BAJOPAS Volume 12 Number 1, June, 2019

http://dx.doi.org/10.4314/bajopas.v12i1.16

Bayero Journal of Pure and Applied Sciences, 12(1): 118 - 126
Received: May, 2019
Accepted: June, 2019
ISSN 2006 – 6996

BAYESIAN ANALYSIS OF EXPONENTIAL LOMAX DISTRIBUTION USING DIFFERENT LOSS FUNCTIONS

Etim, E.[1*]; Doguwa, S. I. S.[2]; Yahaya, A.[3]
[1,2,3]Department of Statistics, Ahmadu Bello University Zaria, Nigeria

ABSTRACT
The exponential Lomax distribution is an extension of the Lomax distribution proposed by El-Bassiouny et al. (2015). This distribution is very useful and has been found to outperform other extensions of the Lomax distribution such as the exponentiated Lomax, Marshall-Olkin extended-Lomax, beta-Lomax, Kumaraswamy-Lomax, McDonald-Lomax and gamma-Lomax based on some applications to lifetime datasets. In this article, the scale parameter of the exponential Lomax is estimated using the Bayesian method of estimation under two non-informative (Jeffery and Uniform prior) and Informative prior (Gamma prior) distribution and compared to the estimates of maximum Likelihood using three loss functions (Square error, Quadratic, and Precautionary loss function). The posterior distributions of the said parameter were derived and also the Estimators and risks were also obtained using the priors and loss functions. Furthermore, a simulation study was carried out using R software package to assess the performance of the two methods by means of their MSEs.
Keywords: Exponential Lomax distribution; MLE;Bayesian Method; Uniform prior; Jeffrey's prior; Gamma prior; Square error, quadratic and precautionary loss functions; MSE; Sample sizes.

1. INTRODUCTION

The exponential Lomax distribution is an extension of the Lomax distribution proposed by El-Bassiouny*et al*. (2015). The Lomax or Pereto type II distribution was proposed by Lomax in (1954). This distribution has found wide applications such as the analysis of business failure life time data, income and wealth inequality, size of cities, actuariale.t.c.Several Mathematical and statistical properties of Exponential Lomax distribution have been derived and discussed by El-Bassiouny*et al*. (2015). This distribution has been found to out-perform other extensions of the Lomax distribution such as the exponentiated Lomax, Marshall-Olkin extended-Lomax, beta-Lomax, Kumaraswamy-Lomax, McDonald-Lomax and gamma-Lomax based on some applications to life time data sets.

In statistics, we have two basicmethods ofparameter estimationand these are the classical and the non-classical methods. In the classical theory of estimation, the parameters are taken to be fixed but unknown whereas we consider the parameters to be unknown and random just like variables. The most popular and unique method under classical theory is the method of maximum likelihood estimation while the Bayesian estimation method is considered under non classical theory. But, in common real life problemsdescribed by life timedistributions, the parameters cannot be treated as fixed in all the life testing periodaccording to Martz and Waller (1982) as well asIbrahim *et al.* (2001) and Singpurwalla (2006).Based on this fact, it becomes obvious the frequentist or classical approach can no longer handle adequately problems of parameter estimationin life time models and therefore the need for non-classical or Bayesian estimation in life time models.

Due to the stated problem above, a number of research works on Bayesian estimation method of parametersesimates have been conducted and a highlight of some of these studies which dependent on the distribution in question are as follows:Bayesian estimation for the extreme value distribution using progressive censored data and asymmetric loss by Al-Aboud (2009), Bayesian estimators of the shape and scale parameters of modified Weibull distribution using Lindley's approximation under the squared error loss function, LINEX loss function and generalized entropy loss function by Preda*et al.* (2010),comparison of Bayesian estimates of the shape parameter of Generalized Exponential Distribution based on a class of non-informative prior under the assumption of quadratic loss function, squared log-error loss function and general entropy loss function (*GELF*) and maximum likelihood estimates by Dey (2010), Bayesian Survival Estimator for Weibull distribution with censored data by Ahmed *et al.*

