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Evaluation of Long Short-Term Memory and Hidden Markov Model in Predicting the Productivity of Maize in Nigeria

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ABSTRACT

Background: Due to population increase and import constraints, maize, a key cereal crop in Africa, is experiencing a boom in demand. Appropriate models for predicting the productivity of the commodity to meet rising needs is a need of the hour. **Objectives:** The study determined the interactions between maize production in Nigeria and other climatic factors, mostly rainfall and temperature, and predict its outputs using models that encapsulate the relationship.

Methods: The Hidden Markov Model (HMM) and the Long Short-Term Memory neural network (LSTM) are both evaluated and compared in this context to assess their performance in the prediction of the performance of Maize in Nigeria. A variety of performance indicators, such as correlation, mean absolute percentage error (MAPE), standard error of the mean (SEM), and mean square error (MSE), are used to evaluate the models.

Results: The outcomes show that the HMM performs ten times better than the LSTM, with an RMSE of 1.21 and a MAPE of 12.98 demonstrating greater performance. Based on this result, the HMM is then used to forecast maize yield while taking the effects of temperature and rainfall into account.

Conclusion: The estimates highlight the possibility for increasing local output by demonstrating a favorable environment for maize planting in Nigeria. In order to help the Nigerian government in its efforts to increase maize production domestically, these studies offer useful insights.

Keywords: Hidden Markov Model, LSTM, Time Series, Maize, Baum-Welch Algorithm

1. INTRODUCTION

Maize (Zea mays L.) is a vital cereal crop in recent years. Africa and the developing world, including Nigeria, crucial for food security and poverty (Olanivi & Adewale, 2012). reduction However, the problem of low productivity and inadequate use of enhanced technological know-how and machineries has hampered maize production in Nigeria (Olaniyi & Adewale, 2012). Sequel to the purported low-maize problem is a greater challenge of providing sufficient staple foods, predominantly in the urban areas. The amount of locally produced staple food is not enough to satisfy demand given the technological state of rural and urban areas today and the pricing structure of basic food items. In order to satisfy several agricultural items local demand,

exported before I960 have been imported in recent years.

Maize, one of Nigeria's most popular food crops is consumed by millions of Nigerians and is also used to make animal feed. On July 13th, 2020, the Central Bank of Nigeria directed all authorised dealers to stop processing Form M for maize importation with immediate effect. The unpredictable wealth conditions and the negative effect of the Covid -19 pandemic have been added to the unknown factors that as caused a shortfall in maize production. Mohammed & Adam,(2021) did a stochastic model for rice yield forecast employing the Hidden Markov Model (HMM) on data about Niger state. However, in most

Anchor University Journal of Science and Technology , Volume 5 Issue 1

cases, the production is hidden or invisible, and each state randomly generates one out of every visible k observation.

However, the present literature did not compare the model used with another model with a dataset emanating from Niger state. Therefore, this study will demonstrate the comparative of process the prediction of Hidden Markov models underlying unobservable process (X_t) and an (HMM) and Long Short-Term Memory Neural Networks (LSTM-NN) with yearly datasets of maize production cases general Nigeria, to the provide necessary information to government to boost maize production in parameters (A, B, \prod) , representing the state Nigeria.

Methodology

Hidden Markov Model

The Hidden Markov Model (HMM), first introduced in 1957, finds extensive applications in various industries such as management, engineering, medicine, signal processing, and production (McDonald & Zuchhini, 2016; Mohammed & Adamu, 2021; Nkemnole & Wulu, 2017). HMM is a discrete stochastic $(X_t, Y_t)_{t \ge 0}$, composed of an observed process $({}^{Y_t})$ generating independent random observations. The components of an HMM include states, transitions, observations, and probabilistic behavior (Fig 1). The HMM's transition probability matrix, state emission probability matrix, and initial state probability matrix, fully describe its behaviour (McDonald & Zuchhini, 2016; Zucchini, MacDonald, &

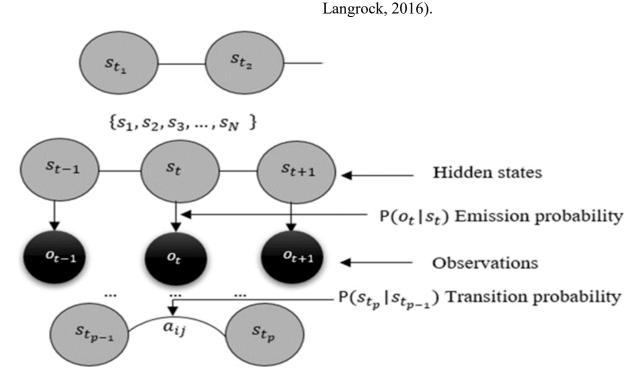


Fig. 1: Illustrates the graphical representation of a hidden Markov model, including hidden states, transition probabilities, observations, and emission probabilities (from Sassi, Anter, & Bekkhoucha, 2021).

