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MATHEMATICAL MODEL ON ILLICIT SUBSTANCE USE AMONG STUDENTS OF TERTIARY INSTITUTIONS IN NIGERIA

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ABSTRACT

The global challenge of dismissal, disability and death of tertiary institution students are attributable to illicit substance use with its addiction. This has become a significant threat to the health and security of people in developed and developing countries. The goal of this research is to formulate a mathematical model for illicit substance (drug) use and addiction. Some control with relevant aspects was considered, which includes psychological, educational, biological and social aspects, as it affects students in tertiary institutions of learning across Nigeria. The model equations were first transformed to obtain the basic reproduction number $\mathbf{R_o}$. The model exhibits the drug-free equilibrium state. The model is analyzed for the existence and stability of the drug-free equilibrium state and endemic equilibrium. In this work, it was established that a drug-free equilibrium state exists and is locally asymptotically stable when the basic reproduction number $\mathbf{R_o < 1}$ and the threshold requirements are met. According to the findings, behavioral modification should be included in the treatment rates with controls in order to lower the rate of psychotropic substance use among students in higher institutions. In addition, students were considered in the formulation of preventive models that uses education and other control strategies with counselling to achieve behavioural change and reduce illicit substance use among the students of the Nigeria tertiary institutions.

Keywords: Addiction, mathematical model, illicit drug use, stability, reproduction number, endemic analysis, tertiary institution, Nigeria.

1. INTRODUCTION

The increasing population tertiary institution students in our nation and the world at large and their progressive addiction to illicit substance (drug) use has become a global concern. This is because illicit substance intake affects the lifestyles of tertiary institution students, which will inevitably, lead to an increase in crime, other social vices and some other negative impacts in our nation and the world (Lyman, 2016; Williams, 2008). Substance abuse issues today are on the rise and calls for intense public health concerns in both developed and developing countries (Holders, 1998). The use of hard substances (drugs) has a negative impact on both the user and the fabric of society. The effects of such an addiction can cause dangerous changes in the mind, body and spirit of the substance addict (Johnston, *et al*, 2011). The most disturbing aspect of the use of substance abuse is that it is reaching epidemic proportions in the whole world. The use and misuse of addictive substances have been an ongoing phenomenon throughout the history of humanity from early civilizations to the present. To promote and preserve the

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the health condition of the population as a whole, the main factor to be considered is the health of the young people in the society (Tsvetkova and Antonova, 2013). University students are the most susceptible to drug use among different youth groups in Nigeria because most of them live outside the watch of their parents or guardians. Previously a mathematical model has been formulated by (Steady and Gift, 2015) to address some biological and social aspects of drug users which include mental drain. However, in this research work, a mathematical model was formulated to address some social menace (such as incarceration, imprisonment etc.) on illicit substance users. It is important to note that the stigma attached to substance abuse and mental disorders often hinders early detection, diagnosis and proper treatment (CMHA, 2005). Based on the various reviews, this current research work is to formulate a mathematical model on illicit substance use among students of Tertiary Institutions in Nigeria. The possibility of this research is based on developing a mathematical model for illicit substance use. The **SEIR** and **SIR** compartmental model approach from the common epidemiological models was adopted and use in the model formulation. The stability properties of the new mathematical model in order to ascertain the model's capability to handle special interest was obtained with, the reproduction number of the model and other relevant results.

Drugs can be classified into two major groups –Legal drugs are those drugs, which are manufactured, produced, bought and sold within the confines of the law (FindLaw, 2019; Sahebi-Fakhrabad, *et al*, 2023). Drugs like aspirin cough syrups, laxatives, antacids, vitamins and certain contraceptives etc. They are legally available psychoactive drugs. These are divided into 3 categories: (a) Non-prescription drugs, (b) Prescription drugs (c) Social drugs – nicotine, caffeine and alcohol (Sharma *et al*, 2022; Better-health, 2022). Illegal drugs are those, which are not used legally but are abused. They can be further divided into two based on their potential to produce high and low dependence: amphetamines, cocaine, depressants and narcotics etc. produces high dependency. Marijuana and other hallucinogens produces low dependency (McLellan, 2017). As a result, these are split into the following three categories: (a) Social drugs, such as nicotine, coffee, and alcohol; (b) Prescription drugs; and (c) Drugs obtained without a prescription (Crocq, 2003). They are studied under various categories as follows: Narcotic Analgesics means 'pain killing' or 'pain relieving'. These drugs slow down a person and create feelings of euphoria. Dentists and doctors mostly prescribe these as painkillers. Codeine. Stimulants Chemicals and drugs, which temporarily stimulate mind and body and excite or speed up the central nervous system, are called stimulants. Stimulants are available in the form of pills and are prescribed by doctors. The younger generation is badly attracted to these drugs. 'They reach the brain through blood and upset the nervous system. Depressants at times called "downers", depress or slow down the functions of mind and

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especially the central nervous system, the heartbeat and respiration. People resort to chemicals to have relaxation, calmness and proper sleep (Preuss *et al*, 2023).Cannabis, which often refers to marijuana and other drugs, produced from Indian hemp-plant, cannabis sativa. It has been cultivated for centuries in different parts of the world for its tough fiber of the stem, for the oil in its seed, and for its psychoactive properties (Shamabadi *et al*, 2023).

In the development of this model, we considered four basic progressive stages to illicit substance use, which are as follows: Experimentation stage**,** Occasional use stage, Regular use stage and Full-blown addiction stage (Gary, 2007). These general characteristics can also be associated to illicit drug, which include feeling that one needs the drug on a regular basis to have fun, relax or deal with your problems. Sudden changes in work or school attendance and quality of work or grades; Doing things one normally would not do. Due to the characteristics observed from illicit drug users the risk factor could be as follows which include Family history of substance abuse, Physical Signs**,** Emotional Signs, Family Dynamics and School Behaviours. The implication of illicit drug include the following; Risk to personal safety, Damage to health, Legal consequences, Destructive behavior, Drug dependency is also a common cause of financial problems and difficulties at work or school.