(2011) as well as Pandey*et al*. (2011), Al-Athari (2011).
Similarly, Aliyu and Yahaya (2016) studied the shape parameter of generalized Rayleigh distribution under non-informative priors with a comparison to the method of maximum likelihood. Besides, a good number of loss functions have been shown to be performing during estimation under Bayesian method in so many studies including Ahmad and Ahmad (2013), Ahmad *et al*. (2015), Ahmad *et al.* (2016), Ieren and Oguntunde (2018), Gupta and Singh (2017), Gupta (2017) and Ieren and Chukwu (2018) and many others.
Since the approach of estimating a parameter differs from one parameter of a distribution to another, this studyaims at estimating the scale parameter of the Exponential Lomax distribution using Bayesian approach and making a comparison between the Bayesian approach and the method of maximum likelihood estimation approach. The rest of this paper is presented in sections and sub-sectionsas follows:

2. MATERIALS AND METHODS

2.1 PDF and Likelihood function

The *pdf* of the Exponential Lomax distributionwith unknown parameter vectorisgiven as:

$$f(x)=\frac{\alpha\omega}{\beta}\left(\frac{\beta}{x+\beta}\right)^{-\alpha+1} e^{-\omega\left(\frac{\beta}{x+\beta}\right)^{-\sigma}} \qquad x\geq\beta,\alpha,\beta,\omega>0 \quad (1)$$

Where, $x\geq\beta,\alpha>0,\beta>0,\omega>0$ α *and* β are the shape parameters respectively and α is the scale parameter of the exponential distribution.
The total log-likelihood function is obtained from *f(x)* as follows:

$$L(\underline{X}\mid\alpha,\beta,\omega)=\left(\frac{\alpha\omega}{\beta}\right)^{n}\prod_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha+1} e^{-\omega\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (2)$$

The likelihood function for the scale parameter, ω, is given by;

$$L(\underline{X}\mid\omega)\propto\omega^{n}e^{-\omega\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (3)$$

2.2 Bayesian Analysis under the Assumption of Uniform Prior Using Three Loss Functions

To obtain the posterior distribution$P(\omega|x)$, the probability distribution of the parameter once the data has been observed, we apply bayes' Theorem

$$p(\omega|X)=\frac{p(\omega)L(x|\omega)}{g(x)} \quad (4)$$

Where g(x) is the marginal distribution of X and

$$g(x)=\begin{cases}\sum_{\infty}^{\cdot}p(\omega)L(x|\omega)\\ \int_{-\infty}^{\infty}p(\omega)L(x|\omega)d\omega\end{cases}$$

where $\sum_{\infty}^{\cdot}p(\omega)L(x|\omega)$ when ω is discrete and $\int_{-\infty}^{\infty}p(\omega)L(x|\omega)d\omega$ when ω is continuous

where $p(\omega)$ and $L(x|\omega)$ are the prior distribution and the Likelihood function respectively.

The uniform prior is defined as:

$p(\alpha)\propto 1;\ 0<\alpha<\infty$

The posterior distribution of the scale parameter ω under uniform prior is obtained from equation (4) using integration by substitution method as

$$p(\omega \mid \underline{X}) = \frac{\omega^{n} e^{-\omega \sum_{i=1}^{n} \left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}}}{\left(\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}\right)^{-(n+1)} \Gamma(n+1)} \quad (5)$$