Parameters of the Hidden Markov Model

Hidden Markov Model (HHM) is structured with the following parameters. The number of states in the model, sometimes referred to as the set of N hidden states $({}^{S^N})$, is the total number of states that the underlying hidden Markov process has: ${}^{S_1, S_2, ..., S_N}$.

The number of unique observations, M. The discrete observation quantity is denoted as

$$V = V_1, V_2, ..., V_M$$
(1)

The Transition probability matrix (A): The elements of the $N \times N$ matrix, where $i, j \in \{1, 2, ..., N\}$, describes the likelihood of transitioning from state $Z_{t-1,j}$ to $Z_{t,i}$ in a single-step. This could be expressed as

$$a_{ij} - P(Z_{t+1} = S_j c Z_t = S_i)$$
(2)

In a Markov model, the transition probabilities denoting the movement from state I to state j and also called the transition probability matrix, is typically represented by $P = [P_{ij}]$. The state transition matrix, given that the states are 1, 2, ..., r is

$$P = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1r} \\ P_{21} & P_{22} & \dots & P_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ P_{r1} & P_{r2} & \dots & P_{rr} \end{bmatrix}$$
(3)

Note that $P_{ij} \ge 0$, and for all i, we have $\sum_{k=1}^{r} P_{ik} = \sum_{k=1}^{r} P(X_{m+1} = k | X_m = i) = 1$ (4)

The observation likelihood or emission probability matrix $\binom{B}{}$ is a $\stackrel{M \times N}{}$ matrix whose members $\stackrel{B_{nj} \ n \in N}{}$ describe the likelihood of making an observation $\stackrel{X_{t,n}}{}$ given a state $\stackrel{Z_{t,j}}{}$ This could be expressed as

$$b_{j}(k) = P(x_{t} = v_{t} | Z_{t} = S_{t})$$
(5)

An emission probability matrix of size $(M + 2) \times (N + 2)$ is given as

	$b_0(k_0)$				••••	••••	••••		
		$b_1(k_1)$	$b_2(k_1)$	$b_{3}(k_{1})$				$b_N(k_1)$	
		$b_1(k_2)$	$b_2(k_2)$	$b_{3}(k_{2})$				$b_N(k_2)$	
P =		8							
		$b_1(k_M)$	$b_2(k_M)$	$b_3(k_M)$				$b_N(k_M)$	
	L								$b_f(k_f)$

Anchor University Journal of Science and Technology , Volume 5 Issue 1

Nkemnole et al 3

 $b_j(k_j)$ is the probability of emitting vocabulary item k_j from state s_i :

$$b_j(k_j) = P(O_t = k_j | X_t = s_i)$$
(7)

Initial probability distribution Π : This is a $N \times 1$ vector of probabilities

$$\prod_{j} = p(z_1 = S_j) \tag{8}$$

The solution to the Likelihood problem

In order to solve the Likelihood problem, we determine the probability of an observation sequence $X = x_1, x_2, x_3$ given a model, or $P(x|\lambda)$. We took into account the state sequence $Q = q_1, q_2, q_3$, where q_1 and q_3 represent the beginning and final states, respectively. For the state sequence Q and a model parameter λ , the probability of observing Xseries can be expressed as

$$P(X \mid Q, \lambda) = \prod_{t=1}^{N} P(x_t \mid q_t, \lambda) = b_{x1}(q_1) \cdot b_{x2}(q_2) \cdot b_{x3}(q_3) \cdot b_{x4}(q_4)$$
(9)

From the property of a Markov chain, we represented the probability of the state

$$\mathbb{P}(X|\lambda) = \sum_{Q} \mathbb{P}(X, Q|\lambda) = \mathbb{P}(X|Q, \lambda).\mathbb{P}(Q|\lambda)$$
(11)

$$\mathbb{P}(X|\lambda) = \sum_{Q} \pi_{q1} \cdot b_{x1}(q_1) \cdot p_{q1,q2} \cdot b_{x2}(q_2) \cdot p_{q2,q3} \cdot b_{x3}(q_3) \cdot b_{x4}(q_4)$$
(12)

Forward Algorithm

Using a forward variable, $\alpha_t(i)$ which indicates the likelihood that, given the HMM model λ , a partial observation sequence will occur up to time $t_{(i)}$ and the underlying Markov process will be in the state $T_{(i)}$.

$$a_{t}(i) = \mathbb{P}(x_{1}, x_{2}, x_{3}, x_{4} = T_{i}|\lambda)$$
(13)

We compute $\alpha_t(i)$ recursively via the following steps:

Initialize the forward probability as a joint probability of state T_i and initial observation m_1 .