According to Muhammed *et al* (2015), the report was done to determine the ratio of the drug abuse among students of private-owned and government universities in his community. From the report, they discovered some factors, which are the main causes of drug abuse among students of medical science, which include depression, schizophrenia, anxiety and peer pressure as well as personality disorder. The study shows that abuse of drug was more common in student of private institution due to abandoned opportunities and also male students were formal more abusive than female. Hence the need for an elaborate research on drug abuse among the student with emphasis on a mathematical model. Oshikoye and Alli (2006) reported on the perception of drug abuse amongst undergraduate in Lagos state university, Nigeria. The survey of the student was carry out within a large percentage of student in different facilities and developments, which show that awareness and other relevant practice is very poor, this research is needed to develop effective prevention strategy that combines school-based interventions with those affecting the family, social institutions and the larger community.

There has been many efforts put in place by the Government in preventing and controlling Drug Abuse in Nigeria. To this effect, The Nigeria Drug Law Enforcement Agency (NDLEA) was created and has been launching nationwide enforcement activities to seize illicit drugs. The 2019 NDLEA report has shown that in the last 10 years of operations, a total of 56, 745, 795, 555 kg of drugs were seized, 85, 058 persons with drug-related offences were arrested and 16, 937 cases were secured and convicted (Abdallah, 2019). Recently, other agencies like

Pharmacists Council of Nigeria (PCN) (Akinkuotu, 2020). The PCN also prohibits the handling of drugs by unlicensed personnel, especially prescription and controlled only drugs (PCN, 2020). The National Agency for Foods and Drugs Administration and Control (NAFDAC), (Reuters, 2020). In 2018, the agency shut down some pharmaceutical companies involved in the manufacturing of codeine-containing syrups in the country (Codeine, 2020). Other strategies by the Federal government include the establishment of the National Drug Control Master Plan (NDCMP, 2020). The NDCMP is a national blueprint for addressing the complex issues of drug trafficking, production, cultivation, and abuse in Nigeria. In 2018, the Federal government constituted a Presidential Advisory Committee for the Elimination of Drug abuse in Nigeria (The Guardian, 2019). However, the literature suggests that the burden of drug abuse may continue to rise in Nigeria due to the involvement of politics in law enforcement and lack of political goodwill (Klantschnig, 2009), (Yakubu et al, 2020).

In (2015), A new mathematical modeling framework to investigate the effects of illicit drug use in the community of south Africa such as Durban, Gauteng, Cape town and Mpumalanga on modeling illicit drug dynamics and its optimal control analysis has been reported by (Steady and Gift, 2015). In this report both epidemic and endemic analysis, which focused on the threshold dynamics of the model was, described the basic reproduction numbers. A new model that will incorporate illicit drug use in tertiary institution can be developed and used to forecast future trends on illicit substance use (drug use).

2. MATHEMATICAL MODEL FORMULATION

The model is divided into a system of ordinary differential equation with eleven different compartmental forms depending on the illicit substance use level. The eleven compartments include the susceptible population $S(t)$ comprises all individuals who are at risk of using any illicit drug but at present are not. All individuals that are subjected to test or detection by screening (counseling) for illicit substance use are grouped under $E(t)$. All students who are tested and found that they are innocent or drug-free are under this group $I_0(t)$. All students who are detected for using drugs for experimental and experience purposes are classified under the group experimental class $I_1(t)$. The students who uses drug in a lighter mode are classified as lightly drug users $I_2(t)$. The students, which uses drugs in a moderate form a little above the light users, are called moderate drug user, $I_3(t)$. The group of students who are using drug of any form at a higher or deeper way are classified under heavy or deep drug users, $I_4(t)$. T(t) comprises of drug addicts receiving treatment, counseling or rehabilitation. The incarceration class F(t) are students who are expel or imprison for correctional services as a result of drug addiction, the damage class D(t) are students who suffer physical and mental illness as a

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result of drug addiction. R(t) comprises of students who are very free from drugs or have stopped taking drug as a result of the treatment they have received. The variables and parameters explained above are presented and described in the Table 1 and Table 2 with a schematic diagram in Figure 1. We assumes that, those students who are illicit substance user, which were treated and have recovered may not be involved in illicit substance addiction. The parameter β measures the strength of interaction between the susceptible students and illicit substance users that have been detected, that is the detected influence of I_1, I_2, I_3 and I_4 on S, $\eta \geq 4$ is a modification factor which accounts for the increased chances of heavy illicit substance users to influence more new drug users compared to experimental users I_1. The following system of equations (1-11) are obtained from the model schematic diagram in figure1.

2.1 Model diagram: A Compartmental representation of the model.

Figure 1 shows the diagram representing the transmission of drug addiction using arrows that describes movement of individuals to from one compartment to the others.