The Bayes estimators and posterior risksunder uniform prior using *SELF*, *QLF* and *PLF* are given respectively as follows:

$$\omega_{SELF} = E(\omega \mid \underline{X}) = \frac{n+1}{\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (6)$$

$$P(\omega_{SELF}) = \frac{n+1}{\left(\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}\right)^{2}} \quad (7)$$

$$\omega_{QLF} = \frac{n-1}{\left(\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}\right)} \quad (8)$$

$$P(\omega_{QLF}) = \frac{1}{n} \quad (9)$$

$$\omega_{PLF} = \left\{E(\omega^2 \mid \underline{X})\right\}^{\frac{1}{2}} = \frac{\left[(n+2)(n+1)\right]^{\frac{1}{2}}}{\left(\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}\right)} \quad (10)$$

$$P(\omega_{PLF}) = 2\left\{\frac{\left\{(n+2)(n+1)\right\}^{\frac{1}{2}} - (n+1)}{\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}}\right\} \quad (11)$$

2.3 Bayesian Analysis under the Assumption of Jeffrey's Prior Using Three Loss Functions

Also, the Jeffrey's prior is defined as:

$$p(\alpha) \propto \frac{1}{\alpha}; 0 < \alpha < \infty \quad (12)$$

The posterior distribution of the scale parameter ω for a given data under Jeffrey prior is obtained from equation (4) using integration by substitution method as

$$p(\omega \mid \underline{X}) = \frac{\omega^{n-1}\left(\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}\right)^{n} e^{-\omega \sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}}}{\Gamma(n)} \quad (13)$$

The Bayes estimators and posterior risks under Jeffrey's prior using *SELF*, *QLF* and *PLF* are given respectively as follows:

$$\omega_{SELF} = E(\omega \mid \underline{X}) = \frac{n}{\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (14)$$

$$P(\omega_{SELF}) = \frac{n}{\left(\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}\right)^2} \quad (15)$$

$$\omega_{QLF} = \frac{n-2}{\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (16)$$

$$P(\omega_{QLF}) = \frac{1}{n-1} \quad (17)$$

$$\alpha_{PLF} = \frac{\left[n(n+1)\right]^{\frac{1}{2}}}{\omega} \quad (18)$$

$$P(\omega_{PLF}) = 2\left\{\frac{\left\{n(n+1)\right\}^{\frac{1}{2}} - (n)}{\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}}\right\} \quad (19)$$

2.4 Bayesian Analysis under the Assumption of Gamma Prior Using Three Loss Functions

Also, the gamma prior is defined as:

$$P(\omega) = \frac{a^b}{\Gamma(b)}\omega^{b-1}e^{-a\omega}; a,b,\omega > 0 \quad (20)$$

The posterior distribution of the scale parameter ω for a given data under gamma prior is obtained from equation (4) using integration by substitution method as

$$P(\omega \mid \underline{X}) = \frac{\left(a + \sum_{i=1}^{n}\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)^{n+b}\omega^{n+b-1}e^{-\omega\left(a+\sum_{i=1}^{n}\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)}}{\Gamma(n+b)} \quad (21)$$

The Bayes estimators and posterior risks under gamma prior using *SELF*, *QLF* and *PLF* are given respectively as follows:

$$\omega_{SELF} = E(\omega \mid \underline{X}) = \frac{(n+b)}{\left(a+\sum_{i=1}^{n}\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)} \quad (22)$$

$$P(\omega_{SELF}) = \frac{(n+b)}{\left(a+\sum_{i=1}^{n}\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)^2} \quad (23)$$

$$\omega_{QLF} = \frac{(n+b-2)}{\left(a+\sum_{i=1}^{n}\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)} \quad (24)$$

$$P(\omega_{QLF}) = \frac{1}{(n+b-1)} \quad (25)$$

$$\omega_{PLF} = \left\{E\left(\omega^2 \mid \underline{X}\right)\right\}^{\frac{1}{2}} = \frac{\left[(n+b+1)(n+b)\right]^{\frac{1}{2}}}{\left(a+\sum_{i=1}^{n}\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)} \quad (26)$$

$$P(\omega_{PLF}) = 2\left\{\frac{\left[(n+b+1)(n+b)\right]^{\frac{1}{2}} - (n+b)}{\left(a+\sum_{i=1}^{n}\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)}\right\} \quad (27)$$

