http://dx.doi.org/10.4314/bajopas.v12i1.22

Bayero Journal of Pure and Applied Sciences, 12(1): 155 - 160
Received: June, 2019
Accepted: June, 2019
ISSN 2006 – 6996

α-CUTS AND INVERSE α-CUTS IN FUZZY SOFT SETS

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Abstract
In this paper, the concepts of α-cuts, strong α-cuts, inverse α-cuts and weak inverse α-cuts of fuzzy soft sets were introduced together with some of their properties. Some distinctive features between α-cuts and inverse α-cuts were established. Some related theorems were formulated and proved. It is further demonstrated that both α-cuts and inverse α-cuts of fuzzy soft sets were useful tools in decision making.
Key Words: Fuzzy set, Fuzzy soft set, α-cut, strong α-cut

1 INTRODUCTION

A fuzzy set which is a collection of objects with various degrees of membership was first introduced by (Zadeh, 1965) and later studied by various scholars such as (Goguen, 1967; Wygralak, 1989; Gattwald, 2006) to mention a few. It is often useful to consider those elements that have at least some minimal degree of membership, say $\alpha \in [0,1]$. This is something like asking who has a passing grade in a class or a minimal height to ride on a roller coaster. This process can be better comprehended by using the notion of alpha-cuts introduced in (Zadeh, 1965). Sun and Han (2006) introduced the notion of inverse α-cut to improve the usability of the concept of α-cut to real life problems. For more details on α-cuts and its application refer to (Sun and Han, 2006; Kreinovich, 2013; Singh *et al.*, 2014; Singh *et al.*, 2015).

Soft set was introduced by (Molodtsov, 1999), thereafter many researchers study the theory in various context as in (Maji and Roy, 2002; Majumdar and Samanta, 2008; Isah and Tella, 2018; Isah, 2018). In this paper, the idea of α-cuts and inverse α-cuts in fuzzy soft sets were introduced and their applications were illustrated.

2 PRELIMINARIES

2.1 Fuzzy Sets

In this section, preliminaries for fuzzy set, soft set and fuzzy soft set which were needed for the concept of $\alpha -$ cuts of fuzzy soft sets were presented.

Definition 2.1.1 (Zadeh, 1965) Let X be a non-empty universe set, then a fuzzy set A over X is defined by a membership function

$\mu_A: X \to [0,1]$, that is, $A = \{(x, \mu_A(x)) \mid x \in X\}$.

Let A and B be two fuzzy sets, then

(i) $A \subseteq B$ if and only if

$$\mu_A(x) \leq \mu_B(x), \quad \forall x \in X .$$

(ii) $A \cup B = C$, with membership function

$$\mu_C(x) = \max[\mu_A(x), \mu_B(x)], \qquad \forall x \in X$$

(iii) $A \cup B = D$, with membership function

$$\mu_D(x) = \min[\mu_A(x), \mu_B(x)], \qquad \forall x \in X$$

Definition 2.1.2 (Zadeh, 1965)

Let X be a non-empty set and $F(X)$ the set of all fuzzy sets of X. Let $A \in F(X)$ and $\alpha \in [0,1]$. Then the non-fuzzy set (or crisp set)

$${}^{\alpha}A = \{x \in X \mid \mu_A((x) \geq \alpha)\}$$

is called the *α-cut* or *α-level* set of A.

If the weak inequality ≥ is replaced by the strict inequality >, then it is called the *strong α-cut,* denoted by ${}^{\alpha+}A$. That is,

$${}^{\alpha+}A = \{x \in X \mid \mu_A((x) > \alpha)\}.$$

Definition 2.1.3 (Sun and Han, 2006)

Let $A \in F(X)$ and $\alpha \in [0,1]$. Then the non-fuzzy set

$${}^{\alpha}A^{-1} = \{x \in X \mid \mu_A(x) < \alpha\}$$

is called an *inverse* α *-cut* or *inverse* α *-level set* of A.

