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ORIGINAL RESEARCH ARTICLE

BASIC CONSTITUTIVE EQUATIONS IN ELASTO-PLASTICITY – A REVIEW

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ABSTRACT

The present paper reviews the constitutive equations of engineering materials in the elasto-plastic regime. Here the plastic flow rules, strain rate, yield function, overstress function as well as evolution equations for the internal parameters are discussed. Similarly, the implementation of the elasto-viscoplastic constitutive model in a standard finite element code is also discussed and it has been concluded that an appropriate choice of material model is key in accurately simulating the behaviour of an engineering material to applied loading. It is also recommended that the appropriateness of the selected constitutive response is crucial for mimicking precise the material behaviour.

1.0 Introduction

Constitutive equations tend to characterize and establish the macroscopic behaviour of materials when loaded. They are ideally mathematical equations that substantially approximate the physical observations of real materials responses over limited range of parameters for example, temperature and deformations. Real materials, generally behave in a complex manner under various parameters (of temperature and deformation) as such separate equations of elasticity, viscosity, visco-elasticity, visco-plasticity as well as plasticity are used to describe the different observed material responses (Marlvern 1969; Stein 1993).

Here we will be looking at the elastoplastic constitutive equations for materials such as metals, concrete, soils and so on which exhibit elastic-plastic material behaviour. These materials are classified as either rate-independent or rate-dependent (Belytschko et al., 2014) with their constitutive equations grouped into infinitesimal and finite elastoplasticity. The finite elastoplasticity constitutive equations are further classified as either hyperelastic-base plasticity or hypoelastic-base plasticity; this has been widely studied by Hashiguchi, (2017) and Hashiguchi, (2019). These elasto-plastic models are however based on a combination of the following plasticity theories namely: decomposition of the strain increments into elastic and plastic parts; yield functions which control the inception and continuation of plastic straining; flow principles of plasticity which governs plastic flow; as well as evolution equations (Grassl and Jirasek, 2006; Marlvern 1969; Belytschko et al., 2014).

2.0 Constitutive Equations for Rate-independent and Rate-dependent Elastic-plastic Material Behaviour

When the stress-strain curve of a material is independent of its rate of deformation, such a material is said to be rate independent; else it is rate dependent. The stress-strain responses of rate-independent and rate-dependent materials in one dimension are illustrated in Figure 1.

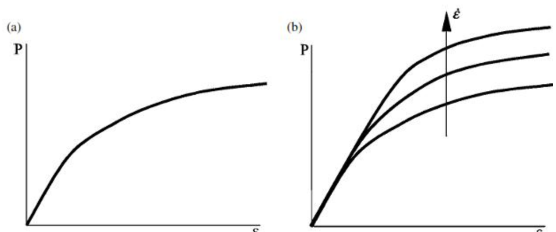

Figure 1: Engineering stress-strain curve a) rate-independent b) rate-dependent adapted from Belytschko et al. 2014.

In the following sections, the basic frame work for the rate-independent and rate-dependent elastoplastic constitutive equations are described concisely.

2.1 Rate-independent Plasticity

Rate-independent plasticity constitutive equations are grouped under the infinitesimal elastoplasticity such as:

A. Here the infinitesimal strains $(d\varepsilon)$ in Equation (1) are additively decomposed into elastic and plastic parts.

$$d\dot{\varepsilon} = d\dot{\varepsilon}^e + d\dot{\varepsilon}^p \qquad (1)$$

Hence, in terms of strain rates it is expressed as shown in Equation (2):

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \qquad (2)$$

B. The stress rate is related by the elastic modulus to the strain rate as given in Equation (3):

$$\dot{\sigma} = E\dot{\varepsilon}^e = E(\dot{\varepsilon} - \dot{\varepsilon}^p) \qquad (3)$$

Here the elastic strain $(\dot{\varepsilon}^e)$ is calculated by subtracting the plastic strain from the effective strain that is: $(\dot{\varepsilon}^e = (\dot{\varepsilon} - \dot{\varepsilon}^p))$ while the stress is calculated by substituting the elastic strain into the hyperelastic equation as given in Equation (4).

$$\dot{\sigma} = E\dot{\varepsilon}^e \qquad (4)$$

where:

E – is the Elastic (Young's) modulus (Figure 2).