Let $\alpha_1(i) = \pi_i b_i (\pi_i)$ for $1 \le i \ge 4$.

Compute $\alpha_3(j)$ for all states j at t = 3, using the induction procedure, substituting t = 1,2,3:

$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) \cdot p_{ij}\right] b_j(x_t), 1 \le t \le (n-1), 1 \le j \le N$$
(14)

3. Using the results from the preceding step, compute $\mathbb{P}(x \mid \lambda) = \sum_{j=1}^{N} \alpha_4(j)$.

Backward Algorithm

This research also constructs a backward variable for the forward algorithm, $\beta_i(i)$, which indicates the likelihood of a partial observation sequence from time t+1 to the end (instead of up to t as in the forward method), where the Markov process is in state s_1 at time t for a specific model, λ . The backward variable can be modeled mathematically as

$$\frac{\beta_i(i)}{(15)} = \mathbb{P}\left(\frac{x_{t+1}, x_{t+2}, x_{t+3}, x_{t+4}}{(15)}\right) = \frac{\beta_i(\lambda)}{(15)}$$

You can compute $a_t(i)$ recursively via the following steps:

Define $\beta_n(i) = 1$ for $1 \le i \ge N$. Compute $\beta_t(i) = \sum_{j=1}^N p_{ij} b_j(x_{t+1}) \beta_{t+1}(j)$.

Solution to Decoding problem

Viterbi algorithm shows that to find the state that maximises the conditional distribution of states given that data is provided a function μ it depends on the previous step, the transition probabilities and the emission proportions. The following equations represent the highest

probability along a single path for first t observations which ends at state i.

$$c = \max s_1, \dots, s_{t-1} p(s_1, s_2, s_3, s_4 = i, v_1, v_2, v_3, v_4 | \theta)$$
(16)

Using the same approach as the forward algorithm, we can calculate w(t+1)

$$w_i(t+1) = \max_i(w_i(t)a_{ij}b_{jk\nu}(t+1))$$
(17)

To find the sequence of the hidden states, we need to identify the state that maximizes w(t) at

each time step ^t.

$\arg max_t w(t)$

Baum-Welch Algorithm

The Baum-Welch Algorithm according to Sassi, *et al.*, (2021), also known as the Forward-Backward Algorithm or the Expectation-Maximisation (EM) Algorithm for fitting the Hidden Markov Models (HMMs), is used to estimate the unknown parameters of an HMM. The algorithm has two steps, the E-step followed by the M-step. These steps are:

- 1. Initialize A and B
- 2. Iterate until convergence.

E-Step

$$\xi_{i}(t) = \frac{\frac{a_{i}(t)a_{ij}b_{jk\nu(t+1)}\beta(t+1)}{\sum_{i=1}^{M}\sum_{j=1}^{M}a_{i}(t)a_{ij}b_{jk\nu(t+1)}\beta(t+1)}}$$
(18)

$$\gamma_{i}(t) = \sum_{j=1}^{M} \xi_{i}(t) \tag{19}$$

Step

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T} \xi_i(t)}{\sum_{t=1}^{T} \sum_{j=1}^{M} \xi_i(t)}$$
(20)

Long short-term memory (LSTM)

The Long Short-Term Memory (LSTM) is a Recurrent Neural Network (RNN) architecture designed to address the disappearing and exploding gradient problem. It uses memory cells, gates (input, output, and forget gates), and a hidden state to recognize long-term For each gate in the network, the learnable speech recognition, robotics, finance, and as a column matrix, as given in Eq 22. healthcare. A vital element of long short-term

memory (LSTM) is the "cell state," which conserves information the periods. over The gating mechanism, implemented using sigmoid functions, decides which data to ignore or retain based on the previous output and new input.

