ON THE APLLICATION OF SINGLE SPECIE DYNAMIC POPULATION MODEL

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ABSTRACT
The Method of mathematical models of Malthus and Verhults were applied on ten years data collected from Magaram Poultry Farm to determine the nature of population growth, population decay or constant growth in a Poultry Farm, and independent t-test as well as one way Anova were used to compare the predicted values and observed values in order to find out whether there is significant difference between the observed and predicted values using these two models.
Keywords: Birth rate, sustainable population, overcrowding, harvesting, independent t-test and one way Anova.

INTRODUCTION
It is feasible that we make reasonable assumptions in order to convert a real problem to mathematical one and also identify the most important variables and their mutual relationships. These assumptions and relations form the mathematical model and generally led to a mathematical problem of some sort, which is solved for the relevant variables using appropriate mathematical techniques.

Once a problem is solved, the solutions must be interpreted back in terms of the real problem and hence try to validate the model by checking if the theoretical solution is in good agreement with the observation from the real situation. If there is a reasonable agreement, then the model can be used either to give theoretical experiment, further observed phenomena to predict further results or to help in making decisions. However, if the agreement between the theoretical and the observed results is inadequate, we must return to our assumptions and deduce which of them need modifying or what additions should be made. The circle is then traversed once more to see if the new model gives adequate descriptions.

Methodology
The study was carried out in Magaram Poultry Farm located along Balangu road in Kafin Hausa Local Government area of Jigawa State, Nigeria. Established in November 2007 with a total population of seven thousands five hundred (7,500) birds including cockerels and pullets purchased from Rano farm, located along Balangu road in Kafin Hausa Local Government area of Jigawa State.

Data Collection
Ten years data starting from 2001 to 2010 has been collected from Magaram Poultry farm and is presented below:

Table 1: Actual population of birds per year 2001 to 2010

15th Jan	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Population	7,500	14,978	29,437	57,103	114,982	209,981	399,804	800,027	1,398484	2,584207

Data Analysis
Two mathematical models were used to predict the number of birds from the actual number of birds and the results were analyzed by SPSS station software. The rate at which a population changes depends on at least three factors:

Net Birth Rate for the Population
The growth of a population depends on many factors and often depends on the way that one population interacts with other population.

We assume that a population changes linearly in proportion to the current value of the population. The constant of proportionality represents the net birth rate for the population and must account for a variety of factors such as:
i)   The proportion of the population that will mate
ii) The number of offspring for each mating pair
iii) The proportion of the population that will die during the next period of time**.**

Overcrowding and Scarcity of Resources
We assume that a population is limited in size by resources such as the availability of food and land, we lump all of these factors into a single overcrowding term that will serve to decrease the population when it grows too large to be supported by the available resources.

Harvesting
The removal of a constant number of individuals from a population during each time period is known as harvesting or sometime fishing. Unregulated harvesting can lead a population to an end.\

Population models that change according to the net birth rate of the current population leads to an exponential growth or decay of the population**.**

Model Analysis Using Malthus Method
Suppose N is the number of birds and t the independent variable, time. Let N(t) represent the number of birds (cockerel and pullets) at time t in the interval (t, t + ∂t).

N(t) changes due to factors affecting population size such as mortality, harvesting, overcrowding, scarcity of resources and so on. Thus we have:

$$N(t + \partial t) = N(t) + bN(t) - cN(t) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

Where b is the birth rate and c is the mortality rate (or harvesting) and are assumed to be constant. (1) gives,

$$N(t + \partial t) - N(t) = bN(t) - cN(t) = N(t)(b-c)\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(2)$$

Thus $$\lim_{\partial t \to 0} \frac{N(t+\partial t) - N(t)}{\partial t} = \lim_{\partial t \to 0} N(t)(b-c) \dots\dots\dots\dots\dots\dots\dots\dots\dots \dots\dots(3)$$

Eqn(3) gives $$N'(t) = \frac{dN}{dt} = DN(t) \; where \; D = (b-c) \dots\dots\dots\dots\dots\dots\dots(4)$$

From eqn(4) we have

$$\int \frac{dN}{N(t)} = \int Ddt, \; hence \; \log_e N(t) = Dt + A, \; \text{where A is a constant}\dots\dots\dots\dots\dots\dots\dots(5)$$

Therefore, we have $$N(t) = e^{Dt+A} = e^A e^{Dt} = Ee^{Dt}, where \; E = e^A \dots\dots\dots\dots\dots\dots\dots\dots(6)$$

When t=0 we have the initial number of birds N_0. Therefore from (6) at t=0, $N=N_0=E$.

