Bayero Journal of Pure and Applied Sciences, 11(2): 229 - 233
Received: November, 2018
Accepted: December, 2018
ISSN 2006 – 6996

AN EVASION DIFFERENTIAL GAME PROBLEM ON THE PLANE

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ABSTRACT
Evasion differential game problem with many pursuers and one evader is studied on the plane. The control functions of the players are subject to integral constraints on each coordinates. Sufficient conditions for evasion to be proposed from many pursuers are obtained. Moreover, evader's strategy is constructed and illustrative example is given.
Keywords: evasion, integral constraint, strategies.

INTRODUCTION

Pursuit and evasion differential game problems involves two controlled dynamical objects called pursuer and evader with conflicting goals. The goal of the pursuit is to catch the evader in a finite time. Whereas, the evader want to escape catch by the pursuer. There is vast literature on pursuit and evasion differential game problems. Few of these are contained in the list of references in this paper. That is, from the first reference Azamov (1964) to the last one (Wah and Ibragimov, 2007).

The study of pursuit problems involves finding conditions for completion of pursuit; optimal pursuit time and construction of pursuer's strategy. In the other hand, finding conditions that guaranteed evader's escape and construction of evader's strategy are the main issues in the study of evasion differential game problem.

A matter of interest in this paper is the evasion differential game problem. There are many publications from researches involving this type of differential game problem. Some of these publications include Azamov (1964), Chodun (1987), Ibragimov and Yusra (2012), Ibragimov and Hasim(2012), Ibragimov *et. al.* (2012), Alias *et.al.* (2016), Idham *et.al*. (2013), Idham et.al. (2016) and Saleh *et. al*. (2013). In the papers Ibragimov and Yusra(2012), Ibragimov *et. al.* (2012) and Alias *et. al*. (2016), evasion problems were investigated on the plane. Therefore, in this regard, these papers are the most relevant once to this research.

In Alias *et. al.* (2016), an evasion differential game problem, involving one pursuer and one evader in the plane, is studied. Control functions of the players are subject to geometric constraints. Maximum speed of the pursuers is equal to 1 and maximal speed of the evader is$\alpha > 1$. Control set of the evader is a sector *S* whose radius is greater than 1. Sufficient conditions are obtained that guarantee the evasion, regardless of the location of the initial positions of the players.

Evasion differential game problem, with many but finite number of pursuers and one evader in the plane, is studied by Ibragimov *et al.* (2012) . Player's motion is described by simple equations. Control functions of the players are subjected to integral constraints. The main result in this paper is the sufficient condition that guaranteed evasion in the problem considered.

Ibragimov and Yusra (2012) studied evasion differential described by simple equations which involved many pursuer and one evader in the plane. Each coordinate of the control functions of the players is subjected to integral constraints. Sufficient condition for which evasion to be possible is found. Evader's strategy is constructed based on controls of the pursuers with lag. Moreover, illustrative example is given.

In the present paper, we investigate an evasion differential game problem of many pursuers and one evader with coordinate-wise integrals constraints in the space $\mathbf{R}^2$. We obtain sufficient conditions for which evasion is possible in the game. The problem considered in Ibragimov and Yusra(2012) is a special case of the problem considered in this paper. That is, the present work is a generalization in the work Ibragimov and Yusra(2012).

Statement of the Problem

We consider an evasion differential game problem with many pursuers and one evader whose equations of motion are given by:

$$Pi: \; x_i \; = a(t)u_i, \; x_i(0) = x_i^0, \quad i = 1, \dots, m; \qquad (1)$$
$$E: \; y^{0\cdot} \; = b(t)v, \quad y(0) = y^0,$$

where $x_i^0, y^0, u_i, v, x_i, y \in \mathbf{R}^2$; $u_i(t) = (u_{i1}(t), u_{i2}(t))$ and $v(t) = (v_1(t), v_2(t))$ are control functions of the pursuer and evader respectively, satisfying the inequalities

$$\int_0^\infty |u_{i1}(t)|^2\,dt \le \rho_{i1}^2, \; \int_0^\infty |u_{i2}(t)|^2\,dt \le \rho_{i2}^2, \qquad (2)$$
$$\int_0^\infty |v_1(t)|^2\,dt \le \sigma_1^2, \qquad \int_0^\infty |v_2(t)|^2\,dt \le \sigma_2^2, \qquad (3)$$

where $\rho_{i1}, \rho_{i2}, \sigma_1, \sigma_2, i = 1, \dots, m$ are given positive numbers.