2.4 Maximum Likelihood Estimation

Let $x_1, x_2, ..., x_n$ be a random sample from a population *X* with probability density function $f(x),$. The likelihood function, $L(\underline{X} \mid \alpha, \beta, \omega)$, is defined to be the joint density of the random variables $x_1, x_2, ..., x_n$. The *pdf* of the *ELD* is given as

$$f(x) = \frac{\alpha\omega}{\beta}\left(\frac{\beta}{x+\beta}\right)^{-\alpha+1} e^{-\omega\left(\frac{\beta}{x+\beta}\right)^{-\alpha}}$$

$$L(\underline{X} \mid \alpha, \beta, \omega) = \left(\frac{\alpha\omega}{\beta}\right)^{n} \prod_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha+1} e^{-\omega\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (28)$$

The likelihood function for the scale parameter, ω, is given by;

$$L(\underline{X} \mid \omega) \propto \omega^{n} e^{-\omega\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (29)$$

Let the log-likelihood function, $l = \log L(\underline{X} \mid \omega)$, therefore

$$l = n\log\omega - \omega\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha} \quad (30)$$

Differentiating l partially with respect to ω, the scale parameter and solving for $\hat{\omega}$ gives;

$$\frac{\partial l}{\omega} = \frac{n}{\omega} - \sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha} = 0 \Rightarrow \hat{\omega} = \frac{n}{\sum_{i=1}^{n}\left(\frac{\beta}{x_i+\beta}\right)^{-\alpha}} \quad (31)$$

Which is the MLE of the scale parameter, ω.

3. RESULTS AND DISCUSSIONS

3.1 Simulation and Comparison

In this section, a package in R software is considered to generate random samples of sizesn = (10, 15, 20, 27, 35, 55, 95, 125) from Exponential Lomax distributionunder the following combination of parameter values: $\alpha=1,\beta=1$ and $\omega=0.5$; $\alpha=1,\beta=3$ and $\omega=0.5$; $\alpha=3,\beta=1$ and $\omega=0.5$ $\alpha=0.5,\beta=0.5$ $\omega=1$. The following tables present the results of our simulation study by listing the estimates of the scale parameter with their respective Mean Square Errors (MSEs) under the appropriate estimation methods such as the Maximum Likelihood Estimation (MLE), Squared Error Loss Function (SELF), Quadratic Loss Function (QLF), and Precautionary Loss Function (PLF) under Uniform Jeffrey and gamma priors respectively.

Table 1: Estimators their Estimates and Mean Squared Errors based on the replications and sample sizes where $\alpha=1,\beta=1$ and $\omega=0.5$