Hence, $$N(t) = N_o e^{Dt}\dots\dots\dots\dots\dots\dots\dots\dots(7)$$

Validation of Malthus Model

In the validation, the analysis of the formulation above would be interpreted to give the behavior of the mathematical model.

Based on the data in table 1 applying eqn(7), we have for the year 2001 at t=0,

$N(0) = 7500e^0 = 7,500$.

For the year 2002 that is at   t = 1 we have

$$N(t) = 7500e^{Dt} \Rightarrow 14978 = 7500e^{Dt} = 7500e^D \Rightarrow e^D = \tfrac{14978}{7500} = 1.9971 \approx 2.$$

$$Therefore, \; D = \log_e 2 \approx 0.7, \; thus \; finaly \; we \; have$$

$$N(t) = 7500e^{0.7t}\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(8)$$

After calculating the value of D we now calculate the remaining values for N(t) at the corresponding time.

2002, at t = 1, N(1) = $7,500e^{0.7}$ = 15,103
2003, at t = 2, N(2) = $7,500e^{0.7(2)}$ = $7,500e^{1.4}$=30,413.999 ≈ 30,414
2004, at = 3, N(3) = $7,500e^{0.7(3)}$ = $7,500e^{2.1}$=61,246.2743 ≈ 61,246
2005, at = 4, N(4) = $7,500e^{0.7(4)}$ = $7,500e^{2.8}$=123,334.8508≈ 123,335
2006, at = 5, N(5) = $7,500e^{0.7(5)}$ =$7,500e^{3.5}$= 248,365.8897 ≈ 248,366
2007, at = 6, N(6) = $7,500e^{0.7(6)}$ = $7,500e^{4.2}$=500,147.4828≈ 500,147
2008, at = 7, N(7) = $7,500e^{0.7(7)}$ = $7,500e^{4.9}$=1,007,173.348≈ 1,007,173
2009, at = 8, N(8) = $7,500e^{0.7(8)}$ = $7,500e^{5.6}$=2,028,198.056≈ 2,028,198
2010, at = 9, N(9) = $7,500e^{0.7(9)}$ = $7,500e^{6.3}$=4,084,289.326≈ 4,084,289

By the Malthus model we can now compare the calculated result with real one in table

Table 2: Predicted values against the actual number of birds (Malthus Model)

15th Jan.	Actual No. of Birds	Predicted No. of Birds
2001	7,500	
2002	14,978	15,103
2003	29,437	30,414
2004	57,103	61,246
2005	114,982	123,355
2006	209,981	248,366
2007	399,804	500,147
2008	800,027	1,007,173
2009	1,398,485	2,028,198
2010	2,584,207	4,084,289

Verhulst Model (Bounded Population)

Verhulst population model is the modification of Malthus model. He assumed that there is upper limit denoted by N_∞ to the population, which, can be sustained. The change in population of size N with respect to change in time t given by dN/dt is, now assumed to be proportional to the current population level N and the fraction of the population resources still not utilized that is $(1-N/N_\infty)$. Thus we have

$$\frac{dN}{dt} = \gamma(1 - \tfrac{N}{N_\infty}).$$

$\frac{dN}{dt} = \gamma(1 - \frac{N}{N_\infty})$. $\frac{dN}{N\left(1 - \frac{N}{N_\infty}\right)} = \gamma dt$. To analyse the collected and tabulated data using Verhulst model, the

following assumptions were made.