C. The plastic strain rate is given by the plastic flow rule as expressed in Equation (5):

$$\dot{\varepsilon}^p = \dot{\bar{\lambda}}\frac{\partial \Psi}{\partial \sigma}; \dot{\bar{\varepsilon}} = \dot{\bar{\lambda}}; \ \sigma = \acute{\sigma} - \alpha; \Psi = / \acute{\sigma} / \qquad (5)$$

D. The evolution equation for the back stress (α also known as the kinematic hardening variable) is given as in Equation (6):

$$\dot{\alpha} = k\dot{\varepsilon}^p \qquad (6)$$

E. The yield condition (f) with isotropic and kinematic hardening is expressed as given in Equation (7) :

$$f = /\sigma - \alpha/ - \sigma_y(\bar{\varepsilon}) = 0 \qquad (7)$$

F. The loading and unloading condition is expressed as in Equation (8):

$$\dot{\lambda} \geq 0; f \leq 0; \dot{\lambda} f = 0 \qquad (8)$$

where:

$\dot{\lambda} > 0 -$ is the condition for plastic loading

$\dot{\lambda} = 0 -$ is the condition for elastic loading

$\alpha -$ is the internal variable called the back stress

$\dot{\bar{\varepsilon}} -$ is the effective plastic strain rate

$\bar{\varepsilon} -$ is the effective plastic strain

$k -$ is the kinematic hardening variable

$\dot{\varepsilon}^p -$ is the plastic strain rate

$\Psi -$ is the plastic flow potential

For plastic flow to occur the yield condition $f = 0$ must be met.

G. The tangent modulus (E^{tan}) during the plastic loading is given as:

$$E^{tan} = E - \beta \frac{E^2}{E + (H + k)} \tag{9}$$

where:

$\beta -$ is the plastic switch parameter having a value of 1 for plastic loading and zero for purely elastic loading or unloading.
$H -$ is the plastic modulus.
$E^{tan} -$ is the elastic-plastic tangent modulus, (Belytschko et al., 2014; Hashiguchi and Ueno, 2017; Hashiguchi 2019).

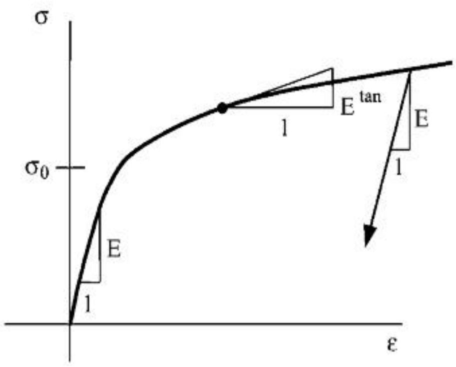

Figure 2: Stress-strain curve for a typical elastic–plastic material adapted from Belytschko et al. (2014)

2.2 Rate-dependent Plasticity

For rate-dependant materials:

A. The plastic response depends on the rate of loading which can be described by an over stress model given as:

$$\dot{\bar{\varepsilon}} = \frac{\phi(\sigma\bar{\varepsilon}\alpha)}{\eta} = \dot{\lambda} \tag{10}$$

where:

$\phi -$ is the overstress

$\eta -$ is the viscosity

$\bar{\varepsilon} -$ is the effective plastic strain

$\dot{\bar{\varepsilon}} -$ is called the effective plastic strain rate and it is equivalent to the plastic rate parameter ($\dot{\lambda}$).

B. The plastic flow rule and the evolution equation is given by:

$$D^P = \dot{\lambda} r(\sigma, q); \quad \dot{q} = \dot{\lambda} h \tag{11}$$

C. The above plastic deformation in rate-dependant materials occur when the yield condition giving in Equation (12)

$$f = /\sigma - \alpha/ - \sigma_y(\bar{\varepsilon}) = 0 \tag{12}$$

is met or exceeded. (Belytschko et al., 2014; Hashiguchi and Ueno, 2017; Hashiguchi 2019).

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D. The evolution equation is given by Belytschko et al. (2014) as:

$$\dot{\alpha} = k\dot{\bar{\varepsilon}}\, sing\, (\acute{\sigma}) \quad (13)$$

3 Elasto-viscoplastic Constitutive Model

For the above rate dependent model in Equation 10, the plastic deformation (that is the viscoplastic deformation which is unrecoverable) is described by an overstress model (Hashiguchi 2014). In this section, the original overstress model called the Bingham model is modified so that it is applicable for use in loading rates ranging from quasi-static to impact loads.

3.1 Elasto-viscoplastic Constitutive Equations

Generally, constitutive equation adopted in describing rate dependence at a stress level below yield is called the viscoelastic constitutive equation also known as the Maxwell model. Here, the strain rate $\dot{\varepsilon}$ is additively decomposed into the elastic and viscous strain rates.

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^v \quad (14)$$

The above viscoelastic constitutive equation is applicable with rate dependent deformation at stress below the yield stress (Hashiguchi, 2014).