It is also required that the scalar measurable functions $a(t), b(t)$ are such that for all $t \ge 0$, there exists a positive number ζ satisfying the inequality

$$\begin{cases} |a(t)| \le \zeta \le b(t), & if\ a(t) \ne b(t), \\ |a(t)| = |b(t)| \le \zeta, & if\ a(t) = b(t). \end{cases} \quad (4)$$

Definition 0.1 *A measurable function* $u_i(t) = (u_{i1}(t), u_{i2}(t)),\ t \ge 0$ *is called an admissible control of the pursuer* P_i, *if the inequalities (2) are satisfied.*

Definition 0.2 *A measurable function* $v(t) = (v_1(t), v_2(t)),\ t \ge 0$ *is called an admissible control of the evader E if the inequalities (3) are satisfied*

Definition 0.3 *A strategy of the evader* $V(\cdot)$ *is the function of the form*

$$V(t) = \begin{cases} (0,0) & 0 \le t \le \epsilon, \\ (f_1(u_1(t-\epsilon), \dots, u_m(t-\epsilon),\ f_2(u_1(t-\epsilon), \dots, u_m(t-\epsilon) & t > \epsilon, \end{cases} \quad (5)$$

where ϵ *is a positive number,* f_1, f_2 : $\mathbf{R}^m \longrightarrow \mathbf{R}$ *are a continuous functions and* $ui(t)$, *for all* $i = 1,2,3,\dots,m$, $t \ge 0$ *are admissible controls of the pursuers.*

Definition 0.4 *If there exist a strategy of the evader* $V(\cdot)$ *which ensure that* $x_{i1}(t) \ne y_1(t)$ *and/or* $x_{i2}(t) \ne y_2(t)$, *for all* $i = 1, \dots, m,\ t \ge 0,$ *and for any admissible controls* $u_i(\cdot)$ *of the pursuers then we say that evasion possible from the initial positions of the pursuers and evader* $x_i^0 = (x_{i1}^0, x_{i2}^0)$, $i = 1,2,\dots,m$ *and* $y^0 = (y_1^0, y_2^0)$, *respectively, in the game described by (1)-(4).*

Research Question: What are the sufficient conditions for evasion to be possible in the game described by (1)-(4).

SOLUTION OF THE PROBLEM

The theorem and it's proof below, which is the main result of this research, provides the sufficient conditions for the possibility of evasion in the game problem considered in this paper.

Theorem 0.1 For *evasion to be possible in the game described by (1)-(4) it is sufficient that*

1. *There exists is a partition* $\{I_1, I_2\}$ *of the set* $I = \{1,2,3,\dots,m\}$, *for which the following inequalities hold*

$$\sum_{i \in I_1} \rho_{i1}^2 \le \sigma_1^2, \quad \sum_{i \in I_2} \rho_{i2}^2 \le \sigma_2^2,$$

2. $y_1^0 \in (-\infty, a_1) \cup (b_1, \infty), \qquad y_2^0 \in (-\infty, a_2) \cup (b_2, \infty),$

where

$$a_1 = \min_{i \in I_1}\{x_{i1}^0\},\ b_1 = \max_{i \in I_1}\{x_{i1}^0\},\ a_2 = \min_{i \in I_2}\{x_{i2}^0\},\ b_2 = \max_{i \in I_2}\{x_{i2}^0\}.$$

Proof:

Suppose the condition 1 and 2 of the theorem holds. Let ϵ be a positive number such that

$$\epsilon < \frac{1}{4\,\zeta^2 \rho^2} \min\{(y_1^0 - b_1)^2, (y_2^0 - b_2)^2\},$$

where $\rho_i = \rho_{i1}^2 + \rho_{i2}^2$ and $\sum_{i=1}^m \rho_i = \rho^2$.