dependencies in sequential data. LSTMs are weights W (input weights), R (recurrent widely used in natural language processing, weights), and b (bias) are individually initialized

$$W = \begin{bmatrix} W_i \\ W_f \\ W_g \\ W_0 \end{bmatrix}, R = \begin{bmatrix} R_i \\ R_f \\ R_g \\ R_0 \end{bmatrix}, b = \begin{bmatrix} b_i \\ b_f \\ b_g \\ b_0 \end{bmatrix}$$
(22)

The cell state and concealed state at time 't' are provided by Eq 23. The Hadamard product is represented by \odot and σ_c is the state activation function-hyperbolic tangent function (tanh) respectively (element-wise multiplication).

$$C_t = f_t \odot C_{t-1} + i_t \odot g_t \tag{23}$$

$$H_t = o_t \odot \sigma_c(C_t) \tag{24}$$

At each timestep 't', equation 23 illustrates the update process of values in the network. The activation function for the gates, depicted 'g', is a sigmoid function.

$$i_{t} = \sigma_{g}(W_{i} * x_{i} + R_{i}H_{t-1} + b_{i})$$
(25)

$$f_t = \sigma_g (W_f * x_i + R_f H_{t-1} + b_f)$$
(26)

$$g_t = \sigma_c (W_g * x_i + R_g H_{t-1} + b_g)$$
(27)

$$o_t = \sigma_g (W_o * x_i + R_o H_{t-1} + b_o)$$
(28)

Anchor University Journal of Science and Technology, Volume 5 Issue 1

Nkemnole et al 6

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These gates may allow information to enter and batches once. The best model due to their exit the cell. It has a sigmoid neural network performance comparison was used to forecast. layer and a pointwise multiplication operation that helps the mechanism (Mishra, 2017). Nigeria statistics database website on climate In analysing the result, we contrast the change and the production of maize. The factors taken into account were the annual temperatures, annual rainfall, and the annual (LSTM) network and the Hidden Markov production of maize in Nigeria from 1990 to Model (HMM) technique. The datasets were 2022. In this experiment, the performance of the

models was evaluated using some measures. The Hidden Markov Model (HMM) was then trained using the Forward-Backward Algorithm models using various metrics are shown in the and then predicted the most likely states with Viterbi Algorithm and also forecasted using Baum Welch Algorithm. The Long short-term memory model (LSTM) was trained also using shown in Table 1. the means square error, the ADAM optimizer

Gates in LSTMs control the addition and was used for the weight updates, and 200 removal of information from the cell state. epochs, where one epoch looped over all

Experimental Results and Discussion

effectiveness of the Long short-term model partitioned so that we could train and test our models using 80 % and 20 % of the same data set. Measures for the predictions of both results reported in the following subsection. The values of the measures for both models are

Table 1: This table shows the values for all the measures for the prediction of both models.

	MAPE	RMSE	Corr	SEM	MSE
HMM	1.26	0.37	-0.85	0.18	0.13
LSTM	12.98	1.21	-0.85	4.19	0.87

HMM performed better than LSTM according Maize production to the study's conclusions. HMM had lower MAPE and RMSE values (1.26 and 0.37) compared to LSTM (12.98 and 1.21). HMM also had lower SEM and MSE values (0.18 and 0.87) compared to LSTM (4.19 and 0.87). The two models showed a negative correlation of -0.85. This finding was consistent with the submission of other studies that found a HMM better model (Amiens & Osamwonyi, 2022; de Wit, 2019). However, this results was at variance for the findings of Wang & Yuan (2023) who got a better model with LSTM.

LSTM training and evaluation analysis on

The input layer passes the data onto the LSTM layer that has 200 nodes at the output. The output of the LSTM layer is passed onto a dense layer 1 (one) that has 200 nodes at its input. The dense layer 1 uses mean square error (MSE) as the loss function and ADAM as the optimizer. The dense layer 1 is finally connected to the output layer that is also a fully-connected layer.

Table 2: This table shows some parameters of the LSTM in training and evaluation

Hidden Layer	Node or Neurons	Epochs	Loss	Val_Loss	Test Accuracy
1	32	200	0.7100	0.1055	0.5813

From Table 2, the training loss is 0.7100, indicating reasonable performance on the training data. The validation loss is 0.1055, lower than the training loss, indicating good generalisation to new data. The test accuracy is 0.5813, suggesting poor performance on test data and limited generalization. In comparing the HMM with LSTM for predicting maize yield, the HMM outperforms LSTM. Therefore, the HMM model is used for future predictions. Maize Yields Forecast Hidden Markov Model unsupervised learning observed symbols are vihidden states is unknow future years, this section yield statistics and a I The summary of the da 3.

