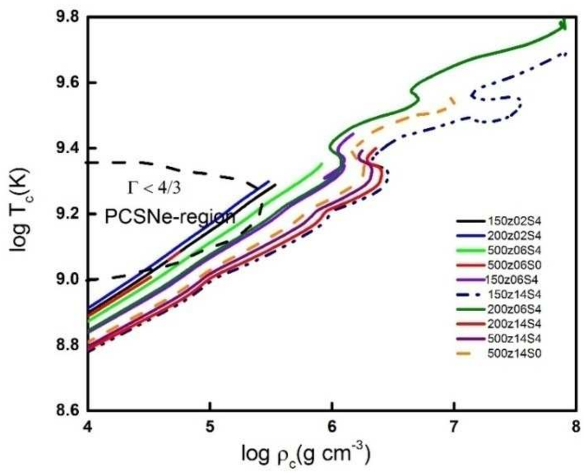

Figure 4: The central temperature $T_c$against the central density evolution, for the rotating 150 M$_\odot$(black), 200 M$_\odot$(blue) and 500 M$_\odot$ (green) and the non-rotating 500 M$_\odot$ (red) showing the electron-positron pair creation regime. Taken from Yusof et al., 2013.

In all the induced rotating models, the mass of the oxygen core is shrinking by the degrees of rotation, and we can see from Fig. 5 how the rotation significantly affects the evolution of the stars. The rotating models are more luminous than the non-rotating, and therefore, the nucleosynthesis might have altered rotation occurring in the stars. This might be due to the reduction in the effective gravity by the centrifugal force during the rotation and also the hydrogen-burning core when the main sequence becomes enlarged due to rotationally-induced chemical mixing. The increase in rotation brings about chemically homogeneous evolution and produce higher oxygen core mass which is necessary for the pair-creation in the core of the stars, thus higher degrees of rotation brings the star much closer to the density-temperature regime where the adiabatic index is below 4/3, this trend was also noted by Chatzopoulos & Wheeler, 2012a.

In table 2 we summarize the fundamental properties of the models under consideration and give other works for comparison. The first two columns are the initial masses ZAMS and final masses (in $M_{\odot}$), the third, fourth, fifth and sixth columns are the initial metallicity, the critical rotation, the log of maximum central density (g cm^{-3}) and the maximum central temperature (K) encountered due to the pair-creation instability respectively. The remaining columns seventh, eighth and ninth represents the helium core mass ($M_{\odot}$), the mass of the oxygen core ($M_{\odot}$) and finally, the fate of the models observed by various hydrodynamic codes.

MODELING OF ELECTRON-POSITRON EQUATION OF STATE TABLE

Numerical hydrodynamic and hydrostatic modeling of many astrophysical phenomena requires the use of appropriate tabular equations of state. This is essential due to the time consuming, instability, and lack of suitability for many computer codes that are commonly used in the direct numerical evaluation of the EOS involved (Swesty, 1996). One particular difficulty is the complexity that often arise when solving the many-body problems which describe the interactions between the constituents of the gas or liquid and secondly, the behavior of a particular EoS, one is interested, with respect to the range of temperature and density, which evidently shows discontinuities in thermodynamic variables at the phase transitions and coexistence boundaries (Swesty, 1996).

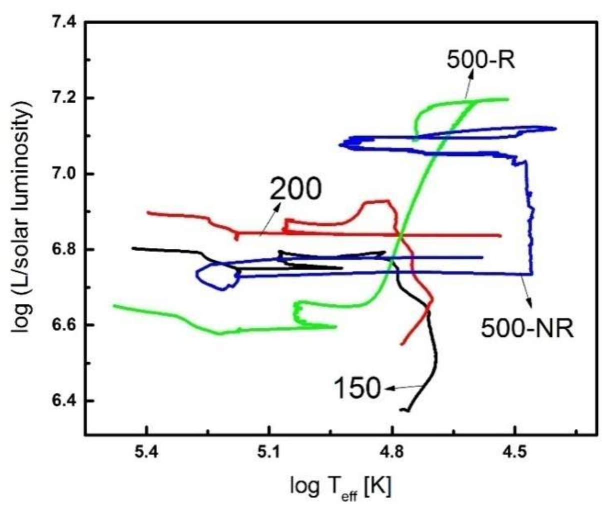

Figure 5. H-R diagram for 150 $M_{\odot}$(black), 200 $M_{\odot}$(red) at SMC and 500$M_{\odot}$for rotating(green) and non-rotating (blue) at LMC models respectively.