We construct the evader's strategy as follows:

$$V(t) = \begin{cases} (0,0), & 0 \le t \le \epsilon, \\ \left(\gamma\sqrt{\sum_{i \in I_1} u_{i1}^2(t-\epsilon)},\ \gamma^* \sqrt{\sum_{i \in I_2} u_{i2}^2(t-\epsilon)},\right), & t > \epsilon, \end{cases} \quad (6)$$

where $\gamma = \frac{y_1^0 - x_{k1}^0}{|y_1^0 - x_{k1}^0|}$, $k \in I_1$; $\quad \gamma^* = \frac{y_2^0 - x_{j2}^0}{|y_2^0 - x_{j2}^0|}$, $j \in I_2$ and u_i, $i = 1,2,3,\dots,m,$ are admissible controls of the pursuers.

Now our goal is show that if the evader uses the strategy defined by (6) then evasion is possible. Which means that $x_i(t) \ne y(t)$, for all $i = 1,2,3,\dots,m$, and for all $t \ge 0$. That is $(x_{i1}(t), x_{i2}(t)) \ne (y_1(t), y_2(t))$, which is equivalent to $x_{i1}(t) \ne y_1(t)$ and/or $x_{i2}(t) \ne y_2(t)$, for all $t \ge 0$.

To show this, we need to consider the four different possibilities with respect to the initial position of the evader, which arose from condition 2 of the theorem. One of these possibilities is that $y_1^0 > b_1, y_2^0 > b_2$. Other three are $y_1^0 > b_1,\ y_2^0 < a_2$; $y_1^0 < a_1,\ y_2^0 < a_2$ and $y_1^0 < a_1,\ y_2^0 > b_2$.

We now consider the first case, that is $y_1^0 > b_1, y_2^0 > b_2$. This implies that $\gamma = \gamma^* = 1$. Therefore the strategy defined by (6) reduces to the form

$$V(t) = \begin{cases} (0,0), & 0 \le t \le \epsilon, \\ \left(\sqrt{\sum_{i \in I_1} u_{i1}^2(t-\epsilon)},\ \sqrt{\sum_{i \in I_2} u_{i2}^2(t-\epsilon),}\right), & t > \epsilon, \end{cases} \quad (7)$$

The strategy defined by (7) is admissibility. This can be seen from the following:

$$\int_0^\infty |v_1(t)|^2 dt = \int_0^\epsilon |v_1(t)|^2 dt + \int_\epsilon^\infty |v_1(t)|^2 dt$$
$$= \int_0^\infty \left|\sqrt{\sum_{i \in I_1} u_{i1}^2(t-\epsilon)}\right|^2 dt$$

$$= \int_0^\infty \left|\sum_{i\in I_1} u_{i1}^2(t)\right| dt = \sum_{i\in I_1}\left(\int_0^\infty |u_{i1}^2(s)|\right) ds$$
$$= \sum_{i\in I_1} \rho_{i1}^2 \le \sigma_1^2.$$

In a similar way, we can show that

$$\int_0^\infty |v_2(t)|^2 dt \le \sigma_2^2.$$

If the evader uses this admissible strategy then we show that

1. Evasion is possible from the pursuers P_i , $i \in I_1$ on the both intervals $[0, \epsilon]$ and (ϵ, ∞).
2. Evasion is possible from the pursuers P_i $i \in I_2$, on the both intervals $[0, \epsilon]$ and (ϵ, ∞) .

To show (1), we first consider evasion in the interval $[0, \epsilon]$. Observe that

$$\int_0^t \zeta |u_{i1}(s)|\, ds \le \left(\int_0^t \zeta^2 ds\right)^{\frac{1}{2}} \left(\int_0^t |u_{i1}(s)|^2 ds\right)^{\frac{1}{2}} \le \zeta\sqrt{t}\rho_i \le \zeta\sqrt{\epsilon}\rho \qquad (8)$$

Let the evader uses the strategy (7) and using the inequalities (4) and (8), with $i \in I_1$, we have

$$\begin{aligned} y_1(t) - x_{i1}(t) &= y_1^0 + \int_0^t b(s)v_1(s)ds - x_{i1}^0 - \int_0^t a(s)u_{i1}(s)ds \\ &\ge y_1^0 - b_1 - \int_0^t |a(s)||u_{i1}(s)|ds \\ &\ge y_1^0 - b_1 - \int_0^t \zeta |u_{i1}(s)|ds \\ &\ge y_1^0 - b_1 - \zeta\sqrt{t}\rho \\ &\ge y_1^0 - b_1 - \zeta\rho\sqrt{\epsilon} \ge \tfrac{1}{2}(y_1^0 - b_1) > 0 \end{aligned}$$

This is by the choice of ϵ, we have $\rho\sqrt{\epsilon} \ge \frac{1}{2\zeta}(y_1^0 - b_1)$. This means that evasion is possible from the pursuers $P_i,\ i \in I_1$, in $[0, \epsilon]$.