n	Measures	MLE	Uniform Prior			Jeffrey's Prior			Gamma Prior		
			SELF	QLF	PLF	SELF	QLF	PLF	SELF	QLF	PLF
10	Estimate	0.556	0.6116	0.5004	0.6388	0.556	0.4448	0.5831	0.5548	0.4694	0.5757
	MSE	0.0411	0.0584	0.0308	0.0694	0.0411	0.0274	0.0487	0.0234	0.0155	0.0277
15	Estimate	0.5365	0.5723	0.5008	0.5899	0.5365	0.465	0.5541	0.5411	0.481	0.556
	MSE	0.0228	0.0296	0.0187	0.034	0.0228	0.0173	0.0258	0.0165	0.0121	0.0188
20	Estimate	0.5257	0.552	0.4994	0.565	0.5257	0.4731	0.5387	0.5314	0.4852	0.5428
	MSE	0.016	0.0196	0.0138	0.0219	0.016	0.0131	0.0176	0.0127	0.01	0.0141
27	Estimate	0.5194	0.5386	0.5002	0.5482	0.5194	0.4809	0.5289	0.5248	0.4898	0.5335
	MSE	0.0113	0.0133	0.0102	0.0145	0.0113	0.0098	0.0122	0.0097	0.0081	0.0105
35	Estimate	0.5149	0.5296	0.5002	0.537	0.5149	0.4855	0.5222	0.5198	0.4924	0.5266
	MSE	0.0082	0.0093	0.0075	0.01	0.0082	0.0073	0.0087	0.0073	0.0063	0.0078
55	Estimate	0.5098	0.5191	0.5005	0.5237	0.5098	0.4913	0.5144	0.5134	0.4957	0.5178
	MSE	0.005	0.0054	0.0047	0.0057	0.005	0.0046	0.0052	0.0047	0.0042	0.0049
95	Estimate	0.5047	0.51	0.4994	0.5126	0.5047	0.494	0.5073	0.507	0.4967	0.5096
	MSE	0.0027	0.0029	0.0027	0.003	0.0027	0.0026	0.0028	0.0026	0.0025	0.0027
12	Estimate	0.5042	0.5082	0.5002	0.5102	0.5042	0.4961	0.5062	0.506	0.4981	0.508
5	MSE	0.0021	0.0022	0.002	0.0022	0.0021	0.002	0.0021	0.002	0.0019	0.0021

The results in table 1 show that the estimator of the scale parameter using QLF under Gamma is better than the other estimators (uniform and Jeffrey prior and MLE) with small MSE irrespective of the variation in the samples. This behavior of minimum MSE for Bayesian estimation (using QLF under Uniform, Jeffrey and gamma priors) is an indication that the method for this parameter is better than the Method of Maximum Likelihood estimation (*MLE*) for the chosen parameter values irrespective of small, medium or large sample sizes.

Table 2: Estimators their Estimates and Mean Squared Errors based on the replications and sample sizeswhere $\alpha=1, \beta=3$ and $\omega=0.5$.

n	Measures	MLE	Uniform Prior			Jeffrey's Prior			Gamma Prior		
			SELF	QLF	PLF	SELF	QLF	PLF	SELF	QLF	PLF
10	Estimate	0.5508	0.6058	0.4957	0.6328	0.5508	0.4406	0.5776	0.5507	0.466	0.5715
	MSE	0.04	0.0565	0.0304	0.0671	0.04	0.0275	0.0472	0.0227	0.0155	0.0268
15	Estimate	0.5351	0.5707	0.4994	0.5883	0.5351	0.4637	0.5526	0.5399	0.4799	0.5546
	MSE	0.0227	0.0294	0.0187	0.0337	0.0227	0.0174	0.0257	0.0165	0.0122	0.0187
20	Estimate	0.524	0.5502	0.4978	0.5632	0.524	0.4716	0.537	0.5299	0.4839	0.5413
	MSE	0.0154	0.0189	0.0134	0.0211	0.0154	0.0128	0.017	0.0124	0.0098	0.0137
27	Estimate	0.52	0.5392	0.5007	0.5488	0.52	0.4815	0.5295	0.5253	0.4903	0.534
	MSE	0.0116	0.0136	0.0104	0.0149	0.0116	0.01	0.0125	0.01	0.0082	0.0108
35	Estimate	0.5137	0.5284	0.499	0.5357	0.5137	0.4844	0.521	0.5186	0.4913	0.5254
	MSE	0.0084	0.0095	0.0077	0.0102	0.0084	0.0075	0.0089	0.0075	0.0065	0.008
55	Estimate	0.5089	0.5181	0.4996	0.5227	0.5089	0.4904	0.5135	0.5125	0.4948	0.5169
	MSE	0.005	0.0054	0.0047	0.0057	0.005	0.0047	0.0052	0.0047	0.0042	0.0049
95	Estimate	0.5051	0.5104	0.4998	0.5131	0.5051	0.4945	0.5078	0.5074	0.4971	0.51
	MSE	0.0028	0.003	0.0027	0.0031	0.0028	0.0027	0.0029	0.0027	0.0026	0.0028
12	Estimate	0.504	0.5081	0.5	0.5101	0.504	0.496	0.506	0.5058	0.4979	0.5078
5	MSE	0.002	0.0021	0.002	0.0022	0.002	0.002	0.0021	0.002	0.0019	0.002