The equationof state (EoS) is usedto describe pressure, energy, and entropy as functions of temperature, composition, and density applicable incalculating thermodynamic quantities. In particular, thestellar equation of state determines many aspects of stellar physics, like the electron degeneracy and electron-positron pair production (Arnett, 1996), which is the subject of this work. Many researchersrevealed that proper equation of state for different nuclear densities is crucial in the studies of the explosion mechanisms of core-collapse supernovae (Bethe, 1990; Blinnikov et al., 2011; Janka, et. al., 2007; Suzuki & Totsuka, 1999). In Practical terms, to construct an equation of state table,we need a three-dimensional table of the related thermodynamic quantities as functions of the inputs variables(Lattimer & Swesty, 1991), which are temperature, density, and composition in our case. In massive stars, however, many EoS relating the energy and pressure to temperature, density, and composition have been developed and used, but the difficulties in the computer codes that used them,as we rightlyexplained above, has been bedevilingprevious calculations. Similarly,many electron-positron subroutines have been developed for massive stellar EoS at high temperatures (Timmes & Swesty, 2000; Stanford et al., 2002), which may be used to overcome the challenges faced byEoS in the stellar evolutioncomputer codes.

The electron-positron equation of state subroutines used in this work is HELM-EoS which isbased on the tabular interpolation of Helmholtz free energy. ThisEoS calculates among others, thermodynamic quantities for electron-positron pair formation (that drives the adiabatic index to $\Gamma < 4/3$) as described in details by (Timmes & Arnett, 1999; Timmes & Swesty, 2000),and it is developed such that for an isotope i with Z_i and A_i as its protons and nucleon number respectively, the total isotope i has a mass and number densities to be ρ_c (g cm^{-3}) and n_i (cm^{-3}) and a temperature T (K). For this, the dimensionless mass fraction for individual isotope i is $X_i = \frac{A_i n_i}{\rho N_A}$ and the dimensionless number density is $Y_i = \frac{X_i}{A_i} = \frac{n_i}{\rho N_A}$ where N_A is the Avogadro's number (Timmes & Swesty, 2000; Stanford et al., 2002).

We modeled these subroutines to include14 input variables as some isotopes for 150, 200 and 500 $M_\odot$ stellar models calculated by Yusof et al., 2013. We set the individual mass fractions of the compositions, and their proton and nucleon numbers in the model. Similarly, the average atomic weight per isotope is first calculated in the model after inputting the proton and nucleon numbersper isotope. The respective temperature and density values are then read by the model, such that for each isotope there is a particular temperature, density, average number of thenucleon and theaverage number of protons respectively. The subroutine then proceeds to read the Helmholtz free energy data table only once and then call the EoS.In the EoS the electrons and electron-positron pairs at high temperature are described as perfect thermal gas with arbitrary relativity and degeneracy. And the number density of these electrons -positrons is given, and the single number density for thefree electron as prescribed by Arnett, 1996; Fowler &Hoyle, 1964; Timmes & Arnett, 1999 is given. For positron, the chemical potential must have the rest mass terms which was subtracted in Eq. (2) and is provided by Eq.(3).The chemical potential μ (which is the only unknown in Eqs. (2) & (3)) can be found by applying boundary condition for complete ionization of the matter present (Svensson, 1982) such that;$n_0 = n_{e^-} - n_{e^+} = N_a \frac{\rho Z}{A} = Z n_{ion}$; N_a is Avogadro's number and r, Z and A are the mass density, atomic number and atomic weight of the matter excluding electron-positron pairs. However, many methods can be used for the one-dimensional root finding, but since absolute accuracy and thermodynamic consistency are primarily the major concern, TimmesEoSevaluated the Fermi-Dirac integrals and their derivatives with respect to the chemical potential and relativity parameter (Timmes & Swesty, 2000), whereas, the chemical potential was computed using Newton-Raphson scheme to at least 15 significant figures (Timmes & Arnett, 1999).After finding the value for the chemical potential by the use of Newton-Raphson iteration method, and given temperature [K], density [gcm^{-3}] and a particular isotope characterized by its average nucleon and average proton numbers, the EoSroutine produces many electron-positron thermodynamic quantities. Of prime interest in this work are the pressure [erg cm^{-3}], specific thermal energy [erg g^{-1}] and entropy [erg g^{-1} K^{-1}] along with (what to be the subject of our next paper Garba et al……in preparation) the derivatives of the pressure, energy and entropy, adiabatic indices, specific heats and many more. In the next section, we report the numerical result of these calculations and analyzed them.