Let N_∞ be the maximum population of birds that can be sustained.
And the independent variable time be t

6.1 The Criteria for Estimating the Maximum Population of Birds

The initial population of birds $N_0 = 7500$.

  i)   A bird takes (16 – 18) weeks to start laying from the hatching day.
  ii)   A bird can lay up to 23 eggs in a month.
  iii) A layed egg takes 21 days to hatch out.
  iv)   Egg takes 21 days to hatch out.
  v)   Thus in a year a bird will hatch 15 x 3 = 45 chickens approximately.
  vi)   Out of the total of 7500 birds we start with, 682 are cocks while 6, 818 are hens.
  vii) This is base on the ratio of 1 cock to 10 hens that is, cock: hen 1: 10
  viii) Hence, from 6750 hens we can get a maximum of 6, 818 x 45 = 306,810 chickens in a year. Thus, $N_\infty =$ 306,810

So, using the Verhulst model based on the data in *table 1,* we have:

$$N(t) = \left(\frac{N_\infty}{1 + \left(\frac{N_\infty - N_o}{N_o} \right) e^{-\gamma t}} \right) \ where \ as \ t \to \infty, N \to N_\infty . \ \text{Taking } N_\infty = 306,810.$$

For the year 2001, *at* $t = 0$, $N(0) = \left(\frac{306810}{1 + \left(\frac{306810 - 7500}{7500} \right) e^{-\gamma t}} \right) = \left(\frac{306810}{\frac{10227}{250}} \right) = \frac{306810 \times 250}{10227} = 7,500.$

For the year 2002, *at* $t = 1$, $N(1) = \left(\frac{306810}{1 + \left(\frac{306810 - 7500}{7500} \right) e^{-\gamma t}} \right) = \left(\frac{306810}{1 + \left(\frac{299310}{7500} \right) e^{-\gamma}} \right) = \left(\frac{306810}{1 + (39.908) e^{-\gamma}} \right)$

$\Rightarrow 14978 = \left(\frac{306810}{1 + (39.908) e^{-\gamma}} \right) \Rightarrow 14978 + 14978(39.908) e^{-\gamma} = 306,810 \Rightarrow 291382 = 597742.024 e^{-\gamma}$

$\Rightarrow e^{-\gamma} = \frac{291832}{597742.024} = 0.488224 \Rightarrow \gamma \approx 0.717. \ Hence, \ N(t) = \left(\frac{306810}{1 + (39.908) e^{-0.717}} \right) \dots \dots \dots (9)$

Therefore, f*or the year* 2002, *at* $t = 1$, $N(1) = \left(\frac{306810}{1 + (39.908) e^{-0.717}} \right) = \left(\frac{306810}{20.4837} \right) \approx 14,978$

For the year 2003, *at* $t = 2$, $N(2) = \left(\frac{306810}{1 + (39.908) e^{-1.434}} \right) = \left(\frac{306810}{10.5122} \right) \approx 29.186$

For the year 2004, *at* $t = 3$, $N(3) = \left(\frac{306810}{1 + (39.908) e^{-2.151}} \right) = \left(\frac{306810}{5.6440} \right) \approx 54.360$

For the year 2005, *at* $t = 4$, $N(4) = \left(\frac{306810}{1 + (39.908) e^{-2.868}} \right) = \left(\frac{306810}{3.267} \right) \approx 93.912$

For the year 2006, *at* $t = 5$, $N(5) = \left(\frac{306810}{1 + (39.908) e^{-3.585}} \right) = \left(\frac{306810}{2.1069} \right) \approx 145.622$

For the year 2007, *at* $t = 6$, $N(6) = \left(\frac{306810}{1 + (39.908) e^{-4.302}} \right) = \left(\frac{306810}{1.5404} \right) \approx 199.176$

For the year 2008, *at* $t = 7$, $N(7) = \left(\frac{306810}{1 + (39.908) e^{-5.019}} \right) = \left(\frac{306810}{1.2638} \right) \approx 242.768$

$$\textit{For the year } 2009, \textit{ at } t=8, \ N(8)=\left(\frac{306810}{1+(39.908)e^{-5.736}}\right)=\left(\frac{306810}{1.1289}\right)\approx 271.778$$

$$\textit{For the year } 2010, \textit{ at } t=9, \ N(9)=\left(\frac{306810}{1+(39.908)e^{-6.453}}\right)=\left(\frac{306810}{1.0629}\right)\approx 288,654$$

Table 3.0: *Predicted value against the actual number of birds (Verhults Model)*

15th Jan.	Actual No. of Birds	Predicted No. of Birds
2001	7,500	
2002	14,978	14,978
2003	29,437	29,186
2004	57,103	54,360
2005	114,982	93,912
2006	209,981	145,622
2007	399,804	199,176
2008	800,027	242,768
2009	1,398,485	271,778
2010	2,584,207	288,654