If the strict inequality is replaced by the weak inequality ≤, then it is called a *weak inverse* α*-cut* of A, denoted by ${}^{\alpha^-}A^{-1}$. That is,

$${}^{\alpha^-}A^{-1} = \{x \in X \mid \mu_A(x) \le \alpha\}.$$

2.2 Soft Sets

Definition 2.2.1 (Molodtsov, 1999; Sezgin and Atagun, 2011)

Let *U* be a universe set and *E* a set of parameters or attributes with respect to *U*. Let $P(U)$ be the power set of *U* and $A \subseteq E$. Then, a pair (*F*, *A*) is called a soft set over *U*, where *F* is a mapping given by $F : A \to P(U)$. That is,

$$(F, A) = \{F(e) \in P(U) \mid e \in E,\ F(e) = \varnothing \text{ if } e \notin A\}.$$

Definition 2.2.2 (Maji *et al.*, 2003)

Let (*F*, *A*) and (*G*, *B*) be two soft sets over a common universe *U*, we say that

(i) (*F*,*A*) is a soft subset of (*G*,*B*), denoted $(F, A) \tilde{\subseteq} (G, B)$, if

$A \subseteq B$, and

$\forall\ e \in A, F(e) \tilde{\subseteq} G(e)$.

(ii) (*F*, *A*) is soft equal to (*G*, *B*), denoted (*F*, *A*) = (*G*, *B*), if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$.

Definitions 2.2.3 (Sezgin and Atagun, 2011; Maji *et al.*, 2003)

Let (*F*,*A*) and (*G*,*B*) be two soft sets over a common universe *U*.

(i) The union of (*F*,*A*) and (*G*,*B*), denoted by $(F, A) \tilde{\cup} (G, B)$, is a soft set (*H*,*C*) where $C = A \cup B$ and $\forall\ e \in C$,

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

(ii) The extended intersection of (*F*,*A*) and (*G*,*B*), denoted $(F, A) \tilde{\cap} (G, B)$, is a soft set (*H*,*C*) where $C = A \cup B$ and $\forall\ e \in C$,

$$H(e) = \begin{cases} F(e), \text{ if } e \in A - B \\ G(e), \text{ if } e \in B - A \\ F(e) \cap G(e), \text{ if } e \in A \cap B. \end{cases}$$

(iii) The restricted intersection of (*F*,*A*) and (*G*,*B*), denoted (*F*,*A*) $\cap_R$ (*G*,*B*), is a soft set (*H*,*C*) where $C = A \cap B$ and $\forall\ e \in C,\ H(e) = F(e) \cap G(e)$. If $A \cap B = \phi$ then (*F*,*A*) $\cap_R$ (*G*,*B*)= $\tilde{\Phi}_\phi$.

(iv) The restricted union of (*F*,*A*) and (*G*,*B*), denoted (*F*,*A*) $\cup_R$ (*G*,*B*), is a soft set (*H*,*C*) where $C = A \cap B$ and $\forall\ e \in C,\ H(e) = F(e) \cup G(e)$. If $A \cap B = \phi$ then (*F*,*A*) $\cup_R$ (*G*,*B*) = $\tilde{\Phi}_\phi$.

2.3 Fuzzy Soft Set

Definition 2.3.1 (Maji *et al.*, 2001)
Let U be an initial universal set and E be a set of parameters. Suppose I^U denote the power set of all fuzzy subsets of U, and $A \subseteq E$. Then, a pair $(\mathcal{F}, E\}$ is called a fuzzy soft set over U, where $\mathcal{F}$ is a mapping given by $\mathcal{F}: A \to I^U$.

3 α-CUTS AND ITS PROPERTIES IN FUZZY SOFT SETS

In this section, the concepts of α-cuts and strong α-cuts of fuzzy soft sets were introduced together with some of their properties.

