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OPTIMAL PREVENTIVE REPLACEMENT MODEL FOR A SYSTEM SUBJECT TO TWO TYPES OF FAILURE

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ABSTRACT
This paper considers a preventive maintenance model for an operating system that works for jobs at random times and is imperfectly maintained upon failure. As a failure occurs, the system suffers one of two types of failure: type I (repairable) failure is corrected by a minimal repair and type II (non-repairable) failure is removed by a corrective replacement. A modified random and age replacement model is considered in which the system is replaced at a planned time T or at a random working time Y whichever occurs last. Explicit expression for the total expected cost and optimal schedule of preventive replacement time which minimizes it are presented analytically and discussed numerically. It is also shown that when the mean random working time is small enough, the system can be operating for a longer time and avoid unnecessary replacements when replacement last is done. Finally, the proposed model extends some existing results.
Keywords: Expected cost, Failure, Minimal repair, Reliability, Replacement.

INTRODUCTION

Systems used for the production of goods and services constitute the most critical component of any manufacturing industry. System failure during actual operation is costly and dangerous and brings about high cost of production, production of defective goods, revenue loss due to loss of production, delay in customer services and sometimes causes accidents. To avoid the incidences of system failure during operation, there is the need for developing and implementing optimal maintenance strategies for minimizing maintenance costs and improving system reliability of deteriorating systems. A policy of periodic replacement with minimal repair at failure is the one in which the system is replaced at time T while performing minimal repair at any intervening failures. This is the basic minimal repair policy presented by Barlow and Hunter (1960). In this model, they derived the optimal time T, assuming the cost of minimal repair is constant and using as an optimality criterion the minimization of the total expected cost per unit time over an infinite time horizon. This model has been modified by many authors in different ways. Yakasai (2006) studied two replacement policies for a repairable system whose working time after sequence of repairs follow a geometric process. In policy I, a system is replaced by a new one if the expected working time after $(n-1)$ repairs is within a tolerance limit ε. In policy II, repairs are allowed until the return benefit after the nth repair is a non positive valued function. Zhao and Nakagawa (2012)considered age and periodic replacement last models with working cycles, where a unit is replaced before failure at a total operating time T, or at a random working cycle whichever occurs last. Zaharaddeen and Yakasai (2014) presented a replacement policy of a complex system whose components are grouped into two each with a different type of failure. The first grouped consists of repairable which are minimally repaired upon failure with probability p and non repairable components which are replaced with probability $1-p$ at some threshold number $n+1$, $n>1$, while the second group consists of non repairable components which are replaced at failure.
Chang(2014)considered a preventive maintenance policy with two types of failure. The type I failure (repairable) occurs with probability q and is rectified by a minimal repair, where as the type II failure (non-repairable) occurs with probability $p=1-q$ and is removed by a corrective replacement. The system is replaced at a random working time, or at a planned time T or at a first type II failure whichever occurs first. This model assumed that the average random working time must be very large otherwise; it results in frequent and unnecessary replacement.

This paper modifies the short coming of the Chang model by considering a small enough average random working time. We consider a system subject to two types of failure: type I (repairable) failure occurs with probability q and is corrected by a minimal repair and type II (non-repairable) failure which occurs with probability $p(1-q)$ and is removed by a corrective replacement. A modified random and age replacement model is considered in which the system is replaced at a planned time T or at a random working time Y whichever occurs last.

Using Ross (1970), we define the total expected cost per unit time as follows

$$C(T) = \frac{E(\text{cost per cycle})}{E(\text{length of cycle})} \quad (1)$$

Model formulation and assumptions

We consider a system with two types of failure where repair and replacement takes place according to the following schemes:

1. The failure time X of system has a probability distribution function F(t) and probability density function f(t) with failure rate $r(t) = \frac{f(t)}{\tilde{F}(t)}$ and cumulative failure rate $M(t) = \int_0^t r(u)du.$
2. The system failure at time t can be of two types. A type I failure(repairable) occurs with probability q and is corrected by a minimal repair and a type II failure (non-repairable) which occurs with probability $p = 1-q$, in which the system is correctively replaced.
3. Y is the random working time of the system with probability distribution $G(t)$ which is independent of X and does not take into account any actual failure.
4. A preventive replacement is scheduled to be conducted when the system attains a pre-specified time T $(T > 0)$.
5. Another preventive replacement is scheduled to be conducted at the completion of the working time.
6. In summary, the above model can be analyzed as follows; (i) if type I failure occurs, the system is rectified by a minimal repair. (ii) If type II failure occurs before the job is completed, we replace it immediately; (iii) else if the system is operating satisfactorily for a time duration T but the job has not been completed yet, we continue to operate it until the job is completed, and then replace it; and (iv) else if the system finishes the job using a time duration that is less than T, we start the second job, and so on. When the total operating time of the system has reached T, we shut it down and replace it.
7. The minimal repair cost is C_M. Preventive replacement costs due to time T and random working time Y are C_T and C_Y respectively. The corrective replacement cost due to type II failure is C_Z and it is assumed that $C_Z > C_Y > C_T$.
8. After a replacement, the system is as good as new and the replacement time is negligible.

The problem is to determine a replacement time which balances the cost of unplanned repair/replacements and the cost of planned replacement.

Consider a replacement cycle defined by the interval between replacements of the system caused by type II failure or by a planned replacement at time T or Y. Let Z be the waiting time until the first type II failure occurs.

The reliability function of Z is

$$\tilde{F}(t) = e^{-(1-q)M(t)}. \quad (2)$$

The probability that the system is replaced at age T is

$$P(Z > T, Y \leq T) = \tilde{F}(T)\tilde{G}(T), \quad (3)$$

the probability that the system is replaced at random working time is

$$P(Y > T, Z > Y) = \int_T^\infty \tilde{F}(t)dG(t), \quad (4)$$

and the probability that the system is replaced after a type II failure is equal to the probability that the system fails before time T and Y or the probability that the system fails after completion of at least one job before time T or the Probability that the system fails after T before one job completion.

The probability that the system fails before time T and Y is

$$P(Z \le T, Y > Z) = \int_0^T \tilde{G}(t) dF(t), \tag{5}$$

the probability that the system fails after completion of at least one job before T is

$$P(Z \le T, Z > Y) = \int_0^T G(t) dF(t), \tag{6}$$

and the probability that the system fails after time T before one job completion is

$$P(Z > T, Y > Z) = \int_T^\infty \tilde{G}(t) dF(t). \tag{7}$$

Hence, the probability that the system is replaced at type II failure is

$$= F(T) + \int_T^\infty \tilde{G}(t) dF(t), \tag{8}$$

where,

$$eqn(3) + eqn(4) + eqn(8). \tag{9}$$

The expected length of cycle is

$$T\tilde{F}(T)G(T) + \int_T^\infty t\tilde{F}(t) dG(t) + \int_0^T t dF(t) + \int_T^\infty t\tilde{G}(t) dF(t) = \int_0^T \tilde{F}(t) dt + \int_T^\infty \tilde{F}(t)\tilde{G}(t) dt. \tag{10}$$

The expected number of type I failures in [0, t] using Chang (2014) is $M_q(t) = qM(t)$ and the total expected number of type I failures in a cycle is

$$M_q(T)\tilde{F}(T)G(T) + \int_T^\infty M_q(t)\tilde{F}(t) dG(t) + \int_0^T M_q(t) dF(t) + \int_T^\infty M_q(t)\tilde{G}(t) dF(t)$$

$$= \int_0^T \tilde{F}(t)\, qr(t)\, dt + \int_T^\infty \tilde{F}(t)\tilde{G}(t)\, qr(t) dt. \tag{11}$$

Then, the expected total cost in a renewal cycle is

$$C(T) = \frac{C_Z - (C_Z - C_T)\int_T^\infty G(t) dF(t) + (C_Y - C_T)\int_T^\infty \tilde{F}(t) dG(t)}{\int_0^T \tilde{F}(t) dt + \int_T^\infty \tilde{F}(t)\tilde{G}(t) dt} + \frac{C_M \left[\int_0^T \tilde{F}(t) qr(t) dt + \int_T^\infty \tilde{F}(t)\tilde{G}(t) qr(t) dt\right]}{\int_0^T \tilde{F}(t) dt + \int_T^\infty \tilde{F}(t)\tilde{G}(t) dt} \tag{12}$$