Application of Hidden Markov Model for Maize Yields Forecast

Hidden Markov Models are thought of as an unsupervised learning process where just the observed symbols are visible and the number of hidden states is unknown. In order to anticipate future years, this section uses Nigeria's maize yield statistics and a Hidden Markov model. The summary of the data is presented in Table 3.

Table 3: The States and Observed level of productivity for the Hidden Markov Model for a period of thirty-two years

Year	States	Observa-	Year	States	Observa-
		tion			tion
1990	1	М	2006	2	М
1991	3	М	2007	2	М
1992	1	М	2008	3	М
1993	1	М	2009	2	М
1994	3	М	2010	4	М
1995	1	М	2011	2	М
1996	3	М	2012	3	М
1997	3	L	2013	2	М
1998	2	L	2014	2	Н
1999	3	М	2015	1	Н
2000	1	L	2016	4	Н
2001	1	М	2017	4	Н
2002	1	М	2018	4	Н
2003	3	М	2019	1	Н
2004	1	М	2020	4	Н
2005	2	М	2021	4	Н

NB: L denotes Low productivity, M denoted medium and H denotes high productivity.

Validity Test for the Model

In analysing the result of the Hidden Markov Model (HMM) process, there is a need to determine the transition matrix. Thus, the data was entered into R and the following results were produced for the transition matrix for both temperature and rainfall.

				HL H	
	ĹL	0.3000	0.1000	0.4000 0.3750 0.1250	0.2000
	LH	0.1250	0.3750	0.3750	0.1250
	HL	0.5000	0.3750	0.1250	0.0000
A =	ΗH	0.2000	0.2000	0.0000	0.6000

Anchor University Journal of Science and Technology, Volume 5 Issue 1

Nkemnole et al 8

(28)

While Observation count matrix and Observation probability matrix or the emission probability matrix are given in equations (29) and (30), respectively.

$$B = \begin{bmatrix} L & M & H \\ 0.2000 & 0.1000 & 0.7000 \\ LH \\ 0.1250 & 0.1250 & 0.7500 \\ HL \\ 0.0000 & 0.1250 & 0.8750 \\ 0.8333 & 0.0000 & 0.1667 \end{bmatrix}$$
(30)

The initial state probability is given below

$$\pi = \underbrace{\begin{array}{cccc} LL & LH \\ 0.3125 & 0.2500 \end{array}}_{0.25000} \underbrace{\begin{array}{cccc} HL & HH \\ 0.25000 & 0.1875 \end{array}}_{(31)}$$

After 1000 iteration of Baum-Welch Algorithm, equation (32) settled to (33)

.

$$\lambda_1^* = \left(\hat{A}, \hat{B}, \hat{\pi}\right) \tag{32}$$

Where

$$\pi^* = \underbrace{\begin{array}{cccc} LL & LH & HL & HH \\ 0.2500 & 0.2500 & 0.2500 & 0.2500 \end{array}}_{(33)}$$

This π^* shows the initialisation process. The probabilities assigned is 0.2500 across the four (4) states.

	LL	LH	HL	HH	
	LL [1.0000]	0.0000	0.0000	0.0000	
	LH 0.0924	0.8076	0.0000	0.1000	
	HL 0.0000	0.2246	0.7754	0.000	
=	HH 0.0000	0.0000	0.2052	0.7948	(34)

Â

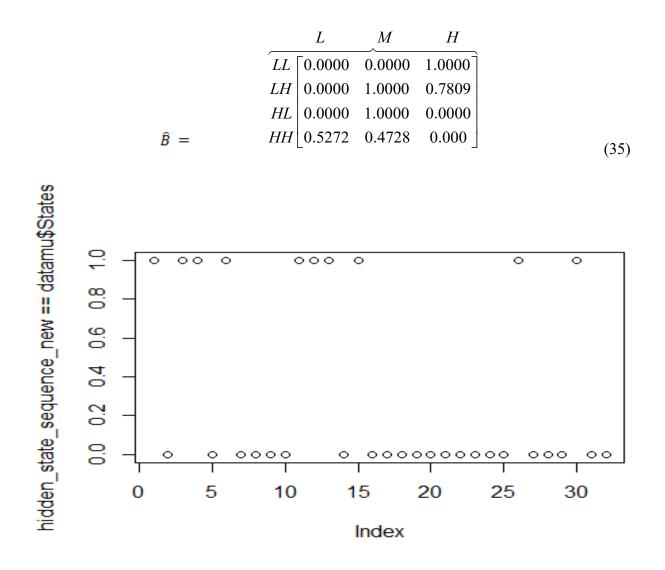


Fig. 2: Plot comparing the model to the actual data

Most Likely States Using Viterbi Algorithm

In finding the most likely states based on A and B estimates from the observed data, the following results were gotten employing the Viterbi Algorithm (Zhang, Wen, & Yang, 2022)

where state 1 is Low Rainfall and Low Temperature and state 3 is High Rainfall and Low Temperature.