Tables of statistical Analysis
OBSVSPREDMALTM

FACTORS	N	Mean	Std. Deviation	Std. Error Mean
OBSERVE	10	555650.4000	840875.41446	265908.15381
PREDVMALM	9	899810.1111	1362645.88312	454215.29437

Table 4.0: *Group statistics*
OBSVSPREDMALTM

Levene's Test of Equality of Variance		t-test of Equality of Means						
F	**Sig.**	**T**	**df**	**Sig. (2-tailed)**	**Mean Difference**	**Std. Error Difference**	**95% confidence interval of the Difference Lower**	**Upper**
1.200	.289	-.670	17	.512	-344159.71111	513315.12454	-1427159.95720	738840.53498
		-.654	13.060	.525	-344159.71111	526325.64056	-1480689.24935	792369.82713

Table 5.0: *T-Test*
OBSVSPREDVERHM

FACTORS	N	Mean	Std. Deviation	Std. Error Mean
OBSERVE	10	555650.4000	840875.41446	265908.15381
PREDVMALM	9	148937.1111	106188.41128	35396.13709

Table 6.0: *Group statistics*
OBSVSPREDVERHM

Levene's Test of Equality of Variance		t-test of Equality of Means						
F	**Sig.**	**T**	**df**	**Sig. (2-tailed)**	**Mean Difference**	**Std. Error Difference**	**95% confidence interval of the Difference Lower**	**Upper**
9.154	0.008	1.437	17	.169	406713.28889	283100.44509	-190576.44025	1004003.01802
		1.516	9.318	.163	406713.28889	268253.67245	-196971.82444	1010398.40222

Table 7.0: *T-Test*
PREDMALTMVSPREDVERHM

FACTORS	N	Mean	Std. Deviation	Std. Error Mean
PREDVVERHM	9	148937.1111	106188.41128	35396.13709
PREDVMALM	9	899810.1111	1362645.88312	454215.29437

Table 8.0: *Group statistics*
PREDMALTMVSPREDVERHM

Leven's Test of Equality of Variance		t-test of Equality of Means							
F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% confidence interval of the Difference		
							Lower		Upper
9.257	.008	-1.648	16	.119	-750873.00000	455592.38379	-1716685.70867		214939.70867
		-1.648	8.097	.137	-750873.00000	455592.38379	-1799280.62302		297534.62302

Table 9.0: *T-Test*
PREDMALTMVSPREDVERHM

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
PREDVVERHM	9	148937.1111	106188.41128	35396.13709	67313.4726	230560.7496	14978.00	288654.00
PREDVMALM	9	899810.1111	1362645.88312	454215.29437	-147612.2360	1947232.4582	15103.00	4.08E+006
OBSV	10	555650.4000	840875.41446	265908.15381	-45875.6348	1157176.4348	7500.00	2.58E+006
Total	28	535543.8929	939891.96118	177622.88490	171091.8372	899995.9485	7500.00	4.08E+006

Table 10.0: *Group Statistics*

Levene Statistic	df1	df2	Sig.
5.232	2	25	0.013

Table 11.0: *Levene Statistics*
PREDMALTMVSPREDVERHM

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	2543434849372.501	2	1271717424686.251	1.492	.244
Within Groups	21308281415522.180	25	852331256620.887		
Total	23851716264894.680	27			

Table 12.0: *Anova table*
PREDMALTMVSPREDVERHM

	Statistic[a]	df1	df2	Sig.
Brown-Forsythe	1.477	2	13.396	.263

Asymptotically F distributed.