Secondly, we consider evasion in the interval (ϵ, ∞) and observe that

$$\int_{t-\epsilon}^t \zeta |u_{i2}(s)|\, ds \le \left(\int_0^t \zeta^2 ds\right)^{\frac{1}{2}} \left(\int_0^t |u_{i2}(s)|^2 ds\right)^{\frac{1}{2}} \le \zeta\sqrt{\epsilon}\rho. \qquad (9)$$

If the evader uses the strategy defined by (7) and using the inequality (9), we have

$$\begin{aligned} y_1(t) - x_{i1}(t) &= y_1^0 + \int_\epsilon^t b(s)v_1(s)ds - x_{i1}^0 - \int_0^t a(s)u_{i1}(s)ds \\ &\ge y_1^0 - b_1 + \int_\epsilon^t b(s)\sqrt{\textstyle\sum_{i\in I_1} u_{i1}^2(s-\epsilon)}\, ds - \int_0^t |a(s)u_{i1}(s)|ds \\ &= y_1^0 - b_1 + \int_0^{t-\epsilon} |b(s+\epsilon)| \sqrt{\textstyle\sum_{i\in I_1} u_{i1}^2(s)}\, ds - \int_0^{t-\epsilon} |a(s)||u_{i1}(s)|ds - \int_{t-\epsilon}^t |a(s)||u_{i1}(s)|ds \\ &\ge y_1^0 - b_1 + \int_0^{t-\epsilon} \zeta \sqrt{\textstyle\sum_{i\in I_1} u_{i1}^2(s)}\, ds - \int_0^{t-\epsilon} \zeta|u_{i1}(s)|ds - \int_{t-\epsilon}^t \zeta|u_{i1}(s)|ds \\ &\ge y_1^0 - b_1 - \int_{t-\epsilon}^t \zeta|u_{i1}(s)|ds \\ &\ge y_1^0 - b_1 - \zeta\sqrt{\epsilon}\rho > \tfrac{1}{2}(y_1^0 - b_1) > 0, \end{aligned}$$

This is by the choice of $\epsilon < \frac{1}{4\zeta^2\rho^2}(y_1^0 - b_1)^2$. This means that evasion from the pursuers $P_i \in I_1$, is ensured in the interval (ϵ, ∞). This proves (1).

To show (2), first we consider evasion from pursuers $P_i,\ i \in I_2$, in the interval $[0.\epsilon]$. Let pursuer uses the strategy defined by (7), then for $i \in I_2$ and $t \in [0, \epsilon]$, we have

$$\begin{aligned} y_2(t) - x_{i2}(t) &= y_2^0 + \int_0^t b(s)v_2(s)ds - x_{i2}^0 - \int_0^t a(s)u_{i2}(s)ds \\ &\ge y_2^0 - b_2 - \int_0^t |a(s)u_{i2}(s)|ds \\ &\ge y_2^0 - b_2 - \int_0^t \zeta|u_{i2}(s)|ds \\ &\ge y_2^0 - b_2 - \zeta\sqrt{t}\rho \\ &\ge y_2^0 - b_2 - \zeta\rho\sqrt{\epsilon} \ge \tfrac{1}{2}(y_2^0 - b_2) > 0. \end{aligned}$$

This is by the choice of *e*, that is $\rho\sqrt{\epsilon} < \frac{1}{2\zeta}(y_2^0 - b_2)$. Thus, evasion from the pursuers $P_i, i \in I_2$, is possible in the interval $[0, \epsilon]$.