Table 2. Characteristic properties of thestellar models used in this work and others: The initial mass, final mass, initial metallicity,therotation rate, the maximum central density and temperature ofHe-core and O-core mass

M_i [$M_\odot$]	M_f [$M_\odot$]	Z_{ini}	$\Omega_{crit\,rot}$	log ρ_c^{max} [g cm^{-3}]	log T_c^{max} [K]	He-core [$M_\odot$]	CO-core [$M_\odot$]	Fate
150	107	0.002	0.4	5.53	9.29	106.5	93	PCSN
200	129	0.002	0.4	5.48	9.30	129.2	124	PCSN
500	75	0.006	0.4	5.92	9.35	74.8	73	PCSN
500	95	0.006	00	4.51	9.01	94.7	93	PCSN
Ref. (Kozyreva et al., 2014**)**								
150	94	0.001	00	6.25	9.54	72	64	PCSN
250	169	0.001	00	6.69	9.71	121	110	PCSN
Ref. (Chatzopoulos & Wheeler, 2012a**)**								
200	-	0.014	00	6.54	9.70	-	120	PCSN
Ref. **(**Langer et al., 2007**)**								
150	93	0.050	10	-	9.36	71	64	PCSN
250	169	0.050	10	-	9.15	121	109	PCSN
Ref. (Heger & Woosley, 2002**)**								
70 $M_\odot$ He	70	0.002	00	6.30	9.55	70	60	-
115 $M_\odot$ He	115	0.002	00	6.67	9.71	115	90	PCSN

RESULTSAND DISCUSSION

The thermonuclear reactions began at the core of the star with hydrogen and helium burningswhich requires only lower energy for nuclear fusion. We focus on the evolution stages where the electron-positron pairs are produced at high temperature especially in the late burning stages. As stated in § 1, the electron-positron pairs are created only in the plane where the adiabatic index is below 4/3 and before the formation of any element heavier than oxygen. On this, our calculation showed thatthere are 14 isotopes in the composition and only during 6 burning stages the stars meet the region for the pair-creation instability. Table 3 summarizes pair creationfinal output from theEoS for all the models under consideration.The first column is the initial mass ($M_\odot$), the second, third, fourth and fifth columns represents the initial metallicity, the critical rotational ratio, the maximum central density (in 10^5 g cm-3) and temperature (in 10^9 K) respectively, while the last three columns are the isotopes(for which the stars encountered the instability regime), themaximum e^+e^-energy (in 10^{17} erg g^{-1}), the maximume^+e^-pressure (in 10^{22} erg cm^{-3}) and the maximume^+e^-entropy (in 10^8erg g^{-1} K^{-1}) at the respective massive stars. The values are almost similar for all the isotopes. The rotating 200$M_\odot$ model indicatehighere^+e^-thermonuclear energyof about 3.29x10^{17} erg g^{-1}while the non-rotating 500$M_\odot$ models, although explode as PCSN, but almost lost its e^+e^- energy andis about to collapse before reaching the pair-creation region, this trend is also depicted in Figure4 where the non-rotating 500$M_\odot$ model was almost unable to enter the instability region which is most likely due to, in addition to non-rotation factor, the small amount of the thermonucleare^+e^- energy that will help the burning process to reach the region.