Definition 3.1 Let the pair $(\mathcal{F}, A\}$ be a fuzzy soft set over U, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : A \to I^U$.
Then the *α-cut* or *α-level* soft set of $(\mathcal{F}, A)$ denoted by ${}^{\alpha}(\mathcal{F}, A)$ is defined as

$$^{\alpha}(\mathcal{F}, A) = \{(e, \{\left(x, \mu_{(\mathcal{F},A)}(x)\right)\} | \mu_{(\mathcal{F},A)}(x) \geq \alpha, \forall e \in A\}.$$

The *strong α-cut,* denoted by ${}^{\alpha+}(\mathcal{F}, A)$ is defined as

$$^{\alpha+}(\mathcal{F}, A) = \{(e, \{\left(x, \mu_{(\mathcal{F},A)}(x)\right)\} | \mu_{(\mathcal{F},A)}(x) > \alpha, \forall e \in A\}.$$

Example 3.2
Let $U = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$, $A = \{e_1, e_2, e_3\}$ and the fuzzy soft set $(\mathcal{F}, A)$ over U be $\mathcal{F}(e_1) = \{(x_1, 0.9), (x_2, 0.3), (x_3, 0.7), (x_4, 0.5)\}$, $\mathcal{F}(e_2) = \{(x_1, 0.6), (x_2, 0.5), (x_3, 0.8), (x_4, 0.1)\}$, $\mathcal{F}(e_3) = \{(x_1, 0.7), (x_2, 0.8), (x_3, 1.0), (x_4, 0.2)\}$. That is,

$$(\mathcal{F}, A) = \{(e_1, \{(x_1, 0.9), (x_2, 0.3), (x_3, 0.7), (x_4, 0.5)\}), (e_2, \{(x_1, 0.6), (x_2, 0.5), (x_3, 0.8), (x_4, 0.1)\}), (e_3, \{(x_1, 0.7), (x_2, 0.8), (x_3, 1.0), (x_4, 0.2)\})\}.$$

For instance, if U is a set of raw materials needed by a manufacturer and A is a set of attributes. This soft set is represented in tabular form as follows:

U	Found within Nigeria (e_1)	Found in forest region (e_2)	Cheap (e_3)
x_1	0.9	0.6	0.7
x_2	0.3	0.5	0.8
x_3	0.7	0.8	1.0
x_4	0.5	0.1	0.2

Then if $\alpha = 0.7$, we have ${}^{0.7}(\mathcal{F}, A) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_3\}), (e_3, \{x_1, x_2, x_3\})\}$ and

$$^{0.7+}(\mathcal{F}, A) = \{(e_1, \{x_1\}), (e_2, \{x_3\}), (e_3, \{x_2, x_3\})\}.$$

If $\alpha = 0.8$, we have ${}^{0.8}(\mathcal{F}, A) = \{(e_1, \{x_1\}), (e_2, \{x_3\}), (e_3, \{x_2, x_3\})\}$ and

$$^{0.8+}(\mathcal{F}, A) = \{(e_1, \{x_1\}), (e_3, \{x_3\})\}.$$

Remark 3.3
α-cut can be use to make a decision. For instance, if a manufacturer stipulates that, the raw material that satisfies all the parameters under the defined α is considered the best then, at $\alpha = 0.8$, there is no best raw material while at $\alpha = 0.7$, x_2 is the best choice.

Proposition 3.4
Let $\alpha \in [0, 1]$ and $(\mathcal{F}, A), (\mathcal{M}, B)$ be fuzzy soft sets over U, the following properties hold:

i. ${}^{\alpha+}(\mathcal{F}, A) \subseteq {}^{\alpha}(\mathcal{F}, A)$
ii. $\alpha \leq \beta \Longrightarrow {}^{\beta}(\mathcal{F}, A) \subseteq {}^{\alpha}(\mathcal{F}, A)$
iii. ${}^{\alpha}[(\mathcal{F}, A) \cup (\mathcal{M}, B)] = {}^{\alpha}(\mathcal{F}, A) \cup {}^{\alpha}(\mathcal{M}, B)$
iv. ${}^{\alpha}[(\mathcal{F}, A) \cap (\mathcal{M}, B)] = {}^{\alpha}(\mathcal{F}, A) \cap {}^{\alpha}(\mathcal{M}, B)$