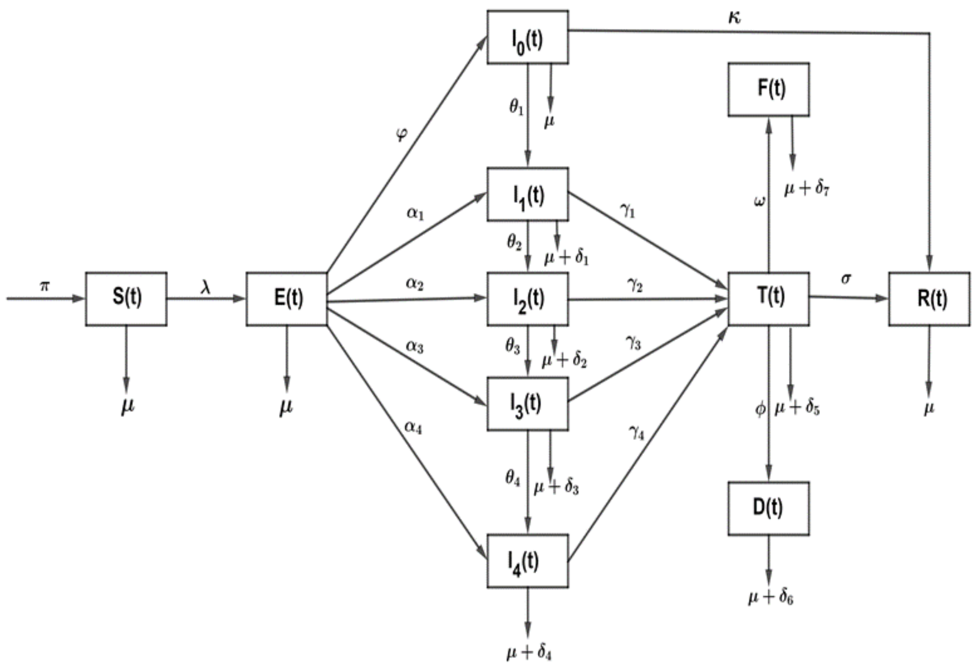

Figure 1: Schematic diagram of the illicit drug addiction and control Model

2.2 Model equations

From the above assumptions and the schematic diagram, differential equation for the model are given as:

$$\frac{dS(t)}{dt} = \pi - \lambda S(t) - \mu S(t) \tag{1}$$

$$\frac{dE(t)}{dt} = \lambda S(t) - (\varphi + \mu + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)E(t) \tag{2}$$

$$\frac{dI_0(t)}{dt} = \varphi E(t) - (\theta_1 + \mu + \kappa)I_0(t) \tag{3}$$

$$\frac{dI_1(t)}{dt} = \alpha_1 E(t) + \theta_1 I_0(t) - (\theta_2 + \mu + \delta_1 + \gamma_1)I_1(t) \tag{4}$$

$$\frac{dI_2(t)}{dt} = \alpha_2 E(t) + \theta_2 I_1(t) - (\theta_3 + \mu + \delta_2 + \gamma_2)I_2(t) \tag{5}$$

$$\frac{dI_3(t)}{dt} = \alpha_3 E(t) + \theta_3 I_2(t) - (\theta_4 + \mu + \delta_3 + \gamma_3)I_3(t) \tag{6}$$

$$\frac{dI_4(t)}{dt} = \alpha_4 E(t) + \theta_4 I_3(t) - (\mu + \delta_4 + \gamma_4)I_4(t) \tag{7}$$

$$\frac{dT(t)}{dt} = \gamma_1 I_1(t) + \gamma_2 I_2(t) + \gamma_3 I_3(t) + \gamma_4 I_4(t) - (\phi + \omega + \mu + \delta_5 + \sigma)T(t) \tag{8}$$

$$\frac{dD(t)}{dt} = \phi T(t) - (\mu + \delta_6)D(t) \tag{9}$$

$$\frac{dF(t)}{dt} = \omega T(t) - (\mu + \delta_7)F(t) \tag{10}$$

$$\frac{dR(t)}{dt} = \sigma T(t) + \kappa I_0(t) - \mu R(t) \tag{11}$$

Where

$$\lambda = \beta \frac{S\left[I_0 + \eta(I_1 + \eta_1 I_2 + \eta_2 I_3 + \eta_3 I_4)\right]}{N} \tag{12}$$

With

$$N(t) = S(t) + E(t) + I_0(t) + I_1(t) + I_2(t) + I_3(t) + I_4(t) + T(t) + D(t) + F(t) + R(t) \tag{13}$$

The feasible region for the illicit substance use or drug addiction model is expressed below

$$\Omega = \{(S, E, I_0, I_1, I_2, I_3, I_4, T, D, F, R) \in \mathbb{R}^{11}, S + E + I_0 + I_1 + I_2 + I_3 + I_4 + T + D + F + R = \frac{\pi}{\mu}\} \tag{14}$$

A description of variables use in the model equation (1- 11) are defined in the table 1 below:

Table 1: The Description of Variables used in the model

Variable	Description
N(t)	Total Population
S(t)	Susceptible Class
$I_0(t)$	Innocent stage
$I_1(t)$	Experimental class
$I_2(t)$	Lightly Drug users class
$I_3(t)$	Moderately Drug users class
$I_4(t)$	Deeply or heavily Drug users class
E(t)	Detected Class (Detected for Counseling)
T(t)	Treatment Class
R(t)	Immune Class (Recovered/ Removed from drug addiction)
D(t)	Damaged Class (Mentally/physically affected Class)
F(t)	Incarceration Class (Detected for imprisonment)

A description of Parameters used in the model as seen in equation (1-11) are defined in the Table 2 below:

Table 2: The Description of Parameters used in the model

Parameter	Description
π	Recruitment rate
η	Modification factors
β	Transmission rate
λ	The Rate at which the susceptible class moves to the detection class.
φ	Detection rate for innocent stage
α_1	Detection rate for experimental class
α_2	Detection rate for light drug users
α_3	Detection rate for moderate drug users
α_4	Detection rate for deep or heavy drug users
θ_1	Escalation of innocent class to experimental users
θ_2	Escalation of the experimental users to light users
θ_3	Escalation of the light users to moderate users
θ_4	Escalation of the moderate users to deep or heavy users
γ_1	Movement from the experimental users to treatment class
γ_2	Movement from the light users to treatment class
γ_3	Movement from the moderate users to treatment class
γ_4	Movement from the heavy users to treatment class
ω	Proportion of the treatment class who were incarcerated
ϕ	Proportion of the treatment class who develop mental illness
σ	Recovery rate
κ	Movement from the innocent class to the recovered class
μ	Natural death rate
$\mu+\delta_1$	Drug use- related death for experimental users
$\mu+\delta_2$	Drug use- related death for light drug users
$\mu+\delta_3$	Drug use- related death for moderate drug users
$\mu+\delta_4$	Drug use- related death for deep or heavy drug users
$\mu+\delta_5$	Drug use- related death for the treatment class
$\mu+\delta_6$	Drug use- related death for the damaged class
$\mu+\delta_7$	Drug use- related death for the incarceration class

3. MODEL ANALYSIS

3.1 Existence, Uniqueness and Boundedness of the systems of equations

Theorem 3.1 for any initial value $q \in \mathbb{R}^{11}$, system (1-11) has a unique nonnegative solution for all $t \geq 0$.