On the other hand, when the stress level is at yield stress (Figure 2) to describe the plastic deformation, the Prandtl elasto plastic constitutive models are used where the strain rate is additively composed of the elastic and plastic parts.

$$\dot{\varepsilon} = \begin{cases} \dot{\varepsilon}^e & for\ \sigma < \sigma_y \\ \dot{\varepsilon}^e + \dot{\varepsilon}^p & for\ \sigma = \sigma_y \end{cases} \quad (15)$$

With rate dependent models where the stress is above the yield stress (Figure 3) the model that describes plastic deformation is called the elasto-viscoplastic model; here the strain rate $\dot{\varepsilon}$ is given by:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{vp}\ for\ \sigma > \sigma_y \quad (16)$$

The above original overstress model in Equation (16); called the Bingham model was first proposed by Bingham (1922) and subsequently extended by Prager (1961) for Von Mises metal and later generalised by Perzyna, (1963) for all materials expressed as given in Equation (17):

$$d = d^e + d^{vp} \quad (17)$$

(Hashiguchi, 2014; Fincato and Tsutsumi, 2019).

where:

The strain rate d is additively decomposed into the elastic strain rates d^e and viscoplastic strain rates d^{vp} with the viscoplastic strain rate expressed as given in Equation (18):

$$d^{vp} = \frac{1}{\bar{\mu}}\left\langle \frac{f(\sigma)}{F(H)} - 1 \right\rangle^{n} N \quad (18)$$

$\bar{\mu}$ – Viscoplastic coefficient

H – Is the hardening variable

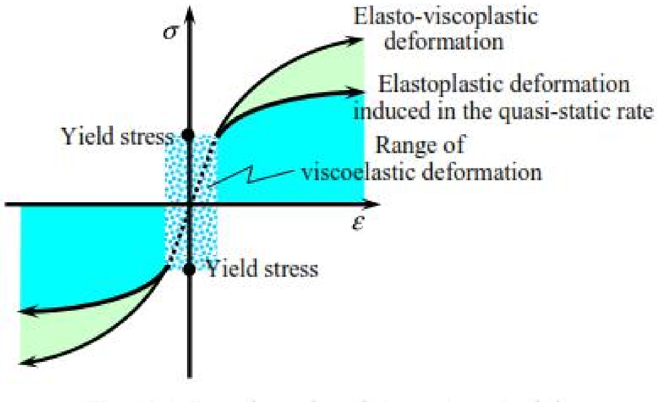


Figure 3: Rate dependent deformation of solids adapted from Hashiguchi, (2014)

The deformation (d) as in Equation (17) above is expressed as:

$$d = E^{-1}: \sigma^{0} + \frac{1}{\mathrm{\mu}}\langle\frac{\mathrm{f(\sigma)}}{\mathrm{F(H)}} - 1\rangle^{\mathrm{n}}\mathrm{N} \tag{19}$$

$$\sigma^{0} = E: d - \frac{1}{\mathrm{\mu}}\langle\frac{\mathrm{f(\sigma)}}{\mathrm{F(H)}} - 1\rangle^{\mathrm{n}}\mathrm{E:N} \tag{20}$$

(Hashiguchi, 2014; Hashiguchi and Ueno 2017)

Equations 14-20 above give a brief history of the advancement in the rate dependent viscoplastic constitutive models.

With infinitesimal rates of deformation, where $\sigma^{0} \to 0$ and $d \to 0$; the yield condition $\mathrm{f(\sigma)} = \mathrm{F(H)}$ **(**Hashiguchi, 2014; Hashiguchi and Ueno, 2017**)** is satisfied that is:

$$\frac{\mathrm{f(\sigma)}}{\mathrm{F(H)}} - 1 \to 0 \tag{21}$$

Equation (21) gives the elastoplastic response in a quasi-static deformation (Figure 4).

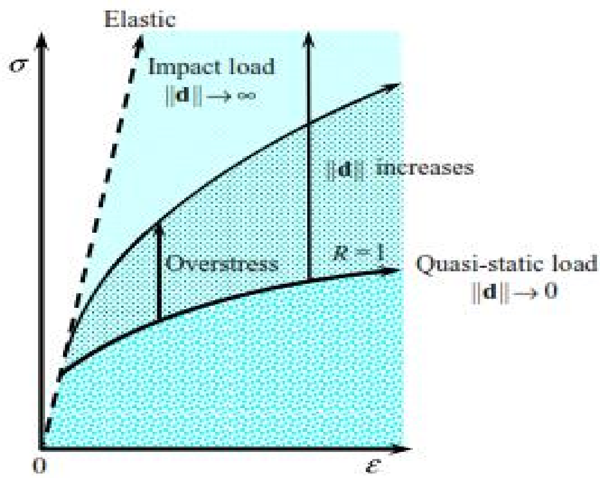


Figure 4: Response past over stress model adapted from Hashiguchi, (2014)