Proof
i. Let $(e, x) \in {}^{\alpha+}(\mathcal{F}, A)$
$\Longrightarrow \mu_{(\mathcal{F},A)}(x) > \alpha, \forall e \in A$
$\Longrightarrow \mu_{(\mathcal{F},A)}(x) \geq \alpha, \forall e \in A$
$\Longrightarrow (e, x) \in {}^{\alpha}(\mathcal{F}, A)$
Therefore, ${}^{\alpha+}(\mathcal{F}, A) \subseteq {}^{\alpha}(\mathcal{F}, A)$
ii. The proof follows from definition.
iii. Let $(e, x) \in {}^{\alpha}[(\mathcal{F}, A) \cup (\mathcal{M}, B)]$

$$\Longrightarrow \mu_{[(\mathcal{F},A)\cup(\mathcal{M},B)]}(x) \geq \alpha, \forall e \in A \cup B$$
$$\Longrightarrow \mu_{(\mathcal{F},A)}(x) \geq \alpha, \forall e \in A \text{ or } \mu_{(\mathcal{M},B)}(x) \geq \alpha, \forall e \in B$$
$$\Longrightarrow (e, x) \in {}^{\alpha}(\mathcal{F}, A) \text{ or } (e, x) \in {}^{\alpha}(\mathcal{M}, B)$$
$$\Longrightarrow (e, x) \in {}^{\alpha}(\mathcal{F}, A) \cup {}^{\alpha}(\mathcal{M}, B)$$

Therefore, ${}^{\alpha}[(\mathcal{F}, A) \cup (\mathcal{M}, B)] \subseteq {}^{\alpha}(\mathcal{F}, A) \cup {}^{\alpha}(\mathcal{M}, B)$

Conversely, suppose $(e,x) \in {}^{\alpha}(\mathcal{F},A)\ \cup\ {}^{\alpha}(M,B)$

$\Rightarrow (e,x) \in {}^{\alpha}(\mathcal{F},A)$ or $(e,x) \in {}^{\alpha}(\mathcal{M},B)$

$\Rightarrow \mu_{(\mathcal{F},A)}(x) \geq \alpha, \forall e \in A$ or $\mu_{(M,B)}(x) \geq \alpha, \forall e \in B$

$\Rightarrow \mu_{(P,C)}(x) \geq \alpha, \forall e \in C$, where $C = A \cup B$ and $(P,C) = (\mathcal{F},A) \cup (\mathcal{M},B)$

$\Rightarrow \mu_{[(\mathcal{F},A)\cup(\mathcal{M},B)]}(x) \geq \alpha, \forall e \in A \cup B$

$\Rightarrow (e,x) \in {}^{\alpha}[(\mathcal{F},A) \cup (\mathcal{M},B)]$

Therefore, ${}^{\alpha}(\mathcal{F},A)\ \cup\ {}^{\alpha}(M,B) \subseteq {}^{\alpha}[(\mathcal{F},A) \cup (M,B)]$

Hence, ${}^{\alpha}[(\mathcal{F},A) \cup (\mathcal{M},B)] = {}^{\alpha}(\mathcal{F},A)\ \cup\ {}^{\alpha}(\mathcal{M},B)$

iv. Let $(e,x) \in {}^{\alpha}(\mathcal{F},A)\ \cap\ {}^{\alpha}(\mathcal{M},B)$

$\Rightarrow (e,x) \in {}^{\alpha}(\mathcal{F},A)$ and $(e,x) \in {}^{\alpha}(\mathcal{M},B)$

$\Rightarrow \mu_{(\mathcal{F},A)}(x) \geq \alpha, \forall e \in A$ and $\mu_{(\mathcal{M},B)}(x) \geq \alpha, \forall e \in B$