Proof.

Using equation (3) and summing up the odes in equation (1) gives,

$\dot{N} = \pi - \mu(S + E + I_0 + I_1 + I_2 + I_3 + I_4 + F + T + D + R)$

$- (\delta_1 I_1 - \delta_2 I_2 - \delta_3 I_3 - \delta_4 I_4 - \delta_5 T - \delta_6 D - \delta_7 F)$ If there is no infection then

$\delta_1 = \delta_2 =. . .= \delta_7 = 0$ hence the equation becomes

$\dot{N} = \pi - \mu N$

Solving for N(t) by integrating factor, accordingly

$$N(t) = \frac{\pi}{\mu} + Ce^{-\mu t}$$

$$N(0) = N_0 = \frac{\pi}{\mu} + C \Rightarrow C = N_0 - \frac{\pi}{\mu}$$

Therefore, $N(t) = \frac{\pi}{\mu} + (N_0 - \frac{\pi}{\mu})\, e^{-\mu t}$ (15)

This completes the required proof. It is very obvious that $N(t) \geq \frac{\pi}{\mu}$ and $\frac{dN(t)}{dt} \leq 0$ therefore, the solution of system (1 – 13) with nonnegative initial value are bounded and exist within $[0, +\infty)$.

3.2 Positivity of the solution

In order to determine the epidemiologically meaningful and well-posed nature of the model, it is we needful to show that the state variables are nonnegative i.e. $\forall\, t \geq 0.$

Theorem 3.1

If $S_0 \geq 0, E_0 \geq 0, I_{0_0} \geq 0, I_{1_0} \geq 0, I_{2_0} \geq 0, I_{3_0} \geq 0, I_{4_0} \geq 0, T_0 \geq 0, D_0 \geq 0, F_0 \geq 0,$ $\text{and } R_0 \geq 0$

It follows that the model equation (1) – (11) remain non – negative for all $t > 0.$

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Proof:

Assume that $S(0) \geq 0$ from equation (1) then equation (1) can be written as (16)

$$\frac{d}{dt}[S(t)\omega(t)] = \varphi\omega(t)$$

where

$$\omega(t) = e^{\int pd} = e^{\int(\lambda+\mu)ds} = e^{\int\left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4]}{\mu}\right]ds} > 0$$

which is the I.F. Therefore, integrating with respect to t, gives

$$\frac{ds(t)}{dt} = \pi - (\lambda+\mu)s(t)$$

$$\frac{ds(t)}{dt} + (\lambda+\mu)s(t) = \pi$$

$$\lambda = \beta\frac{s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4]}{\mu}$$

$$If = e^{\int pd} = e^{\int(\lambda+\mu)ds} = e^{\int\left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4]}{\mu}\right]ds}$$

$$\frac{ds}{dt}\cdot e^{\int\left[\frac{\beta s[I_0+\eta(I_1+\eta_1 I_2+\eta_2 I_3+\eta_3 I_4)]}{\mu}\right]ds} + \left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4+\mu]}{\mu}\right].s(t)$$

$$= \pi e^{\int\left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4+\mu]}{\mu}\right]ds}$$

$$\frac{d}{dt}\left(e^{\int\left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4+\mu]}{\mu}\right]ds.s(t)}\right) = \pi e^{\int\left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4+\mu]}{\mu}\right]ds} + c$$

$$s(t) = \frac{1}{e^{\int\left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4+\mu]}{\mu}\right]ds}}.\pi\left[\int e^{\int\left[\frac{\beta s[I_0+\eta_1 I_1+\eta_2 I_2+\eta_3 I_3+\eta_4 I_4+\mu]}{\mu}\right]ds} + c\right]$$

$$s(0) = 0 \Rightarrow \therefore c = s(0)$$

$$s(t) = \left[s(0) + \int_0^t \pi\varpi(s)\,ds\right] \times \omega^{-1}(s) \qquad (17)$$

This same theorem can be used to show that $E_0 \geq 0, I_{0_0} \geq 0,$ $I_{1_0} \geq 0, I_{2_0} \geq 0, I_{3_0} \geq 0, I_{4_0} \geq 0,$ $T_0 \geq 0$ $D_0 \geq 0,$ $F_0 \geq 0$ and $R_0 \geq 0$

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3.3 Equilibrium points of the drug users' model

These points is obtained by setting the odes equation (1) above to zero which will definitely give equilibrium points namely: the drug free equilibrium points (DFE) and the drug Endemic equilibrium (DEE).

$$\left.\begin{array}{l}
\pi - \lambda S - \mu S = 0 \\
\lambda S - \mu E - \alpha_1 E - \alpha_2 E - \alpha_3 E - \alpha_4 E - \alpha_5 E = 0 \\
\alpha_1 E - \theta_1 I_0 - \mu I_0 - \kappa I_0 = 0 \\
\theta_1 I_0 + \alpha_2 E - \theta_2 I_1 - \mu I_1 - \delta_1 I_1 - \gamma_1 I_1 = 0 \\
\alpha_3 E + \theta_2 I_1 - \theta_3 I_2 - \mu I_2 - \delta_2 I_2 - \gamma_2 I_2 = 0 \\
\alpha_4 E + \theta_3 I_2 - \theta_4 I_3 - \mu I_3 - \delta_3 I_3 - \gamma_3 I_3 = 0 \\
\alpha_5 E + \theta_4 I_3 - \mu I_4 - \delta_4 I_4 - \gamma_4 I_4 = 0 \\
\omega T - \mu F - \delta_7 F = 0 \\
\gamma_1 I_1 + \gamma_2 I_2 + \gamma_3 I_3 + \gamma_4 I_4 - \phi T - \mu T - \delta_5 T - \sigma T - \omega T = 0 \\
\phi T - \mu D - \delta_6 D = 0 \\
\kappa I_0 + \sigma T - \mu R = 0
\end{array}\right\} \quad (18)$$

3.3.1 Drug Free Equilibrium point:

Let us denote the drug free equilibrium points of the system (1) by

$E_f^0 = (S^0, E^0, I_0^0, I_1^0, I_2^0, I_3^0, I_4^0, F^0, T^0, D^0, R^0)$

then solving equations (5), we have

$E_f^0 = (S^0, E^0, I_0^0, I_1^0, I_2^0, I_3^0, I_4^0, F^0, T^0, D^0, R^0) = \left(\frac{\pi}{\mu}, 0,0,0,0,0\right)$

(19)

Stability Analysis Of The $S, E, I_0, I_1, I_2, I_3, I_4, F, D, T, R$ **Model Of Drug-Free Equilibrium**

In this section, we shall examine the nature of stability for the drug – free equilibrium at the Z_0 of the drug users model with the assistance of theorems.