For infinite rate of deformations here $\sigma^{0} \to \infty$ and $d \to \infty$ Equation (20) above becomes

$$\sigma^{0} = E{:}d + 0 \tag{22}$$

Suggesting the material behaves elastically under extreme loading which is not realistic. Implying that Equation 19 and 20 as it is, are not applicable for use in low speeds (high period) impact loading. For use with such loading conditions, Equation (19) is modified as follows:

$$d \; = d^e + d^{vp};\; d = E^{-1}\!:\sigma^0 + \frac{1}{\mu}\langle \mathrm{R}-1\rangle^{\mathrm{n}}\mathrm{N} \qquad (23)$$

where:

$$\mathrm{R} = \frac{\mathrm{f}(\boldsymbol{\sigma})}{\mathrm{F(H)}} \qquad (24)$$

So that when $\sigma > \sigma_y$; R > 1

Equation (23) is better able to model the exact material behaviour under low speeds (high period) impact loading by restricting the dynamic ratio as shown in Figure 5 to a limiting value of $R_m(\gg 1)$ (Hashiguchi, 2014; Fincato and Tsutsumi, 2019).

where:

R- is the dynamic loading ratio

R_m- is the limiting dynamic loading ratio

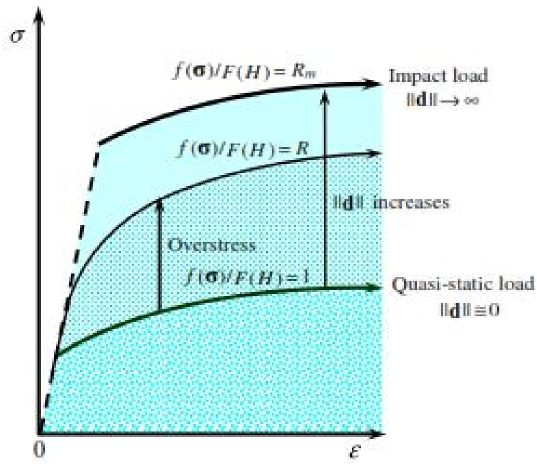


Figure 5: stress-strain cure predicted by the modified overstress model adapted from Hashiguchi, (2014)

4 Hypoelastic-based Plasticity

The constitutive models here relate the rates of stress to the rate-of-deformation. They are appropriate when the elastic strains ($\dot{\varepsilon}^{\mathrm{e}}$) are small in comparison to the plastic deformations ($\dot{\varepsilon}^{\mathrm{p}}$) (Belytschko et al., 2014; Hashiguchi 2019). Here the use of Cauchy (true) stress mainly results in constitutive equations with non-symmetric tangent modulus. Using the Kirchhoff stress on the other hand, leads to constitutive equations with symmetric tangent modulus.

4.1 Hypoelastic-plastic Constitutive Equations (based on the Cauchy stress formulations)

A. Here the rate-of-deformation tensor $(\mathbf{D})$ is additively decomposed into elastic and plastic parts.

$$\mathrm{D} = \mathrm{D}^e + \mathrm{D}^p \qquad (25)$$

B. The (Jaumann) Cauchy stress rate is related by the constant elastic moduli $(C_{el}^{\sigma J})$ to the Elastic part of the rate-of-deformation expressed as:

$$\sigma^{\nabla J} = C_{el}^{\sigma J}\!:\!\mathrm{D}^e = C_{el}^{\sigma J}(\mathrm{D} - \mathrm{D}^p) \qquad (26)$$

Although, the elastic response is hypo-elastic, the work done in a closed cycle of deformation does not vanish hence cannot be expressed precisely (Belytschko et al., 2014; Kim, 2015; Hashiguchi, 2017; Hashiguchi, 2018).

C. The plastic strain rate is given by the plastic flow rule as:

$$D^p = \dot{\lambda} r\,(\sigma,q);\ \dot{q} = \dot{\lambda} h\,(\sigma,q) \tag{27}$$

where:

$\dot{\lambda}$ –is the plastic rate parameter

D. The yield condition (f) expressed as in Equations (28) is required to be isotropic:

$$f(\sigma,q) = 0 \tag{28}$$

E. The loading and unloading condition is expressed as given in Equations (29):

$$\dot{\lambda} \geq 0;\ f \leq 0;\ \dot{\lambda} f = 0 \tag{29}$$

F. The relationship between the (Jaumann) Cauchy stress rate and the total rate-of-deformation tensor for plastic loading is expressed as:

$$\sigma^{\nabla J} = C^{\sigma J}{:}D \tag{30}$$

So that with elastic loading or unloading, $C^{\sigma J} = C^{\sigma J}_{el}$

where:

$C^{\sigma J}$ – Is the continuum elasto-plastic tangent modulus which is a fourth order tensor (Belytschko et al., 2014; Kim, 2015).