Table 2 also gives a similar pattern of the result found in table 1 with lower values of MSEs for the estimators using PLF under Uniform, Jeffrey and gamma priors. This result indicates that *QLF* under gamma prior produces the best estimator more than the *QLF* under Uniform and Jeffrey priors and these effects are found to be continuous despite the different sample sizes used.

Table 3: Estimators their Estimates and Mean Squared Error and based on the replications and sample sizes where $\alpha=3, \beta=1$ and $\omega=0.5$.

n	Measures	MLE	Uniform Prior			Jeffrey's Prior			Gamma Prior		
			SELF	QLF	PLF	SELF	QLF	PLF	SELF	QLF	PLF
10	Estimate	0.554	0.6094	0.4986	0.6365	0.554	0.4432	0.581	0.5621	0.4684	0.585
	MSE	0.0405	0.0575	0.0305	0.0683	0.0405	0.0273	0.0479	0.0298	0.019	0.0353
15	Estimate	0.5357	0.5714	0.5	0.589	0.5357	0.4643	0.5533	0.5448	0.4807	0.5606
	MSE	0.023	0.0298	0.0189	0.0342	0.023	0.0176	0.026	0.0196	0.0141	0.0223
20	Estimate	0.5275	0.5538	0.5011	0.5669	0.5275	0.4747	0.5405	0.5356	0.4869	0.5477
	MSE	0.0165	0.0203	0.0142	0.0227	0.0165	0.0134	0.0182	0.0149	0.0114	0.0165
27	Estimate	0.5199	0.5391	0.5006	0.5487	0.5199	0.4813	0.5294	0.5268	0.4904	0.5358
	MSE	0.0114	0.0133	0.0102	0.0146	0.0114	0.0098	0.0122	0.0106	0.0087	0.0115
35	Estimate	0.5157	0.5304	0.501	0.5377	0.5157	0.4862	0.523	0.5214	0.4933	0.5284
	MSE	0.0082	0.0094	0.0075	0.0101	0.0082	0.0073	0.0088	0.0079	0.0067	0.0085
55	Estimate	0.51	0.5192	0.5007	0.5239	0.51	0.4914	0.5146	0.514	0.4959	0.5184
	MSE	0.0051	0.0056	0.0048	0.0058	0.0051	0.0047	0.0053	0.005	0.0045	0.0052
95	Estimate	0.505	0.5103	0.4997	0.513	0.505	0.4944	0.5076	0.5074	0.497	0.51
	MSE	0.0028	0.0029	0.0027	0.003	0.0028	0.0027	0.0029	0.0028	0.0026	0.0028
12	Estimate	0.5039	0.5079	0.4998	0.5099	0.5039	0.4958	0.5059	0.5058	0.4978	0.5078
5	MSE	0.0021	0.0022	0.0021	0.0022	0.0021	0.002	0.0021	0.0021	0.002	0.0021

Again from table 3, it is confirmed that QLF under gamma priorgave the best estimators for the scale parameter irrespective of the changes in the allocation of sample sizes. This efficiency is again followed by the same *QLF* under Uniform and Jeffrey priors.