$\Rightarrow \mu_{[(\mathcal{F},A)\cap(\mathcal{M},B)]}(x) \geq \alpha, \forall e \in A \cap B$

$\Rightarrow (e,x) \in {}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)]$

Therefore, ${}^{\alpha}(\mathcal{F},A)\ \cup\ {}^{\alpha}(M,B) \subseteq {}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)]$

Conversely, let $(e,x) \in {}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)]$

$\Rightarrow \mu_{[(\mathcal{F},A)\cap(\mathcal{M},B)]}(x) \geq \alpha, \forall e \in A \cap B$

$\Rightarrow \mu_{(\mathcal{F},A)}(x) \geq \alpha, \forall e \in A$ and $\mu_{(\mathcal{M},B)}(x) \geq \alpha, \forall e \in B$

$\Rightarrow (e,x) \in {}^{\alpha}(\mathcal{F},A)$ and $(e,x) \in {}^{\alpha}(\mathcal{M},B)$

$\Rightarrow (e,x) \in {}^{\alpha}(\mathcal{F},A)\ \cap\ {}^{\alpha}(\mathcal{M},B)$

Therefore, ${}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)] \subseteq {}^{\alpha}(\mathcal{F},A)\ \cap\ {}^{\alpha}(\mathcal{M},B)$

Thus, ${}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)] = {}^{\alpha}(\mathcal{F},A)\ \cap\ {}^{\alpha}(\mathcal{M},B)$

4. INVERSE A-CUTS AND ITS PROPERTIES IN FUZZY SOFT SETS

In this section, the concepts of inverse α-cuts and weak inverse α-cuts of fuzzy soft sets together with some of their properties were introduced.

Definition 4.1 Let the pair $(\mathcal{F},A)$ be a fuzzy soft set over U, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : A \rightarrow I^U$. Then the inverse *α-cut* or inverse *α-level* soft set of $(\mathcal{F},A)$, denoted by ${}^{\alpha}(\mathcal{F},A)^{-1}$ is defined as

$${}^{\alpha}(\mathcal{F},A)^{-1} = \{(e, \{\left(x, \mu_{(\mathcal{F},A)}(x)\right)\} | \mu_{(\mathcal{F},A)}(x) < \alpha, \forall e \in A\}.$$

The *weak inverse α-cut* of $(\mathcal{F},A)$, denoted by ${}^{\alpha-}(\mathcal{F},A)^{-1}$ is defined as

$${}^{\alpha-}(\mathcal{F},A)^{-1} = \{(e, \{\left(x, \mu_{(\mathcal{F},A)}(x)\right)\} | \mu_{(\mathcal{F},A)}(x) \leq \alpha, \forall e \in A\}.$$

Example 4.2

Let $(\mathcal{F},A) = \{(e_1, \{(x_1, 0.9), (x_2, 0.3), (x_3, 0.7), (x_4, 0.5)\}),$
$(e_2, \{(x_1, 0.6), (x_2, 0.5), (x_3, 0.8), (x_4, 0.1)\}), (e_3, \{(x_1, 0.7), (x_2, 0.8), (x_3, 1.0), (x_4, 0.2)\})\},$

we have ${}^{o.7}(\mathcal{F},A)^{-1} = \{(e_1, \{x_2, x_4\}), (e_2, \{x_1, x_2, x_4\}), (e_3, \{x_4\})\}$ and

${}^{o.7-}(\mathcal{F},A)^{-1} = \{(e_1, \{x_2, x_3, x_4\}), (e_2, \{x_1, x_2, x_4\}), (e_3, \{x_1, x_4\})\}.$

Remark 4.3

Inverse α-cut can be use to know the most unfavorable selection. For instance, if the manufacturer intends to know the most unsuitable raw material at $\alpha = 0.7$, then x_4 is the most unfavorable choice.

Remark 4.4

Proposition 3.4 (iii) and (iv) fails.