Theorem 3.3: The SE $I_0 I_1 I_2 I_3 I_4 TDFR$ model at $Z_0 = \left(\frac{\pi}{\mu S}, 0,0,0,0,0,0,0,0,0,0\right)$ is locally asymptotically stable when $\Re_0 < 1$ and unstable if $\Re_0 > 1$.

Proof: The Jacobian matrix of system (1 – 13) at Z_0 is given by

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$$
J_{Z_0}=\begin{pmatrix}
-\mu & 0 & \beta & \beta\eta & \beta\eta\eta_1 & \beta\eta\eta_2 & \beta\eta\eta_3 & 0 & 0 & 0 & 0\\
0 & -A & \beta & \beta\eta & \beta\eta\eta_1 & \beta\eta\eta_2 & \beta\eta\eta_3 & 0 & 0 & 0 & 0\\
0 & \varphi & -B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \alpha_1 & \theta_1 & -C & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \alpha_2 & 0 & \theta_2 & -D & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \alpha_3 & 0 & 0 & \theta_3 & -E & 0 & 0 & 0 & 0 & 0\\
0 & \alpha_4 & 0 & 0 & 0 & \theta_4 & -F & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & -G & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi & -(\mu+\delta_6) & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & -(\mu+\delta_7) & 0\\
0 & 0 & \kappa & 0 & 0 & 0 & 0 & \sigma & 0 & 0 & -\mu
\end{pmatrix} \quad (20)
$$

The characteristics equation of the matrix J_{E_0} is represented by $det|J_{Z_0}-\lambda I|=0$ where λ is the eigenvalues of equation (1 – 13). Hence the eigenvalues of equation (20) are:

$$\lambda_1=-\mu\,,\lambda_2=-A,\ \lambda_3=-B,\ \lambda_4=-C,\ \lambda_5=-D,\ \lambda_6=-E,\ \lambda_7=-F,\ \lambda_8=-G,$$

$$\lambda_9=-(\mu+\delta_6),\ \lambda_{10}=-(\mu+\delta_7) \text{ and } \lambda_{11}=-\mu \quad (21)$$

Since all the eigenvalues are negative therefore, the system is locally asymptotically stable

3.3.3 Drug Endemic Equilibrium point:

When substance usage cannot be eliminated but still exists in the community, it is said to be an endemic equilibrium state. At this point, drug usage is present in the susceptible population. Since there are students in this group who are prone to being exposed to substance (drug) use, all compartments in the model will be taken into account in this scenario. In other words, if it is the endemic equilibrium state, then we simultaneously solve equations (1 to 13) while taking into account the outcome for each compartments in consideration as follows:

$$s^*=\frac{\pi}{\lambda+\mu} \quad (22)$$

$$\therefore\ E^*=\frac{\lambda\pi}{(\lambda+\mu)(u+\mu+\alpha_1+\alpha_2+\alpha_3+\alpha_4)} \quad (23)$$

$$\therefore\ I_0^*=\frac{u\lambda\pi}{(\lambda+\mu)(\theta_1+\mu+k)(u+\mu+\alpha_1+\alpha_2+\alpha_3+\alpha_4)} \quad (24)$$

$$\therefore\ I_1^*=\frac{\alpha_1\lambda\pi(\theta_1+\mu+k)+\theta_1 u\lambda\pi}{(\lambda+\mu)(\theta_1+\mu+k)(u+\mu+\alpha_1+\alpha_2+\alpha_3+\alpha_4)(\theta_2+\mu+\delta_1+\gamma_1)} \quad (25)$$

$$\therefore\ I_2^*=\frac{\alpha_2(\lambda\pi)\left[(\theta_1+\mu+k)(\theta_2+\mu+\delta_1+\gamma_1)\right]+\theta_2\left[\alpha_1\lambda\pi(\theta_1+\mu+k)+\theta_1 u\lambda\pi\right]}{(\lambda+\mu)(\theta_1+\mu+k)(u+\mu+\alpha_1+\alpha_2+\alpha_3+\alpha_4)(\theta_2+\mu+\delta_1+\gamma_1)(\theta_3+\mu+\delta_2+\gamma_2)} \quad (26)$$

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$$\therefore\ I_3^* = \frac{\alpha_3(\lambda\pi)\left[(\theta_1+\mu+k)(\theta_2+\mu+\delta_1+\gamma_1)(\theta_3+\mu+\delta_2+\gamma_2)\right]+\theta_3\left[\alpha_2(\lambda\pi)\left[(\theta_1+\mu+k)(\theta_2+\mu+\delta_1+\gamma_1)\right]+\theta_2\left[\alpha_1\lambda\pi(\theta_1+\mu+k)+\theta_1 u\lambda\pi\right]\right]}{(\lambda+\mu)(\theta_1+\mu+k)(u+\mu+\alpha_1+\alpha_2+\alpha_3+\alpha_4)(\theta_2+\mu+\delta_1+\gamma_1)(\theta_4+\mu+\delta_3+\gamma_3)} \tag{27}$$