In formulating the above constitutive equations in terms Kirchhoff stress $(\boldsymbol{\tau})$ **(**which is also called the weighted Cauchy stress expressed as $\boldsymbol{\tau} = \boldsymbol{J\sigma}$ **)** that is using the Kirchhoff stress in the plastic flow equation, will result in a symmetric tangent moduli (Belytschko et al., 2014). The above hypoelastic- based plasticity constitutive equations have been expressed under the limitations of the infinitesimal elastic deformations (Hashiguchi 2017; Hashiguchi 2019).

4.2 Hyperelastic-plastic Constitutive Equations

Hyperelastic constitutive equations are designed to eliminate some of the restrictions on the hypoelastic equations such as the isotropy of its elastic moduli as well as its yield function (Belytschko et al., 2014). Here:

A. The deformation gradient (F = ${}^{\partial x}/_{\partial X}$) is decomposed multiplicatively into elastic (F^e) and plastic $(\boldsymbol{F^p})$ parts as in Equation 31.

$$F = F^e \cdot F^p \tag{31}$$

B. The stress is evaluated by means of hyperelastic potential in terms of the elastic strain and expressed as:

$$\dot{\bar{S}} = C^{\bar{S}}_{el}{:}\dot{\bar{E}}^e \tag{32}$$

C. For plastic flow, the plastic flow rule which determines the plastic flow rate $(\dot{\boldsymbol{F}}^p)$ is given as:

$$\bar{L}^P = \dot{F^p} \cdot \left(\dot{F^p}\right)^{-1} = \dot{\lambda}\bar{r}\,(\bar{S},\bar{q}) \tag{33}$$

Here the internal variable is defined in the intermediate domain $(\bar{\Omega})$ as shown in figure 5.

D. The yield condition in terms of the effective second Piola-Kirchhoff stress $(\bar{S})$ is expressed as:

$$\bar{f}\,(\bar{S},\bar{q}) = 0 \tag{34}$$

E. The rate of evolution of internal variables given in equation 35 can be derived from the hardening (softening) law as:

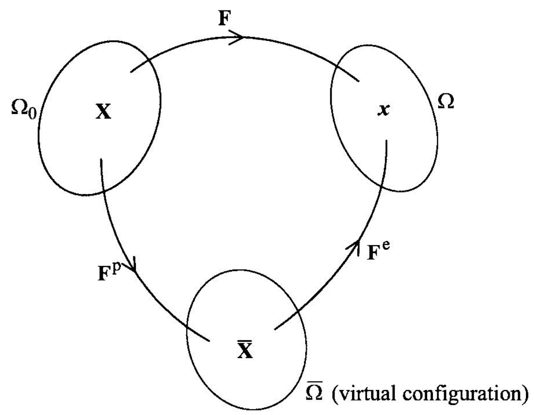
$$\dot{\bar{q}} = \dot{\lambda}\,\bar{h}(\bar{S},\bar{q}) \tag{35}$$

F. While the effective second Piola-Kirchhoff stress rate is given as in equation 36.

$$\dot{\bar{S}} = C_{el}^{\bar{S}}:\bar{D}^e = C_{el}^{\bar{S}}:(\bar{D} - \bar{D}^P) \tag{36}$$

Where:
$\bar{\boldsymbol{L}}^P -$ is the plastic part of the spatial velocity gradient.
$\boldsymbol{C}_{el}^{\bar{S}}$- Is the anisotropic elasticity tensor
$\bar{\boldsymbol{D}}^e$ and $\bar{\boldsymbol{D}}^P$ are the elastic and plastic parts of the rate of deformation.
$\bar{\boldsymbol{r}}$ – is the plastic flow direction
$\bar{\boldsymbol{h}} -$ is the plastic moduli
$\boldsymbol{x} -$ Position vectors in the current configuration
$\boldsymbol{X} -$ Position vectors in the reference configuration (see figure 6 below) (Belytschko et al., 2014; Hashiguchi 2017; Hashiguchi 2019).

For large deformations, it will no longer be viable to additively decompose the strains into elastic and plastic parts, rather the deformation gradient $(\boldsymbol{F})$ is multiplicatively decomposed into elastic and plastic parts (Kim, 2018). This multiplicative decomposition of the deformation gradient into elastic and plastic components as given in equation (31) sets up the configuration as shown in Figure 6 (Belytschko et al., 2014). This configuration shows the mapping of the point $\mathbf{X}$ in the reference domain (Ω_0) to $\bar{\mathbf{X}}$ in the intermediate domain $(\bar{\Omega})$ by the plastic part of the deformation gradient $\boldsymbol{F^p}$. It also shows the mapping of $\bar{\mathbf{X}}$ in the intermediate domain $(\bar{\Omega})$ to $\mathbf{x}$ in the current domain (Ω) by the elastic part $(\boldsymbol{F^e})$ of the deformation gradient (Belytschko et al., 2014; Kim, 2015).