Counter Example

For (iii)

Let $U = \{x_1, x_2, x_3, x_4\},\ E = \{e_1, e_2, e_3, e_4, e_5\},\ A = \{e_1, e_3\}, B = \{e_3, e_5\}$ and
$(\mathcal{F},A) = \{(e_1, \{(x_1, 0.7), (x_2, 0.4), (x_3, 0.6)\}), (e_3, \{(x_1, 0.5), (x_3, 0.2), (x_4, 0.1)\})\},$
$(\mathcal{M},B) = \{(e_3, \{(x_1, 0.9), (x_2, 0.5)\}), (e_5, \{(x_2, 0.6), (x_4, 0.2)\})\}$ be fuzzy soft set over U.

Then ${}^{0.6}(\mathcal{F},A)^{-1} = \{(e_1, \{x_2\}), (e_3, \{x_1, x_3, x_4\})\}$, ${}^{0.6}(\mathcal{M},B)^{-1} = \{(e_3, \{x_2\}), (e_5, \{x_4\})\}$ and
${}^{0.6}(\mathcal{F},A)^{-1} \cup {}^{0.6}(\mathcal{M},B)^{-1} = \{(e_1, \{x_2\}), (e_3, \{x_1, x_2, x_3, x_4\}), (e_5, \{x_4\})\}.$

Also,

$$(\mathcal{F},A) \cup (\mathcal{M},B) = \left\{ \begin{array}{c} (e_1, \{(x_1, 0.7), (x_2, 0.4), (x_3, 0.6)\}), \\ (e_3, \{(x_1, 0.9), (x_2, 0.5), (x_3, 0.2), (x_4, 0.1)\}), (e_5, \{(x_2, 0.6), (x_4, 0.2)\}) \end{array} \right\},$$

and

${}^{0.6}[(\mathcal{F},A) \cup (\mathcal{M},B)]^{-1} = \{(e_1, \{x_2\}), (e_3, \{x_2, x_3, x_4\}), (e_5, \{x_4\})\}.$

Thus, ${}^{0.6}(\mathcal{F},A)^{-1} \cup {}^{0.6}(\mathcal{M},B)^{-1} \neq {}^{0.6}[(\mathcal{F},A) \cup (\mathcal{M},B)]^{-1}$.

Counter Example for (iv)

${}^{0.7}(\mathcal{F},A)^{-1} = \{(e_1,\{x_2,x_3\}),(e_3,\{x_1,x_3,x_4\})\}$, ${}^{0.7}(\mathcal{M},B)^{-1} = \{(e_3,\{x_2\}),(e_5,\{,x_2,x_4\})\}$
and

$$ {}^{0.7}(\mathcal{F},A)^{-1} \cap {}^{0.7}(\mathcal{M},B)^{-1} = \emptyset. $$

However, $(\mathcal{F},A) \cap (\mathcal{M},B) = \{(e_3,\{(x_1,0.5)\})\}$
and

$$ {}^{0.7}[(\mathcal{F},A) \cup (\mathcal{M},B)]^{-1} = \{(e_3,\{x_1\})\}. $$

Thus, ${}^{0.7}(\mathcal{F},A)^{-1} \cap {}^{0.7}(\mathcal{M},B)^{-1} \neq {}^{0.7}[(\mathcal{F},A) \cap (\mathcal{M},B)]^{-1}$.
Moreover, the following hold.

Proposition 4.5

Let $(\mathcal{F},A),(\mathcal{F},B)$ be fuzzy soft sets over U, and $\alpha,\beta \in [0,1]$. The following properties hold:

(i) ${}^{\alpha}(\mathcal{F},A)^{-1} \subseteq {}^{\alpha-}(\mathcal{F},A)^{-1}$

(ii) $\alpha \leq \beta$ implies ${}^{\alpha}(\mathcal{F},A)^{-1} \subseteq {}^{\beta}(\mathcal{F},A)^{-1}$and ${}^{\alpha-}(\mathcal{F},A)^{-1} \subseteq {}^{\beta-}(\mathcal{F},A)^{-1}$