Optimization

In this section, we examine the problem of finding a value of T that minimizes $C(T)$ given by eqn. (12). In other words finding a value T^* if it exists such that

$$C(T^*) = \inf\{C(T) : T > 0\}, \tag{13}$$

where, T^* is the optimal value of T. To do that we state the following;

Theorem1. Suppose the total expected cost is given by $eqn(12)$ and $F(t)$ is strictly increasing failure rate with respect to $t > 0$. If $\tilde{G}(T^*) < 1$, then there exist a finite and unique optimal replacement time T^* $(0 < T^* < \infty)$ that minimizes $C(T)$ and the corresponding total expected cost is $C(T^*) = (C_Z - C_T) r(T^*) - (C_Y - C_T) k(T^*) + C_M qr(T^*)$.

Proof.

The derivative of $C(T)$ is

$$C'(T) = \frac{[(C_Z - C_T) r(T) - (C_Y - C_T) k(T) + C_M qr(T) - C(T)][\tilde{F}(T) - \tilde{F}(T)\tilde{G}(T)]}{\int_0^T \tilde{F}(t) dt + \int_T^\infty \tilde{F}(t)\tilde{G}(t) dt}, \tag{14}$$

where,

$$r(T)=\frac{dF(T)}{\tilde{F}(T)},\ s(T)=\frac{dG(T)}{\tilde{G}(T)} \text{ and } k(T)=\frac{s(T)\tilde{F}(T)\tilde{G}(T)}{\tilde{F}(T)-\tilde{F}(T)\tilde{G}(T)}.$$

At T^*, $C'(T)=0,$

$$\Rightarrow (C_Z-C_T)r(T^*)-(C_Y-C_T)k(T^*)+C_M qr(T^*)-C(T^*)=0. \quad (15)$$

The second derivative of $C(T)$ is

$$C''(T)=[\tilde{F}(T)-\tilde{F}(T)\tilde{G}(T)]((C_Z-C_T)r'(T)-(C_Y-C_T)k'(T)+C_M qr'(T)-C'(T)-C'(T))$$
$$\frac{+[\tilde{F}(T)-\tilde{F}(T)\tilde{G}(T)]'((C_Z-C_T)r(T)-(C_Y-C_T)k(T)+C_M qr(T)-C(T))}{\int_0^T \tilde{F}(t)dt+\int_T^\infty \tilde{F}(t)\tilde{G}(t)dt}.$$

At T^*, $C''(T^*)$ becomes

$$C''(T^*)=\frac{[\tilde{F}(T^*)-\tilde{F}(T^*)\tilde{G}(T^*)]((C_Z-C_T)r'(T^*)-(C_Y-C_T)k'(T^*)+C_M qr'(T^*))}{\int_0^{T^*} \tilde{F}(t)dt+\int_{T^*}^\infty \tilde{F}(t)\tilde{G}(t)dt}. \quad (16)$$

If $\tilde{F}(T^*)-\tilde{F}(T^*)\tilde{G}(T^*)>0,$

$$\Rightarrow \tilde{G}(T^*)<1, \quad (17)$$

then $C''(T^*)>0$ and T^* is a minimizer of $C(T)$, otherwise $T^*=\infty$. The corresponding total expected cost using (15) is

$$C(T^*)=(C_Z-C_T)r(T^*)-(C_Y-C_T)k(T^*)+C_M qr(T^*). \quad (18)$$

RESULTS

Some replacement models are special cases of this model. They are demonstrated as follows;

Case I

If $q=1$, $\tilde{F}(t)=1$ and $C_Y=C_T$. This case is considered by Nakagawa(2005), in which the system is replaced at time T, or Y whichever occurs last and undergoes minimal repair at each before replacement. If we set $q=1$, $\tilde{F}(t)=1$ and $C_Y=C_T$ in $eqn(12)$, then the expected total cost is

$$C(T)=\frac{C_T+C_M[H(T)+\int_T^\infty \tilde{G}(t)qr(t)dt]}{T+\int_T^\infty \tilde{F}(t)\tilde{G}(t)dt}. \quad (19)$$