$$\therefore\ I_4^* = \frac{\alpha_4\lambda\pi\left[(\theta_1+\mu+k)(\theta_2+\mu+\delta_1+\gamma_1)(\theta_3+\mu+\delta_2+\gamma_2)(\theta_4+\mu+\delta_3+\gamma_3)\right]+\theta_4\left[\theta_3\alpha_2\lambda\pi(\theta_1+\mu+k)(\theta_2+\mu+\delta_1+\gamma_1)+\theta_3\theta_2\left[\alpha_1\lambda\pi(\theta_1+\mu+k)+\theta_1 u\lambda\pi\right]\right]}{(\lambda+\mu)(\theta_1+\mu+k)(u+\mu+\alpha_1+\alpha_2+\alpha_3+\alpha_4)(\theta_2+\mu+\delta_1+\gamma_1)(\theta_3+\mu+\delta_2+\gamma_2)(\theta_4+\mu+\delta_3+\gamma_3)(\mu+\delta_4+\gamma_4)} \tag{28}$$

$$T^*(t) = \frac{\gamma_1 I_1+\gamma_2 I_2+\gamma_3 I_3+\gamma_4 I_4}{(\phi+\omega+\mu+\delta_5+\delta)} \tag{29}$$

$$D^* = \frac{\phi\left[\gamma_1 I_1+\gamma_2 I_2+\gamma_3 I_3+\gamma_4 I_4\right]}{(\phi+\omega+\mu+\delta_5+\delta)(\mu+\delta_6)} \tag{30}$$

$$F^* = \frac{\omega\left[\gamma_1 I_1+\gamma_2 I_2+\gamma_3 I_3+\gamma_4 I_4\right]}{(\phi+\omega+\mu+\delta_5+\delta)(\mu+\delta_7)} \tag{31}$$

$$R(t) = \frac{\sigma T(t)+KI_0(t)}{\mu} \tag{32}$$

$$R^* = \frac{\sigma\left[\dfrac{\gamma_1 I_1+\gamma_2 I_2+\gamma_3 I_3+\gamma_4 I_4}{(\phi+\omega+\mu+\delta_5+\delta)}\right]+K\left[\dfrac{u\lambda\pi}{(\lambda+\mu)(\theta_1+\mu+k)(u+\mu+\alpha_1+\alpha_2+\alpha_3+\alpha_4)}\right]}{\mu} \tag{33}$$

3.4 Basic Reproduction number for the DFE

We shall determine the basic reproduction number of the drug addiction using the next generation method. Consider the following matrices for finding the basic reproduction number. Using the notation found in (Eguda et al, 2022), the matrices F and V, for the new addiction and remaining transfer are represented respectively as:

Obtaining the partial derivatives of $\mathcal{F}_i$ with respect to E, I_0, I_1, I_2, I_3 and I_4 evaluated at the drug free addiction equilibrium gives:

$$F = \begin{pmatrix} 0 & \beta & \beta\eta & \beta\eta\eta_1 & \beta\eta\eta_2 & \beta\eta\eta_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} A & 0 & 0 & 0 & 0 & 0 \\ -\alpha_1 & B & 0 & 0 & 0 & 0 \\ -\alpha_2 & -\theta_1 & C & 0 & 0 & 0 \\ -\alpha_3 & 0 & -\theta_2 & D & 0 & 0 \\ -\alpha_4 & 0 & 0 & -\theta_3 & E & 0 \\ -\alpha_5 & 0 & 0 & 0 & -\theta_4 & F \end{pmatrix} \tag{34}$$

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Where

$A = \mu + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$

$B = \mu + \theta_1 + k$

$C = \mu + \theta_2 + \delta_1 + \gamma_1$

$D = \mu + \theta_3 + \delta_2 + \gamma_3$

$E = \mu + \theta_4 + \delta_3 + \gamma_3$

$F = \mu + \delta_4 + \gamma_4$

$$V^{-1} = \frac{1}{ABCDEF}\begin{pmatrix} BCDEF & 0 & 0 & 0 & 0 & 0 \\ \propto_1 CDEF & ACDEF & 0 & 0 & 0 & 0 \\ \propto_1 \theta_1 DEF & A\theta_1 DEF & ABDEF & 0 & 0 & 0 \\ \propto_1 \theta_1\theta_2 EF & A\theta_1\theta_2 EF & AB\theta_2 EF & ABCEF & 0 & 0 \\ \propto_1 \theta_1\theta_2\theta_3 F & A\theta_1\theta_2\theta_3 F & AB\theta_2\theta_3 F & ABC\theta_3 F & ABCDF & 0 \\ \propto_1 \theta_1\theta_2\theta_3\theta_4 & A\theta_1\theta_2\theta_3\theta_4 & AB\theta_2\theta_3\theta_4 & ABC\theta_3\theta_4 & ABCD\theta_4 & ABCDE \end{pmatrix}$$

(35)

$$FV^{-1} = \begin{pmatrix} 0 & \beta & \beta\eta & \beta\eta\eta_1 & \beta\eta\eta_2 & \beta\eta\eta_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{A} & 0 & 0 & 0 & 0 & 0 \\ \frac{\propto_1}{AB} & \frac{1}{B} & 0 & 0 & 0 & 0 \\ \frac{\propto_1\theta_1}{ABC} & \frac{\theta_1}{BC} & \frac{1}{C} & 0 & 0 & 0 \\ \frac{\propto_1\theta_1\theta_2}{ABCD} & \frac{\theta_1\theta_2}{BCD} & \frac{\theta_2}{CD} & \frac{1}{D} & 0 & 0 \\ \frac{\propto_1\theta_1\theta_2\theta_3}{ABCDE} & \frac{\theta_1\theta_2\theta_3}{BCDE} & \frac{\theta_2\theta_3}{CDE} & \frac{\theta_3}{DE} & \frac{1}{E} & 0 \\ \frac{\propto_1\theta_1\theta_2\theta_3\theta_4}{ABCDEF} & \frac{\theta_1\theta_2\theta_3\theta_4}{BCDEF} & \frac{\theta_2\theta_3\theta_4}{CDEF} & \frac{\theta_3\theta_4}{DEF} & \frac{\theta_4}{EF} & \frac{1}{F} \end{pmatrix}$$