(iii) ${}^{\alpha}[(\mathcal{F},A) \cup (\mathcal{M},B)]^{-1} \subseteq {}^{\alpha}(\mathcal{F},A)^{-1} \cup {}^{\alpha}(\mathcal{M},B)^{-1}$

(iv) ${}^{\alpha}(\mathcal{F},A)^{-1} \cap {}^{\alpha}(\mathcal{M},B)^{-1} \subseteq {}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)]^{-1}$

Proof

i. Let $(e,x) \in {}^{\alpha}(\mathcal{F},A)^{-1}$

$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) < \alpha, \forall e \in A $$
$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) \leq \alpha, \forall e \in A $$
$$ \Longrightarrow (e,x) \in {}^{\alpha-}(\mathcal{F},A)^{-1} $$

Therefore, ${}^{\alpha+}(\mathcal{F},A) \subseteq {}^{\alpha}(\mathcal{F},A)$

ii. Let $(e,x) \in {}^{\alpha}(\mathcal{F},A)^{-1}$

$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) < \alpha, \forall e \in A $$
$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) < \beta, \forall e \in A \text{ Since } \alpha \leq \beta $$
$$ \Longrightarrow (e,x) \in {}^{\beta}(\mathcal{F},A)^{-1} $$

Therefore, ${}^{\alpha}(\mathcal{F},A)^{-1} \subseteq {}^{\beta}(\mathcal{F},A)^{-1}$

Also, suppose $(e,x) \in {}^{\alpha-}(\mathcal{F},A)^{-1}$

$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) \leq \alpha, \forall e \in A $$
$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) \leq \beta, \forall e \in A \text{ Since } \alpha \leq \beta $$
$$ \Longrightarrow (e,x) \in {}^{\beta-}(\mathcal{F},A)^{-1} $$

Therefore, ${}^{\alpha-}(\mathcal{F},A)^{-1} \subseteq {}^{\beta-}(\mathcal{F},A)^{-1}$

iii. Let $(e,x) \in {}^{\alpha}[(\mathcal{F},A) \cup (\mathcal{M},B)]^{-1}$

$$ \Longrightarrow \mu_{[(\mathcal{F},A)\cup(\mathcal{M},B)]}(x) < \alpha, \forall e \in A \cup B $$
$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) < \alpha, \forall e \in A \text{ or } \mu_{(\mathcal{M},B)}(x) < \alpha, \forall e \in B $$
$$ \Longrightarrow (e,x) \in {}^{\alpha}(\mathcal{F},A)^{-1} \text{ or } (e,x) \in {}^{\alpha}(\mathcal{M},B)^{-1} $$
$$ \Longrightarrow (e,x) \in {}^{\alpha}(\mathcal{F},A)^{-1} \cup {}^{\alpha}(\mathcal{M},B)^{-1} $$

Therefore, ${}^{\alpha}[(\mathcal{F},A) \cup (\mathcal{M},B)]^{-1} \subseteq {}^{\alpha}(\mathcal{F},A)^{-1} \cup {}^{\alpha}(\mathcal{M},B)^{-1}$

iv. Let $(e,x) \in {}^{\alpha}(\mathcal{F},A)^{-1} \cap {}^{\alpha}(\mathcal{M},B)^{-1}$

$$ \Longrightarrow (e,x) \in {}^{\alpha}(\mathcal{F},A)^{-1} \text{ and } (e,x) \in {}^{\alpha}(\mathcal{M},B)^{-1} $$
$$ \Longrightarrow \mu_{(\mathcal{F},A)}(x) < \alpha, \forall e \in A \text{ and } \mu_{(\mathcal{M},B)}(x) < \alpha, \forall e \in B $$
$$ \Longrightarrow \mu_{[(\mathcal{F},A)\cap(\mathcal{M},B)]}(x) < \alpha, \forall e \in A \cap B $$
$$ \Longrightarrow (e,x) \in {}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)]^{-1} $$