In Figure 6 we can see the rise in the e^+e^- energy for centraltemperature and density(we choose onlyone isotope since it is almost equal for all the isotopes) for all the massive stars. The rotating models at SMC showed higher energies due to their ability to reach the instability region as a result of low mass loss and higher oxygen core that is contrary to their LMC counterparts which experienced more mass loss than them. This affects the fate of the stars, such that the SMC stars will fully explode as PCSNe without any remnant.

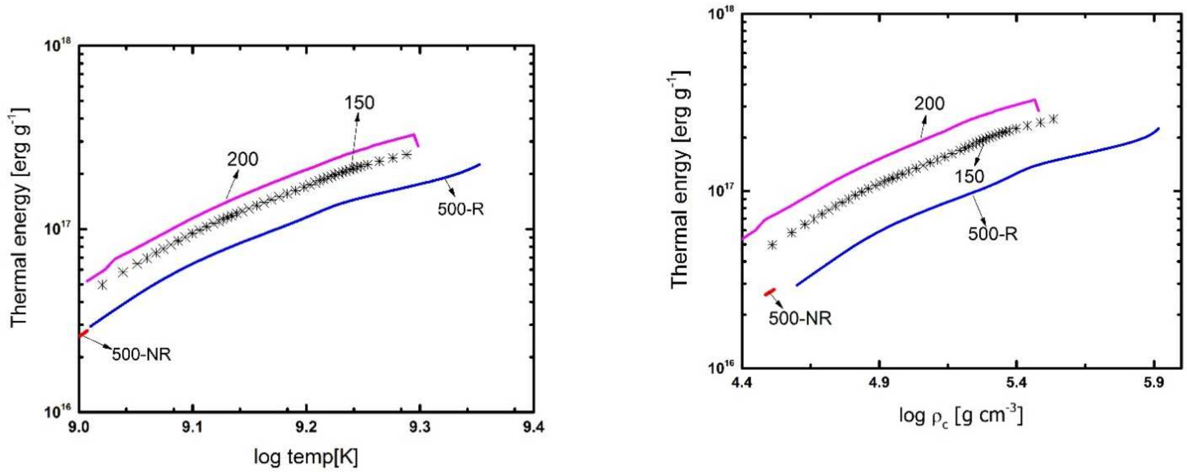

Figure 6: Electron-positron thermonuclear energy for 150M$_\odot$ (black), 200M$_\odot$(magenta) and 500M$_\odot$ rotating (blue) and non-rotating (red) models with respect to temperature (left) and density (right).

As strong mass loss distance the stars further away from the pair-creation region, so also, the thermonuclear energy of these pairs is reduced due to this mass loss and low oxygen core as can be seen in Fig. 7. However, the metallicity also affects the thermal energy in the way that the SMC having lower metallicity than the LMC suppresses the mass loss with agreateramount of energy and since the 200 M$_\odot$ has greater oxygen core mass it must, therefore,hasgreater thermal energy in the region to correspond its low metallicity and mass loss. On the other hand, the non-rotating 500 M$_\odot$ might have only reached the instability region due to its lower metallicity (comparing with solar metallicity) which is capable of suppressing the mass loss such that the thermonuclear energy would maintaincertain amount of elements into the pair production region. The graph in Figure 6 indicate that the instability is set in at a temperature of around 1.01x10^9 K, 1.02x10^9, 1.02x10^9 K and 1.00x10^9 K corresponding to density 4.56x10^4 g cm^{-3}, 2.44x10^4 g cm^{-3}, 3.99x10^4 g cm^{-3} and 3.06x10^4 g cm^{-3} respectively for 150, 200 and 500 M$_\odot$ rotating and non-rotating models. Inside the pair-creation region, however, the nuclear reaction rates increased by a certain amount such that energy is released due to the production of higher oxygen core mass that is necessary for the pair-creation.