The formulation of the above constitutive equations is done on the virtual configuration as shown in Figure 6.

4.3 Structural Model

The rate dependent elasto-viscoplastic material model has been used in the transient finite element analysis of a steel beam (UB 533x210x92 with a thickness of 10.1mm) subject to impact loading tested by Aliyu, 2019. This is because mild steels are generally strain rate sensitive with its stress strain relationship highly dependent on the rate of loading. The mild steel beam was subjected to impact loads at both the mid span and quarter span respectively where the loading was applied incrementally. According to Aliyu, 2019 loading of this nature tends to generate strain rates in the range of 10^2/sec. Hence, the material model specification adopted from the Ansys material library used in the transient nonlinear finite element analysis of the beam to determine the plastic deformation capacity is as given below in the material model parameter section.

4.4 Material Model Parameters

For this particular finite element (FE) simulation, the ANSYS nonlinear finite element code adapted for rough nonlinear elasto-plasticity problems was used; with the following material model specification: Non-linear > Inelastic > Rate Dependent > Visco-Plasticity > Isotropic Hardening Plasticity > Mises Plasticity > Bilinear Isotropic This was to ensure that the beam is properly modelled in both the elastic and the plastic range.

The end condition of the beam was fixed. The mesh discretion was done using the Beam4 3-D finite element (Figure 7) which has two nodes per element and six degrees of freedom per node. This particular element had tension, compression, torsion and bending capabilities as well as special features of stress stiffening and large deformation capabilities.

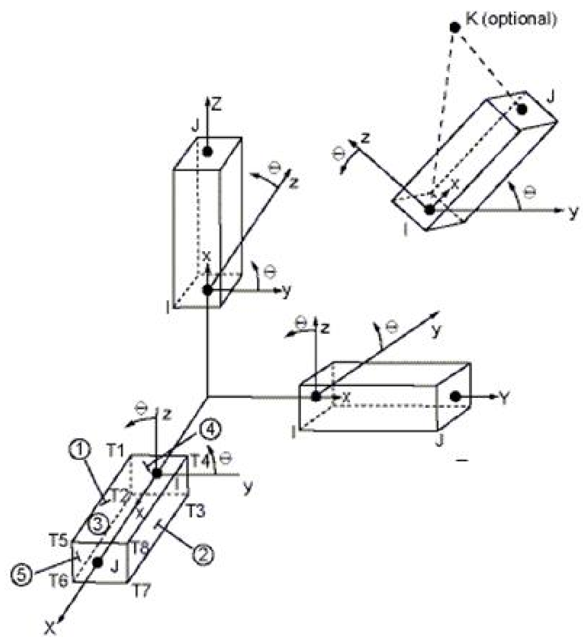
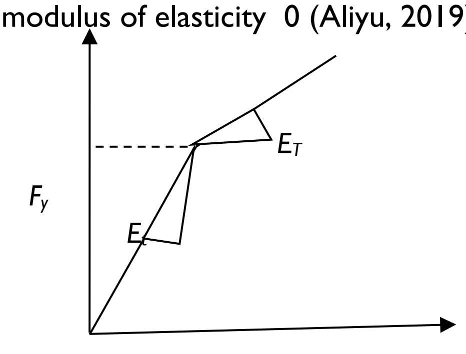


Figure 7: Details of the Beam 4 3D geometry adapted from the ANSYS material library

Figure 8 shows the Idealized elasto-plastic relationship assumed in Aliyu, 2019; the values of the material properties adopted were:

Modulus of elasticity 2.1 x 10^{11} N/m^2 ; Passion ratio 0.3; Yield stress 4.10 x 10^8 N/m^2 Tangent modulus of elasticity 0 (Aliyu, 2019)



where:

E_T = tangent modulus

E_t = elastic modulus

For the adopted idealized elasto-plastic stress strain curve in Figure 8, the initial behaviour was assumed elastic until yield stress f_y was reached after which the plastic phase was assumed to have begun. The results from the nonlinear transient finite element simulation using the above elasto-viscoplastic material model showed good agreement with the results obtained from the analytical model. Figures 9 and 10 compares the resulting displacements (from both the simulation and analytical model) of the steel beam after being struck at mid-spans and quarter-spans respectively by varying impactors from height ranges of 15m, 20m, and 25m respectively. The resulting displacements obtained from the mid span appeared to match exactly both the allowable and actual deflections predicted from the analytical calculations as given in Aliyu, (2019) which was arrived at by using the energy momentum balance technique suggested by Mughal et al. (1994). However, a very minor variation was observed with the actual quarter span deflections.