Table 13.0: *Robust Tests of Equality of Means*
PREDMALTMVSPREDVERHM

	(I) FACTORS	(J) FACTORS	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	PREDVVERHM	PREDVMALM	-750873.00000	435209.08299	.216	-1834904.8933	333158.8933
		OBSV	-406713.28889	424189.34289	.609	-1463296.8816	649870.3038
	PREDVMALM	PREDVVERHM	750873.00000	435209.08299	.216	-333158.8933	1834904.8933
		OBSV	344159.71111	424189.34289	.700	-712423.8816	1400743.3038
	OBSV	PREDVVERHM	406713.28889	424189.34289	.609	-649870.3038	1463296.8816
		PREDVMALM	-344159.71111	424189.34289	.700	-1400743.3038	712423.8816
Games-Howell	PREDVVERHM	PREDVMALM	-750873.00000	455592.38379	.281	-2049432.1282	547686.1282
		OBSV	-406713.28889	268253.67245	.327	-1150986.4765	337559.8987
	PREDVMALM	PREDVVERHM	750873.00000	455592.38379	.281	-547686.1282	2049432.1282
		OBSV	344159.71111	526325.64056	.793	-1044785.3612	1733104.7834
	OBSV	PREDVVERHM	406713.28889	268253.67245	.327	-337559.8987	1150986.4765
		PREDVMALM	-344159.71111	526325.64056	.793	-1733104.7834	1044785.3612

Table 14.0: *Post HOC test multiple comparison*

DISCUSSION

In this study we have predicted the number of birds from 2001 to 2010 using Malthus Model and Verhulst Model from the actual number of birds. Independent t-test was applied to compare the mean value of actual birds with mean value of predicted birds using Malthus Model under equal variance assumed. The significance level of Levene's Test for Equality of Variance was found to be greater than 0.05 indicating that there is no significant difference between the actual number of birds and predicted number of birds using Malthus Model since the P_{value} of sign(2-tailed) is 0.512 greater than 0.05.

Similar type of Independent T-test was also carried out under Equal Variance not assumed. The significance level of Levene's Test for Equality of Variance was found to be less than 0.05 indicating that there is no significant difference between the actual number of birds and predicted number birds using Verhulst Model since the P_{value} of sign(2-tailed) is 0.169 greater than 0.05.

The predicted values using the two models under Equal Variance assumed were also compared. The significance level of Levene's Test for Equality of Variance was found to be less than 0.05 indicating that there is no significant difference between the predicted number of birds using Malthus Model and Verhulst Model since the P_{value} of sign(2-tailed) is 0.119. Also we compared actual number of birds, predicted number of birds using Malthus Model and predicted number of birds using Verhulst Model by Analysis of Variance(Anova), from the Anova result, Levene Statistic will tell us wether to use Anova table or Robust Test of Equality of Means. In our own case, since the significance level of Levene's Statistic is 0.012 less than 0.05 so Anova table will be used. there is no signifiant difference between the number of actual birds, predicted number of birds using Malthus Model and predicted number of birds using Verhulst Model since the significant level of the Anova table is 0.244 greater than 0.05 and this confirm the Turkey HSO results of(multiple comparison).

CONCLUSION

In this work we may conclude that either of the two models can be use for prediction of birds population, since we do not find the significance difference between the observe and predicted number of birds using the two models after subjecting the result into statistical test of significance of independence t-test as well as one-way Anova.

REFERENCES

C. I. Krebs; Ecology-*The exponential analysis of distribution and abundance,* Harper and Row, New York, (1972)

C.S. Holing-*Some Characteristics of Simple types of prediction and parasitism*, Canada Entomologist, 91, (1959), 385

D. Ludwing, D. D. Jones and C.S Holing- *Qualitative analysis of insect outbreaks systems: the spruce budworm and forest*, J. Anim. Ecol. 47 (1978), 315

F. R. Oliver-*Note on the logistic curve for human populations,* Journal of Royal Statistical Society, 1982, 145.

Gotelli Nicholas, *A primer of Ecology,* Sinauer, (2001)..

J.J.O.Cannor, and E.F. Robertson-*http://www-history.mcs.sc-andrews. ac.uk/Biographies/Verhulst.htm]* (December, 1996)

Kingsland Sharon- *Modeling Nature: Episodes in the History of Population Ecology*, University of Chicago Press, (1995), 127 – 146.

Renshaw Eric -*Modeling Biological Population in space and Time*, Cambridge University Press, (1991), 6 – 9.

McIntosh Robert -The *background of Ecology*, Cambridge, University Press, (1985), 171 – 198.

Montgomery D.C- *design and analysis of experiment,* John Wiley and sons, New York, (1997)

Udom A.U-*Essential of Statistic,* Louis Chumez Primary Enterprises, Nsukka Nigeria, (2011).

R. Pearl-*The growth of Population*, Quart. Rev. Biol.2, (1927), 532.

W. C. Allee-*The Social Life of animals*, W. W Norton and Co. New York (1938)