Lastly, let $t \in (\epsilon, \infty)$, then according to (7), for any $i \in I_2$ and using the inequality (9), we have:

$$\begin{aligned} y_2(t) - x_{i2}(t) &= y_2^0 + \int_\epsilon^t b(s)v_2(s)ds - x_{i2}^0 - \int_0^t a(s)u_{i2}(s)ds \\ &\ge y_2^0 - b_2 + \int_\epsilon^t |b(s)| \sqrt{\textstyle\sum_{i\in I_2} u_{i2}^2(s-\epsilon)} - \int_0^t |a(s)u_{i2}(s)|ds \\ &\ge y_2^0 - b_2 + \int_0^{t-\epsilon} |b(s+\epsilon)| \sqrt{\textstyle\sum_{i\in I_2} u_{i2}^2(s)}\, ds - \int_0^{t-\epsilon} |a(s)||u_{i2}(s)|ds - \int_{t-\epsilon}^t |a(s)||u_{i2}(s)|ds \\ &\ge y_2^0 - b_2 + \int_0^{t-\epsilon} \zeta \sqrt{\textstyle\sum_{i\in I_2} u_{i2}^2(s)}\, ds - \int_0^{t-\epsilon} \zeta|u_{i2}(s)|ds - \int_{t-\epsilon}^t \zeta|u_{i2}(s)|ds \\ &\ge y_2^0 - b_2 - \int_0^t \zeta|u_{i2}(s)|ds \end{aligned}$$

$$\geq y_2^0 - b_2 - \zeta\rho\sqrt{\epsilon} \geq \tfrac{1}{2}(y_2^0 - b_2) > 0,$$

Thus, evasion from the pursuers $P_i, i \in I_2$ is possible.

The proof of the remaining cases i.e. $y_1^0 > b_1,\ y_2^0 < a_2$; $y_1^0 < a_1,\ y_2^0 < a_2$ and $y_1^0 < a_1, y_2^0 > b_2$, is similar. This completes the proof of the theorem. ■

Illustrative Example

Example. Consider an evasion differential games problem described below

$$P_1:\ \dot{x_1} = (\sin t)\, u_1(t), \qquad x_1(0) = (0,1),$$
$$P_2:\ \dot{x_2} = (\sin t)\, u_2(t), \qquad x_2(0) = (1,2),$$
$$E:\ \ \dot{y} = (1 + e^{-t})v(t), \quad y(0) = (1,3),$$

in which control functions of the pursuers $u_i(t) = \big(u_{i1}(t), u_{i2}(t)\big),\ i = 1,2,$ and that of the evader $v(t) = (v_1(t), v_2(t))$ are such that

$$\int_0^\infty |u_{11}(t)|^2 dt \leq \rho_{11}^2, \qquad \int_0^\infty |u_{12}(t)|^2 dt \leq \rho_{12}^2,$$
$$\int_0^\infty |u_{21}(t)|^2 dt \leq \rho_{21}^2, \qquad \int_0^\infty |u_{22}(t)|^2 dt \leq \rho_{22}^2,$$
$$\int_0^\infty |v_1(t)|^2 dt \leq \sigma_1^2, \qquad \int_0^\infty |v_2(t)|^2 dt \leq \sigma_2^2,$$

where $\rho_{11}^2 = 3;\ \rho_{12}^2 = 4;\ \rho_{21}^2 = 1;\ \rho_{22}^2 = 2;\ \sigma_1^2 = 3$ and $\sigma_2^2 = 2$.

Observe that the two conditions in the theorem are satisfied for the partition $\{\{1\},\{2\}\}$ of the set $I = \{1,2\}$.That is

1. $\rho_{11}^2 = 3 = \sigma_1^2;\ \rho_{22}^2 = 2 = \sigma_2^2$, for $I_1 = \{1\}$ and $I_2 = \{2\}$.
2. $y_1^0 = 1 \in (-\infty, 0) \cup (0, \infty)$ and $y_2^0 = 3 \in (-\infty, 2) \cup (2, \infty)$.

(This is the case $y_1^0 = 1 > b_1 = 0$ and $y_2^0 = 3 > b_2 = 2$).