Therefore, ${}^{\alpha}(\mathcal{F},A)^{-1} \cap {}^{\alpha}(\mathcal{M},B)^{-1} \subseteq {}^{\alpha}[(\mathcal{F},A) \cap (\mathcal{M},B)]^{-1}$

5 CONCLUSION

The concept of α-Cuts, inverse α-Cuts and their properties in fuzzy soft sets were introduced and their applications were highlighted. It is shown that α-Cut of fuzzy soft sets can be used to determine the best choice while inverse α-Cut of fuzzy soft sets can be used to determine unfavorable alternative. Some related results were presented.

REFERENCES

Ali, M.I., Feng, F., Lui, X.Y., Min, W.K. and Shabir, M. (2009). *On some new operations in soft set theory*, Computers and mathematics with applications, 57 (9) 1574-1553.

Gattwald, S. (2006). *Universes of Fuzzy Sets and Axiomatization (part 1), Studia Logica,* An International J. Symbolic Logic, 82 (2) 211 – 244.

Goguen, J. A. (1967). *L-fuzzy Sets*, Journal of Mathematical Analysis and Applications, (18) 145 – 174.

Isah, A. I. and Tella, Y. (2018). Relations and Functions in Soft Multiset Context, *Science World Journal, 13 (3) 10-14.*

Isah, A. I. (2018). Some properties of soft multiset, *Science World Journal, 13 (4) 44-46.*

Klir, G. J. and Yuan, B. (1995). *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice Hall, Upper Saddle River, NJ.

Kreinovich, V. (2013). Membership Functions or α-Cuts? Algorithmic (Constructivist) Analysis Justifies an Interval Approach, *Applied Mathematical Sciences, 7 (5) 217–228.*

Khan, N., Khan, F. H. and Thakur, G. S. (2013). Weighted Fuzzy Soft Matrix Theory and its Decision Making, *International Journal of Advances in Computer Science and Technology, 2 (10) 214-218.*

Maji, P. K., Biswas, R. and Roy, A. R. (2001). Fuzzy soft sets, *Journal of Fuzzy Mathematics, 9 (3) 589–602.*

Maji, P. K., Biswas, R. and Roy, A. R. (2003). Soft set theory, *Computers and mathematics with applications, 45 (4-5) 555-562.*

Maji, P. K and Roy, A. R. (2002). *An application of Soft sets in a Decision Making Problem*, Computers and Mathematics with Applications, 44, 1077-1083.

Majumdar, P. and Samanta, S.K. (2008). Similarity measure of Soft sets, *New math. and Natural Computation, 4 (1) 1-12.*

Molodtsov, D. (1999). Soft set theory-First results, *Computers and Mathematics with Applications, 37, 19-31.*

Sezgin, A. and Atagun, A.O. (2011). On operations of soft sets, *Computers and mathematics with applications, (60) 1840-1849.*

Singh, D., Alkali, A. J. and Isah, A. I. (2014). Some Applications of α-Cuts in Fuzzy Multiset Theory, *Journal of Emerging Trends in Computing and Information Sciences, 3 (4) 328-335.*

Singh, D., Alkali, A. J. and Isah, A. I. (2015). An extension of the properties of inverse α - cuts to fuzzy multisets, *Annals of Fuzzy Mathematics and Informatics, 9 (6) 917-923.*

Sun, Z. and Han, J. (2006). *Inverse alpha-Cuts and Interval [a, b)-Cuts*, Proceedings of the International Conference on Innovative Computing, Information and Control (ICICIC2006), 30 August - 1 September, Beijing, IEEE Press, 441-444.

Wygralak, M. (1989). Rough Sets and Fuzzy Sets – Some Remarks on Interrelations, *Fuzzy Sets and Systems, (29) 241 – 243.*

Zadeh, L. A. (1965). Fuzzy Sets, *Information Control, (8) 338 – 353.*