(36)

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Where

$$a_{11} = \frac{\beta\alpha_1}{AB} + \frac{\beta\eta\alpha_1\theta_1}{ABC} + \frac{\beta\eta\eta_1\alpha_1\theta_1\theta_2}{ABCD} + \frac{\beta\eta\eta_2\alpha_1\theta_1\theta_2\theta_3}{ABCDE} + \frac{\beta\eta\eta_3\alpha_1\theta_1\theta_2\theta_3\theta_4}{ABCDEF}$$

$$a_{12} = \frac{\beta}{B} + \frac{\beta\eta\theta_1}{BC} + \frac{\beta\eta\eta_1\theta_1\theta_2}{BCD} + \frac{\beta\eta\eta_2\theta_1\theta_2\theta_3}{BCDE} + \frac{\beta\eta\eta_3\theta_1\theta_2\theta_3\theta_4}{BCDEF}$$

$$a_{13} = \frac{\beta\eta}{C} + \frac{\beta\eta\eta_1\theta_2}{CD} + \frac{\beta\eta\eta_2\theta_2\theta_3}{CDE} + \frac{\beta\eta\eta_3\theta_2\theta_3\theta_4}{ABCDE}$$

$$a_{14} = \frac{\beta\eta\eta_1}{D} + \frac{\beta\eta\eta_2\theta_3}{DE} + \frac{\beta\eta\eta_3\theta_3\theta_4}{DEF} \quad (37)$$

$$a_{15} = \frac{\beta\eta\eta_2}{E} + \frac{\beta\eta\eta_3\theta_4}{DE}$$

$$a_{16} = \frac{\beta\eta\eta_3}{F}$$

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Therefore, the eigenvalue of FV^{-1} for the equation $Z = \ | \mathrm{FV}^{-1} - I\lambda| = 0$ gives

$$\mathcal{R}_0 = \beta \left[\frac{\alpha_1}{AB} + \frac{\eta\alpha_1\theta_1}{ABC} + \frac{\eta\eta_1\alpha_1\theta_1\theta_2}{ABCD} + \frac{\eta\eta_2\alpha_1\theta_1\theta_2\theta_3}{ABCDE} + \frac{\eta\eta_3\alpha_1\theta_1\theta_2\theta_3\theta_4}{ABCDEF}\right] \quad (38)$$

Therefore, equation (38) is the basic reproduction number of the model in equation (1-11)

5. Discussions

The essence of the formulated model in figure 1 with the model equations (1 - 11), explain the progression of admitted students from the point of entering the university and undergoing various test on illicit drug use, with the various stages of drug addiction as it relates to the variables and parameters in Table1 and Table 2. This show the reality that illicit substance use among the territory institution students with various stages of drug addiction will promote many vices among youth in any giving population as it to support (Eguda *et al*, 2022) therefore, the need to for prevention among the students and youths becomes very necessary Hafiruddin (2019). By the process of evaluate and derivation we obtain R_0 seen in equation (38). We use this R_0 as a key variable in our research to determine if substance addiction can be eradicated or will continue to be pervasive in the community. R_0 is a threshold below which the generation of secondary cases is insufficient to maintain the spread of illicit substance in tertiary institution. If $R_0 < 1$, the number of initiated individuals will decrease from one generation to next and the illicit substance use will die out, and if $R_0 > 1$ the number of initiated individuals will increase from one generation to the next and the spread of illicit substances will persist. (Driesche and Watmough, 2022). From the analysis of the model, it is clear that the model satisfies stability, positivity, drug free equilibrium and other properties. The model can also become endemic as see in equation (22 to 33).

Whether experimental, light, moderate or heavy users, those who have been treated need to be fully cut off from their former friends or gangs to avoid relapsing into damage class, incarceration and addiction. To keep the number of students at a minimum or zero, it is crucial for governmental, non-governmental, and religious groups to get involved in this campaign against illicit drug abuse. This can be improved by all parties involved stepping up their public education efforts on the risks drug usage poses to human health (Chinnadurai, 2020)

6. Conclusion

This study of drug addiction behavior among university students uses an eleven compartmental model that was developed using ordinary differential equations. When the fundamental reproductive number is smaller than unity, it was demonstrated that the model exhibited a drug-free equilibrium point that was locally asymptotically stable.

This indicates that if all parties involved do not adequately implement control mechanisms, drug addiction will continue to affect the human population as seen in equation (22 to 33).

References

Arriola, L (2009): "Sensitivity analysis for quantifying uncertainty in mathematical models," in *Mathematical and Statistical Estimation Approaches in Epidemiology*, Chowell, G., J. M. Hyman, L. M. A. Bettencourt, and C. Castillo-Chavez, Eds., pp. 195–248, Springer, New York, NY, USA, 2009.

Abdallah, M.M. (2019). Text of the pre-event press briefing to flag-off the commemoration of 2019 international day against drug abuse and illicit drug trafficking. Available at:https:// www.vanguardngr.com/2019/06/drug-abuse-ndlea-takes-campaign-to-s reets-markets-schools-chairman/(Accessed 12/09/2020).

Akinkuotu E. (2020): FG to stop open sale of drugs next month -PCN. Available at: https://punchng.com/fg-to-stop-open-sale-of-drugs-next-month-pcn/(Accessed March 21, 2020).

Betterhealth, (2022): https:// www.betterhealth.vic.gov.au/health/ healthyliving/pregnancy-medication-drugs-and-alcohol

Codeine. NAFDAC Syrup crisis. Available at:https://www.nafdac.gov.ng/codeine-syrupcrisis-nafdac-shuts-down-peace-standard-pharmaceutical-limited-bioraj-pharmaceutical-limited-both-in-ilorin-and-emzor-pharmaceuticals-ind-ltd-lagos/ (Accessed March 21, 2020).