According to our theorem, when the evader uses the strategy

$$V(t) = \begin{cases} (0,0), & 0 \leq t \leq \epsilon, \\ \left(\sqrt{u_{11}^2(t-\epsilon)}, \quad \sqrt{u_{22}^2(t-\epsilon)}\right), & t > \epsilon, \end{cases} \tag{10}$$

then evasion from the pursuer P_1 and P_2 is possible. This can be seen from the following: Observe that in this example, $\rho^2 = \rho^2_{11} + \rho^2_{12} + \rho^2_{21} + \rho^2_{22} = 10$, and let $\epsilon < \frac{1}{40}$

1. Firstly, we show evasion from P_1 in possible in both intervals $[0, \epsilon]$ and (ϵ, ∞).
   If $t \in [0, \epsilon],\ i = 1,$ and evader uses the strategy (10), then
$$\begin{aligned} y_1(t) - x_{11}(t) &= 1 + \int_0^t (1 + e^{-s}) v_1(s)\, ds - 0 - \int_0^t \sin(s) u_{11}(s)\, ds \\ &\geq 1 - \int_0^t |\sin(s) u_{11}(s)|\, ds \\ &\geq 1 - \left(\int_0^t ds\right)^{\frac{1}{2}} \left(\int_0^t |u_{11}(s)|^2 ds\right)^{\frac{1}{2}} \\ &\geq 1 - \sqrt{t}\sqrt{10} \\ &\geq 1 - \sqrt{\epsilon}\sqrt{10} = 1 - \tfrac{\sqrt{10}}{\sqrt{40}} = \tfrac{1}{2} > 0, \end{aligned}$$
   This is by the choice of $\epsilon \leq \frac{1}{40}$.
   Also, if $t \in (\epsilon, \infty)\ i = 1,$ and evader uses the strategy (10), we have
$$\begin{aligned} y_1(t) - x_{11}(t) &= 1 + \int_\epsilon^t (1 + e^{-s}) v_1(s)\, ds - 0 - \int_0^t \sin(s) u_{11}(s)\, ds \\ &\geq 1 + \int_\epsilon^t |1 + e^{-s}||u_{11}(s-\epsilon)| ds - \int_0^t |\sin(s)||u_{11}(s)| ds \\ &\geq 1 + \int_0^{t-\epsilon} |u_{11}(s)| ds - \int_0^{t-\epsilon} |u_{11}(s)|\, ds + \int_{t-\epsilon}^t |u_{11}(s)| ds \\ &\geq 1 - \int_{t-\epsilon}^t |u_{11}(s)| ds \\ &\geq 1 - \left(\int_0^t ds\right)^{\frac{1}{2}} \left(\int_0^t |u_{11}(s)|^2 ds\right)^{\frac{1}{2}} \\ &\geq 1 - \sqrt{\epsilon}\sqrt{10} = 1 - \tfrac{\sqrt{10}}{\sqrt{40}} = \tfrac{1}{2} > 0. \end{aligned}$$
2. We can show that evasion from the pursuer P_2 is possible in both the intervals $[0, \epsilon]$ and (ϵ, ∞).

   For $i \in I_2$ and $t \in [0, \epsilon]$, we can easily show that
$$y_2(t) - x_{22}(t) = 3 + \int_0^t (1 + e^{-s}) v_2(s)\, ds - 2 - \int_0^t \sin(s) u_{22}(s)\, ds \geq 1 - \frac{1}{4} > 0.$$

Also, for $i \in I_2$ and $t \in (\epsilon, \infty)$, we can easily show that

$$y_2(t) - x_{22}(t) = 3 + \int_{\epsilon}^{t} (1 + e^{-s}) v_2(s)\, ds - 2 - \int_{0}^{t} \sin(s) u_{22}(s)\, ds \geq 1 - \frac{1}{4} > 0.$$

CONCLUSION

We have studied evasion differential game of many pursuers and one evader on the plane. Motions of players are described by first order differential equations. Control functions of the players are subjected to integral constraints. We formulated and proved a theorem that provides sufficient conditions for evasion to be possible in the problem considered. In the proof of the theorem, we used the idea in [**4**]. Furthermore, we demonstrated the result of this paper by an illustrative example.

The evasion problem studied in [**4**] is a special case to the one considered in this paper. That is, the case in which $a(t) = b(t) = 1$, and therefore, $\zeta = 1$. This means that there are some problems solvable with the result of this paper, but not solvable using the result in [**4**]. This point is supported by the illustrative example.

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