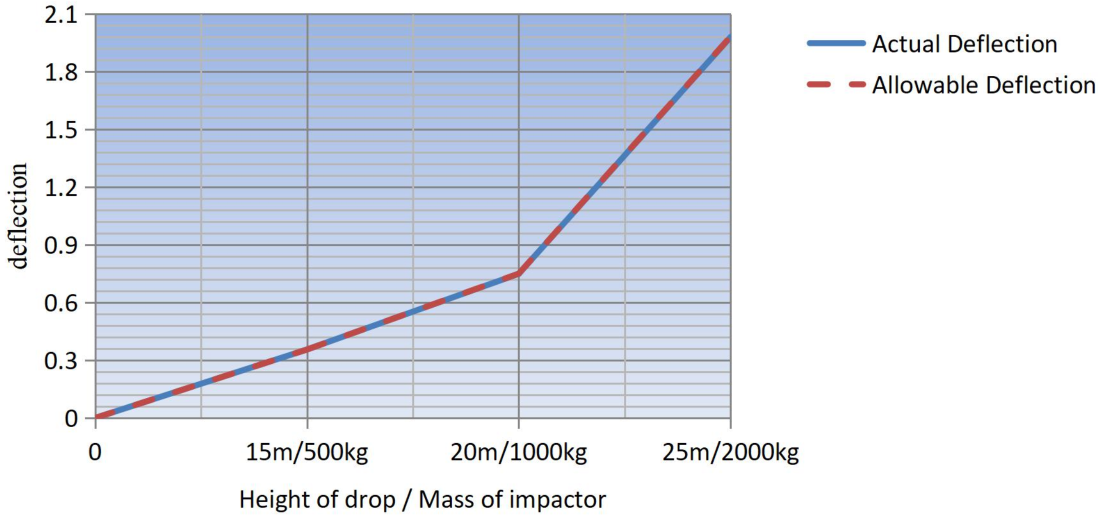


Figure 9: A plot of actual versus allowable displacement considering the effects of other loads for mid span deflection adapted from Aliyu, (2019)

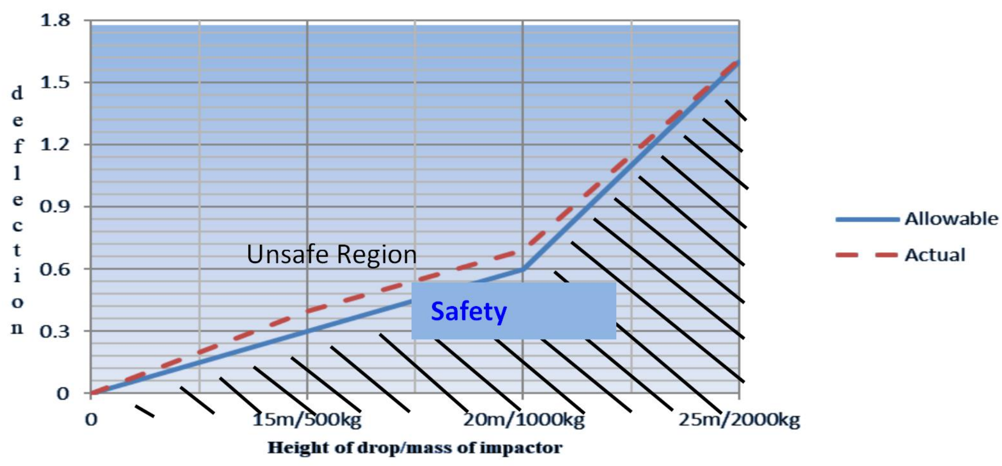


Figure 10: A graph of actual versus allowable displacement considering the effects of other loads for quarter span deflection adapted from Aliyu, (2019)

5 Conclusion

It can be concluded that, the choice of a suitable constitutive material model is considered key in any analysis simulation in order to properly model the exact material behaviour under loading conditions. This recommendation has been verified in the study carried out by Aliyu, 2019 where the rate dependent elasto-viscoplastic material model was selected from the material library of the Ansys standard finite element code as the appropriate model for use in the deformation regime where the mild steel beam behaves ductile for low speed (high period) loading and becomes brittle for high speed (low periods) loading. The model predicted with reasonable accuracy the behaviour of mild steel beams under impact loading when compared to comparable results from an analytical model reported by Aliyu, 2019. An improvement in the simulation results for the quarter span deflection may be achieved by using higher order finite elements such as SOLID 186 which is an eight noded finite element with six degrees of freedom per node. This particular element, supports plasticity, hyperelasticity, creep, stress stiffening, large deflection, and has large strain capabilities. The use of this higher order finite element may result in an increased simulation time and cost.