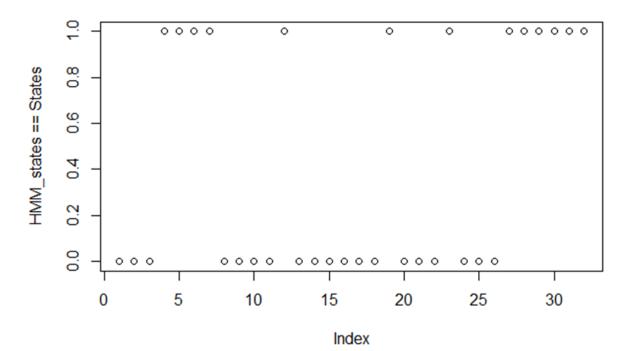


Fig. 3: This plot shows that the hidden states are solely responsible for the observable factors.

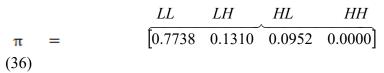
Sum (hidden state sequence) / length (States) = 0.3125

This shows that the hidden state (LL, LH, HL, HH) are 31.3% responsible for the observable factors of maize production (Low, Moderate, High).

Steady State

The likelihood that a Markov chain will remain in each state over the long run is its steady-state behavior. The initial probabilities are assumed to be equal for all states. The

resulting steady state probabilities are:

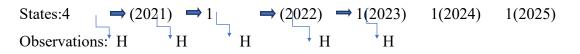


which indicate that in the long run the system will be in state 1 about 77.4 % of the time, in state 2 about 13.1% of the time in state 3 about 9.5 % of the time, and in state 4 never. This

Hidden Markov Model for future forecast

HMM was developed to forecast maize yield for future years, the parameters of the HMM were determined utilising rainfall, temperature and maize yield data from 1990 to 2021, at the point, we made forecast for 2022, 2023, 2024, and 2025.

Future Forecast States and Observations



Anchor University Journal of Science and Technology , Volume 5 Issue 1

Baum Welch algorithm, λ_1 , settled to another model, Λ_1 . This new model was then used to make a forecast for future years. From the References forecast, the HMM was in state 4 at time T (2021) emitting High rice maize, at that point, it then makes move to state 2 at time T+1 (2022) emitting High maize yield. Similar interpretation is given to move to state 2 at time T+2 (2023), move to state 2 at time T+3 (2024), move to state 2 at time T+4 (2025) all emitting High rice maize.

Conclusion

This experiment gives a quick overview of the HMM and LSTM, highlighting the salient McDonald, S., & Zuchhini, W. (2016). Hidden features of stochastic modeling and deep learning for crop production. We also show the individual effectiveness of each model and the similarities between the HMM and LSTM using multiple measures respectively. The study discovered from the measurement that there are some broad similarities, particularly at lower dimensionalities. The study found that HMM was the best model after checking their Mohammed, Y., & Adamu, L. (2021). A performances. The hidden Markov Model was then applied for future forecasts on observed maize production data and used to develop a model that depicts sequences of observable factors given some hidden state and to estimate the predicted maize production in Nigeria. The stable state probabilities were estimated transition alongside the and emission probabilities. The hidden state showed to be 21.9 % responsible for the variables which can be observed and the estimated model predicts a 31.3 % chance that the observable factors are influenced by the hidden states. Given the results, the HMM is the higher-performing model when predicting the production of Sassi, I., Anter, S., & Bekkhoucha, A. (2021). maize when comparing it with the LSTM. More than twice as low as the MAPE and RMSE of the LSTM forecast were the MAPE and RMSE of the HMM prediction. It is concluded that the Hidden Markov Model is suitable for modelling maize production as it identifies the production levels as either low, moderate, and/or high production in Nigeria. It

reveals the probability of changing from one state to another and the probability of emitting observations when in specific states.

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