In figure 8, we examined the effect of the helium core mass on the e^+e^- energy. Remembering that, the helium mass directly affects the explosion mechanism such that higher helium core mass reduced the pair-creation threshold. In this figure, the thermal energy slides down when the star approaches its final explosive mass (helium core mass), and it is interesting to note that, all the three rotating models showed similar dynamic, differing only with the non-rotating counterpart.

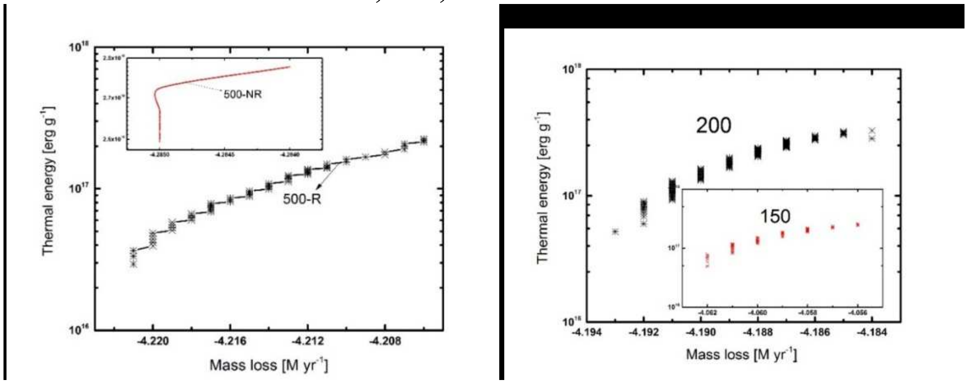

Figure 7: Electron-positron thermal energy [in erg g^{-1}] for mass loss (in $M_\odot$ yr^{-1}]. The left outer graph is for the rotating 500 $M_\odot$ models which showalow mass loss due totherotation and its thermonuclear energy of the e^+e^- pairs is greater than the non-rotating 500 $M_\odot$ (inner graph) that showsahigher mass losswhile the right outer and inner graph are for rotating 200 and 150 models respectively.

This situation is also noted whenfraction of critical rotation is put into consideration, figure 9. The higher helium mass (200 $M_\odot$), which is a resultof this rotation maintained higher continues thermal energyfor thethermonuclearprocess in the region.In the low helium core mass (500 $M_\odot$) the thermal energy is steady and suddenly decay with respect to fractional rotation.Thee$^+$e$^-$ thermonuclear energy, produced vibrational instability in temperature density region and is large enough to expand the star further. Thisvibrationdies very quickly, and the star evolves backto enter the instability region. The process continues until all the oxygen is completely exhausted from the center of the star and completely explode without any remnant.

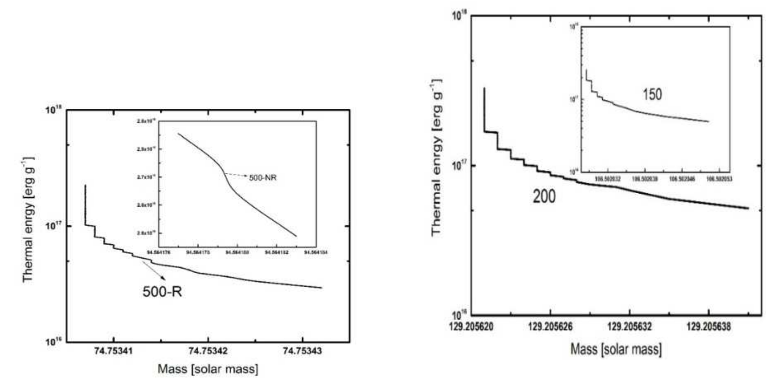

Figure 8: Thermal energies due to e^+e^- pair-creation w.r.t. Helium core mass of the star. 150$M_\odot$ (inner left) and 200$M_\odot$ (outer left), and rotating 500 $M_\odot$ (outer right) and non-rotating 500 $M_\odot$ (inner right). The rotating models almost show similar dynamic w.r.t. the mass.