Generally, the elastoplastic constitutive equations discussed can be used in modelling the behaviour of geomaterials (such as: soils, concrete and so on) where permanent/irreversible strains are developed upon unloading.

Similarly, the stress strain relationships discussed in the text, postulates that plastic strain rate develops when stress approaches yield surface. Hence, models developed from this principle can be used to describe rate-independent/dependent elasto-plastic deformations of materials under monotonic/cyclic loading and proportional/non-proportional loading.

Furthermore, elasto-plastic constitutive equations can also be employed in describing friction between solids; formulating damage and plastic models used in fracture and damage analysis of composite material subject to dynamic loadings.

The above applications of elasto-plastic constitutive equations are however, not exhaustive but only to mention a few.

6 Future trends

Elasto plastic constitutive equations in formulating damage and plastic models for simulating the behaviour of geomaterial have also been proposed in literature but this is still a long standing challenge as more studies are required particularly in understanding the influence of combined damage and plasticity on analysing failure patterns in these material under monotonic/cyclic loading and proportional/non-proportional loading. Improvement in this area would be highly invaluable in automobile, aerospace industries and so on.

Glossary

E – Elastic (Young's) modulus (Pa or kPa)

β – Plastic switch parameter with a value of 1($\beta = 1$) corresponding to plastic loading and ($\beta = 0$) corresponding to pure elastic loading

α – Back stress 1Pa

E^{tan} – Slope of a line tangent to the stress-stain cure at a point of interest it is expressed as a percentage of the Young's modulus E

H – Plastic modulus (cm^3)

ϕ – Overstress which is the stress above the yield surface which describes viscoplastic ` deformation

η – Viscosity with S.I unit of pascal-second (Pa s)

$\dot{\alpha}$ – Super imposed dot denotes the time derivative of the backstress

$\bar{\varepsilon}$ – Effective plastic strain rate

k – Kinematic hardening parameter

$\acute{\sigma}$ – Difference between the stress on the yield surface and the backstress

$\mathbf{N}$ – Directional vector

d^{vp} –Viscoplastic rate

$\bar{\mu}$ – Viscoplastic coefficient

$f(\sigma)$ – Yield condition

$\boldsymbol{F}^e, \boldsymbol{F}^p$ – Elastic plastic parts of the deformation gradient $\boldsymbol{F} = \boldsymbol{F}^e \cdot \boldsymbol{F}^p$

$\boldsymbol{\tau}$ – Kirchhoff stress tensor

$\boldsymbol{\sigma}$ –Cauchy stress tensor

$\dot{\bar{\boldsymbol{S}}}$ – Effective second Piola-Kirchhoff stress rate

$\dot{\bar{\boldsymbol{E}}}$ – Effective Green strain rate

$\bar{\boldsymbol{q}}$ – internal variable

References

Aliyu, O. 2019. Design Recommendations for Steel Beams to Prevent Brittle Fracture. Nigerian Journal of Technologycal Development, 16(2): 78-84.

Belytschko, T. Liu, WK. Moran, B. and Elkhoday, KI. 2014. Nonlinear Finite Element for Continuua and Structures. Second Edition ed. West Sussex United Kingdom: John Wiley and Sons.

Fincato, R. and Tsutsumi, S. 2018. Coupled damage viscoplasiticity model for metals under cyclic loading conditions. Elsevler Procedia Structual Integrity, 18: 75-85.

Grassl , P. and Jirasek, M. 2006. Damage Plastic Model for Concrete Failure. International Journal of Solids and Structures , 43:. 7166-7196.

Hashiguchi, K. and Veno, M. 2017. Elastoplastic Constitutive Equation of Metals Under Cyclic Loading. International Journal of Engineering Science , :86-112.

Hashiguchi, K. 2014. Elastoplasticity Theory. 2nd Edition ed. Hannover Germany: Springer.

Hashiguchi, K. 2017. Basic Constitutive Equations in Multiplicative Hyperelastic-based plasticity. The Japan Society of Mechanicl Engineers , 3.

Hashiguchi, K. 2019. Multiplicative Hyperelasticity Based Plasticity for Finite Elastoplastic Deformations/Sliding: A Comprehensive Review. Archives of Computational Methods Engineering, 26: 597-637.

Malvern, LE. 1996. Introduction to the Mechanic of a Continous Medium. United States of America: Prentice-Hall, Inc.

Mughal, MA. Smith, SJ. and Roberts, AC. 1994. Design Guide for Structures Subject to Impulse and Impactive Loads. PPG Report No. SXB-IC-096565; NDA Report N0. PWR/92/161 Bechtel Guide N0. C-2.45

Kim, NH. P. 2015. Introduction to Nonlinear Finite Elements Analysis. Springer, New York.