Crocq M.A.(2003): Alcohol, nicotine, caffeine, and mental disorders. Dialogues Clin Neurosci. 2003 Jun;5(2):175-85. doi: 10.31887/DCNS.2003.5.2/macrocq. PMID: 22033899; PMCID: PMC3181622.

Fatokun Johnson Oladele; Abegye Shehu Yakubu; Oduwole Henry Kehinde (2020) Numerical Analysis of a prey harvesting of One Predator – Two Prey Dynamics in Tropical Zone Conference in Mathematics, Computer Engineering and Computer Science, ICMCECS 2020, 2020, 9077655 SCOPUS

FindLaw (2019) ; Drug Cultivation and Manu facturing by FindLaw team https:// www.findlaw.com/criminal/criminal-charges/drug-manufacturing-and-cultivation.html

Fikiri .L.M and Dmirty .K (2019): Mathematical modelling on the effect of drug abuse to the societies. Journal of Annals of Pure and Applied Mathematics 19(1),21-35doi:10.22457apam.591v19n1a4

Gary R. Collin (2007) Christian counselling: A comprehensive guide, third edition ISBN-139-781-4185-0329-1 printed in the United States-of-America.

http://hzpaperjgzi.moodleesvnemesio.info/ short-essay-on-drug-addiction.html

http://evirtualguru.com/essay-on-drug-abuse-complete-essay-for-class-10-class-12-and-graduation-and-other-classes/

https://www.chegg.com/homework-help/ definitions/model-formulation-18

Interlandi J. (2008):What Addicts Need. Newsweek. 2008: p. 36-42.

Klantschnig G. (2009). The politics of law enforcement in Nigeria: lessons from the war on drugs. *J Mod Afr Stud*, 47, 529–549. 10.1017/s0022278x09990036

Lyman, M. D. (2016). *Drugs in society: Causes, concepts, and control*. Routledge.

McLellan, A.T (2017): Substance Misuse and Substance use Disorders: Why do they Matter in Healthcare? Trans Am Clin Climatol Assoc. 2017;128:112-130. PMID: 28790493; PMCID: PMC5525418.

Mushayabasa, S. and Bhunu, C. P. (2014): "Hepatitis C virus and intravenous drug misuse: a modeling approach," *International Journal of Biomathematics*, vol. 7, no. 1, Article ID 1450006, 2014.

Mushayabasa, S. (2014:) "On the role of optimal counseling and antiviral therapy on controlling HCV among intravenous drug misusers," *Journal of Computational Medicine*, vol. 2014, Article ID 347827, 7 pages, 2014.

Mushayabasa, S. (2015): "The role of optimal intervention strategies on controlling excessive alcohol drinking and its adverse health effects," *Journal of Applied Mathematics*, vol. 2015, Article ID 238784, 11 pages, 2015.

National Drug Laws Enforcement Agency (2020). Available at: https://nigeria.tradeportal.org/media/NDLEA%20Act.pdf (Accessed 13/11/2020).

National drug control master plan-2015 to 2019. Available at: https://www.unodc.org/documents/nigeria/Drug- response/NDCMP_book_1706152.pdf.(Accessed 21/03/2020).

Ogunsola S. O., Fajemisin, E. A. , Aiyenuro, A.E., Tunde, A. A. (2020). Experiences and projections for Drug Abuse Sensitization and Eradication among youths in South West, Nigeria. Journal of Alcoholism Drug Abuse & Substance Dependence, 6, 018.

Pharmacists Council of Nigeria (2020). Council of Nigeria. Available at:http://www.pcn.gov.ng/index.php (Accessed January 25, 2020).

Preuss, C.V., Kalava .A and King, K.C.(2023): Prescription of Controlled Substances: Benefits and Risks. [Updated 2023 Apr 29]. In: StatPearls [Internet]. Treasure Island (FL):StatPearls Publishing; 2023 Jan- .Available from: https://www.ncbi.nlm.nih.gov/books/NBK537318/

Reuters.(2020) Nigeria recalls 2.4 million. Available at: https://www.reuters.com/article/us- nigeria-drugs/nigeria-recalls-24-million-bottles-of-codeine-cough-syrup-idUSKBN1KL207 (Accessed 21 March, 2020).

Sahebi-Fakhrabad, A., Sadeghi, A. H., and Handfield, R. (2023). Evaluating state-level prescription drug monitoring program (PDMP) and pill mill effects on opioid consumption in pharmaceutical supply chain. In *Healthcare* (Vol. 11, No. 3, p. 437). MDPI.

Shamabadi, A., Ahmadzade, A., Pirahesh, K., Hasanzadeh, A., and Asadigandomani, H. (2023). Suicidality risk after using cannabis and cannabinoids: An umbrella review. *Dialogues in clinical neuroscience*, *25* (1), 50-63.

Sharma, S., and Samanta, G. P. (2015). Analysis of a drinking epidemic model. International Journal of Dynamics and Control, 3(3), 288–305. doi:10.1007/s40435-015-0151-

Sharma, J., Garima, N. S., & Sharma, P. (2022). Drug Abuse in India: A Review. *Contemporary Advances in Science & Technology-V: Pharmaceutical Chemistry and Technology*, 19.

Steady M. and Gift T.(2015): Modeling Illicit Drug Use Dynamics and Its Optimal Control Analysis. Journal of Computational and Mathematical Methods in Medicine, volume 2015.https://doi.org/10.1155/2015/383154

The Guardian (2019). Buhari receives report of marwa's presidential committee on drug abuse. Available at: https://guardian.ng/news/buhari-receives-report-of-marwas presidential-committee-on-drug-abuse/ (Accessed March 21, 2020).

Van den Driessche, P. and Watmough, J. (2002): "Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission," *Mathematical Biosciences*, vol.180, pp. 29–48, 2002.

Yakubu R, Isa A.M, Abubakar I.J, Oreagba I, Awaisu A. (2020). *Drug safety in developing countries*. Amsterdam, Netherlands: Elsevier, 525–556.

Williams, D. R. (2008). The health of men: structured inequalities and opportunties. *American journal of public health*, *98*(Supplement_1), S150-S157.