Table 4: Estimators their Estimates and Mean Squared Errors based on the replications and sample sizes where $\alpha = 0.5, \beta = 0.5$ and $\omega = 1$.

n	Measures	MLE	Uniform Prior			Jeffrey's Prior			Gamma Prior		
			SELF	QLF	PLF	SELF	QLF	PLF	SELF	QLF	PLF
10	Estimate	1.1095	1.2204	0.9985	1.2747	1.1095	0.8876	1.1636	1.1828	1.0436	1.2171
	MSE	0.1630	0.2313	0.1223	0.2748	0.1630	0.1093	0.1929	0.0946	0.0495	0.1119
15	Estimate	1.0712	1.1426	0.9998	1.1778	1.0712	0.9284	1.1063	1.1418	1.0380	1.1675
	MSE	0.0914	0.1186	0.0752	0.1360	0.0914	0.0700	0.1034	0.0690	0.0418	0.0792
20	Estimate	1.0497	1.1022	0.9972	1.1282	1.0497	0.9448	1.0757	1.1129	1.0304	1.1333
	MSE	0.0625	0.0766	0.0542	0.0858	0.0625	0.0517	0.0688	0.0529	0.0353	0.0594
27	Estimate	1.0419	1.0805	1.0034	1.0997	1.0419	0.9648	1.0611	1.0941	1.0297	1.1100
	MSE	0.0449	0.0529	0.0400	0.0580	0.0449	0.0382	0.0485	0.0413	0.0296	0.0455
35	Estimate	1.0298	1.0593	1.0004	1.0739	1.0298	0.9710	1.0445	1.0736	1.0225	1.0864
	MSE	0.0337	0.0383	0.0310	0.0412	0.0337	0.0300	0.0358	0.0319	0.0246	0.0346
55	Estimate	1.0180	1.0365	0.9994	1.0457	1.0180	0.9809	1.0272	1.0488	1.0149	1.0572
	MSE	0.0199	0.0217	0.0189	0.0228	0.0199	0.0186	0.0207	0.0197	0.0164	0.0208
95	Estimate	1.0103	1.0209	0.9997	1.0262	1.0103	0.9890	1.0156	1.0295	1.0093	1.0345
	MSE	0.0108	0.0114	0.0105	0.0118	0.0108	0.0104	0.0111	0.0109	0.0097	0.0113
125	Estimate	1.0056	1.0137	0.9976	1.0177	1.0056	0.9895	1.0096	1.0206	1.0051	1.0244
	MSE	0.0082	0.0085	0.0080	0.0086	0.0082	0.0080	0.0083	0.0082	0.0075	0.0084

The above table, table 4 also reveals finally that gamma prior with QLF is the most efficient for the scale parameter, and looking at all the results presented in the tables, we can conclude that Bayes estimates using Quadratic loss function (*QLF*) are associated withminimum MSE when compared to those obtained using *MLE*, *SELF* and PLF irrespective of the parameter values as well as the allocated sample sizes of n=10, 15, 20, 27, 35, 55, 95 and 125.

4. CONCLUSIONS

In this article, we obtain Bayesian estimators of the scale parameter of Exponential Lomax distribution. The Posterior distributions of this parameter are derived by using Uniform, Jeffrey and gamma priors. Bayes estimators and their risks have been derived by using three loss functions under the three prior distributions. The three loss functions are Squared Error Loss Function (SELF), Quadratic Loss Function (QLF) and Precautionary Loss Function (PLF). The performance of these estimators is assessed on the basis of their relative Biases and mean square errors. Monte Carlo Simulations are used to compare the performance of the estimators. It is discovered that using the *QLF* produces very minimum measures of MSE under all the priors (gamma, Jeffreys and uniform) and most especially under gamma prior, then the *SELF,MLE* and lastly the *PLF* irrespective of the parameter values and difference in sample size. Most importantly, we found that Bayesian Method using Quadratic Loss Function (*QLF*) under gamma prior produces the best estimators of the scale parameter compared to estimators using Maximum Likelihood method, Squared Error Loss Function (*SELF*) and Precautionary Loss Function (*PLF*) under Uniform and Jeffrey priors irrespective of the values of the parameters and the different sample sizes. It is also discovered that the other parameters have no effect on the estimates of the scale parameter.

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