Case II

If $q=0$ and $C_Y=C_T$. This is the case considered by Zhao (2012), in which corrective replacement is done immediately after failure and preventive replacement is done before failure at time T or Y whichever occurs last. If we set $q=0,\ C_T=c_p,\ C_Z=c_f$ and $C_Y=C_T$ in $eqn(12)$, then the expected total cost is

$$C(T)=\frac{c_f-(c_f-c_p)\int_T^\infty G(t)dF(t)}{\int_0^T \tilde{F}(t)dt+\int_T^\infty \tilde{F}(t)\tilde{G}(t)dt}. \quad (20)$$

Numerical example 1.

Let the failure time Z follow a Gamma distribution $F(t)=1-(t+1)e^{-t}$, the random working time follow the exponential distribution $G(t)=1-e^{-\theta t}$, $C_m=75,\ C_Y=350,\ C_Z=700,\ C_T=150$ and $q=0.7$.

Now,

$$r(t) = \frac{t}{t+1},\ \tilde{\mathrm{F}}(t) = (t+1)e^{-t} \text{ and } \tilde{G}(t) = e^{-\theta t},\ t > 0,\ \theta > 0.$$

Furthermore, $e^{-\theta T^*} < 1,$ for $0 < T^* < \infty.$

Using Theorem 1, a finite and unique T^* minimizes $C(T)$.

Table 1: Optimal T^* and $C(T^*)$ for some values of θ.

θ	Chang Model (2014) T^*	Chang Model (2014) $C(T^*)$	Proposed Model T^*	Proposed Model $C(T^*)$
100	-	-	1.076	380.4
10	-	-	1.076	380.4
2	-	-	1.364	389.4
1	1.556	637.1	1.652	402.6
0.5	1.268	506.1	1.939	415.7
0.1	1.172	405.2	2.323	432.1
0.01	1.172	383.0	2.515	436.9
0.001	1.076	80.60	2.515	437.4

Table 2: Optimal T^* and $C(T^*)$ for some values of C_Y

$\theta = 0.5$	Chang model (2014)		Proposed Model	
C_Y	T^*	$C(T^*)$	T^*	$C(T^*)$
200	1.268	431.1	1.460	406.8
250	1.268	456.1	1.652	410.5
300	1.268	481.1	1.747	413.3
350	1.268	506.1	1.843	415.7
400	1.268	531.1	2.035	417.6
450	1.268	556.1	2.227	419.3
500	1.268	581.1	2.323	420.7
550	1.268	606.1	2.419	421.9

DISCUSSION

From the numerical results in Table 1, we make some conclusion as follows:

- When θ is large enough (2 and above), both T^* and $C(T^*)$ does not exist in respect of Chang model.
- Both T^* and $C(T^*)$ decrease as θ decreases in respect of Chang model and both T^* and $C(T^*)$ increase as θ decreases in respect of the proposed model.
- For a large enough value of θ (0.5 and above), the minimum total expected cost of the proposed model is less than that of the Chang model which shows that our proposed model is better.
- For a small enough value of θ (0.1 and below), the minimum total expected cost of the Chang model is less than that of the proposed model which shows that the Chang model is better.

From the numerical results in Table 2, we make some conclusion as follows:

- When C_Y increases, $C(T^*)$ increases and T^* remains constant in respect of Chang model.
- Similarly, when C_Y increases, both T^* and $C(T^*)$ increase in respect of the proposed model.
- The minimum total expected cost of the proposed model is less than that of the Chang model for each value of C_Y, which shows that the proposed model is better.

Conclusion

In this paper we have proposed an optimal preventive replacement model for a system subject to two types of failures. Explicit expression for the total expected cost and optimal schedule of preventive replacement time which minimizes it are presented analytically and discussed numerically. It is also shown that when the mean random working time is small enough, the system can be operating for a longer time and avoid unnecessary replacements when replacement last is done. Finally, the proposed model extends some existing results.

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