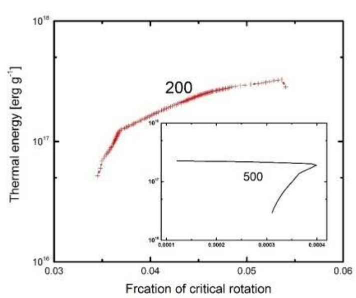

Figure 9: electron-positron thermal energy in the instability region plotted against the fraction of critical rotation for higher helium core mass star (200 $M_\odot$) and lowest helium core mass (500 $M_\odot$)

Table 3: Initial mass, isotopes for which pair-instabilityiscreated.E^{max}, P^{max}&$S^{max}$are maximum electron-positron energy in [ergs g^{-1}],thepressure in [ergs cm^{-3}], and entropy in [ergs $g^{-1}K^{-1}$] respectively.

Massive stars [M⊙]	**Z$_{in}$**	$\Omega_{crit\,rot}$	ρ_c^{max} **x10^5**	**T$_c^{max}$ x10^9**	**Isotope**	**E$^{max}_{e+e-}$ x10^{17}**	**P$^{max}_{e+e-}$ x10^{22}**	**S$^{max}_{e+e-}$ x10^8**
150	0.002	0.4	3.42	1.94	He3	2.57	1.76	1.59
					C13	2.54	1.76	1.59
					O17	2.57	1.76	1.59
					O18	2.55	1.76	1.59
					Be7	2.57	1.76	1.59
					B8	2.57	1.76	1.59
200	0.002	0.4	3.02	1.99	He3	3.29	2.02	1.99
					C13	3.26	2.02	1.99
					O17	3.29	2.02	1.99
					O18	2.83	1.20	1.87
					Be7	3.29	2.02	1.99
					B8	3.29	2.02	1.99
500	0.006	0.4	8.25	2.25	He3	2.31	4.14	1.25
					C13	2.21	4.15	1.25
					O17	2.31	4.14	1.25
					O18	2.25	4.14	1.25
					Be7	2.32	4.14	1.25
					B8	2.31	4.14	1.25
500	0.006	0.0	0.33	1.02	He3	0.40	0.02	0.45
					C13	0.38	0.02	0.45
					O17	0.40	0.02	0.45
					O18	0.28	0.03	0.51
					Be7	0.40	0.02	0.45
					B8	0.40	0.02	0.45

CONCLUSION

Hydrodynamic simulation using KEPLER code, predicts that rotating150 and 200 M⊙massive stars at SMC and LMC 500 M⊙rotating and non-rotating models could end as PCSN. Theelectron-positron pairs that are created in these massive stars have thermal energies that greatly affect their evolution and final fate. We used thethermodynamically consistent equation of state tables to calculate this energy and analyzedits effect onthe evolution and explosion of the massive stars. The mass of the oxygen core playedagreat role in the dynamic of the region, stars with higher oxygen core mass-produced greater thermal energy necessary to keep thermonuclear reaction in the region. Similarly, rotation which decreased the oxygen core mass also showed increasing dynamic with the e^+e^- thermal energies in the region. Although the non-rotating model entered the $\rho_c - T_c$pair-instability region and explode as PCSN, itlacksthe much electron-positron energy threshold to deliver its mass loss fully into the region, just as the high rotation brings the stars much closer the regime.Many thermonuclear processes involved in PCSN can be understood by using appropriate EoS tables. An adiabaticprocess involved in the evolution and explosion of massive stars is one example to be calculated and analyzed for abetter understanding of their final fate.

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