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APPLICATION OF THE MIXED AUTOREGRESSIVE MOVING AVERAGE TIME SERIES MODEL TO TEMPERATURE PATTERN IN LAGOS, NIGERIA

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ABSTRACT

Background: There is the need to take some precautions that will help to reduce the rates at which temperature increases which usually leads to global warming, that is the gradual rise in the mean temperature of the earth and its oceans.
Objectives: The study examines temperature pattern in Lagos state using monthly data for 114 years (1901-2014).
Methods: The univariate Box-Jenkins Autoregressive Moving Average (ARMA) model methodology and the maximum likelihood estimation method were used to obtain the parameters of the Mixed ARMA (MARMA) model of order (P,Q). This was derived from two independent ARMA models of order (p_1, q_1) and (p_2, q_2); where $p=p_1+p_2$ and $Q = max(p_1 + q_2, p_2 + q_1)$. The Lagrange multiplier was used to test for linearity and Augmented Dickey Fuller test for stationarity. Forecast values from the optimum model were compared with the original rainfall series using the Mean Absolute Error (MAE).
Results: The results showed that the data is linear, stationary and non-seasonal. ARMA models (5, 1) and (5, 2) emerged as the best models among others and are then used to obtain the fitted MARMA model (10, 7). Monte Carlo simulation was carried out using the MARMA model at different sample sizes n=250, n=500 and n=1000, each replicated 50 times. Each forecast value gave close estimates to the real temperature values with MAE=1.41535.
Conclusions: The fitted MARMA model performed more excellently than the independent ARMA models in studying the behavior of temperature data and forecasting its future values.

Keywords: Temperature, Monte Carlo, independent ARMA, Mixed ARMA, Fitted MARMA

INTRODUCTION

Temperature is a physical quantity that expresses the degree of hotness or coldness of an object and usually measured with a thermometer historically calibrated in various temperature scales and units of measurement. The most commonly used scales are the Celsius scale, denoted by °C, the Fahrenheit scale (°F), and the Kelvin scale. The kelvin (K) is the unit of temperature in the International System of Units (SI), in which temperature is one of the seven fundamental base quantities". There is the need to take some precautions that will help to reduce the rates at which temperature increases which usually leads to global warming, that is the gradual rise in the mean temperature of the earth and its oceans. The increase in temperature may appear minute but has a great impact. With increase in temperature, there will be a global increase in drought conditions, prolonged droughts, decreased water supplies due to evapotranspiration and an increase in urban and agricultural demand (Oscar, 2014). Studying temperature changes is therefore vital for the Nigerian economy as the nation is gradually shifting focus to agriculture. For the agricultural sector of the nation to survive and thrive, climatic study on temperature is very vital and therefore cannot be over emphasized. As a result of the fact that the rate at which temperature continues to affect plant and human activities on earth through changes in land surface properties as a result of global warming and a host of other factors is at an alarming rate, it poses a major global challenge to human growth and peaceful development. Hence, there is the need to investigate this pattern and obtain a statistical model that best describes the behavior of the series in order to address this problem. Many researchers gave considered application of independent as well as mixed models and discovered that mixed models in most cases perform better than independent models. Granger and Morris (1976) discussed how more complicated time series models can arise from an aggregation of simpler models. For example, addition of *N* independent Autoregressive models of order 1, AR (1) can produce the ARMA(*N,N* − 1) model. This was a starting point to examine aggregation of infinitely many simple AR (1) models with *random* coefficients, which can result in a time series with long memory, Granger (1980) and the earlier contribution by Robinson (1978). The approach was later generalized by a number of authors like Oppenheim and Viano (2004) and Zaffaroni (2004).

METHODOLOGY

The Box-Jenkins method is a modelling strategy for pure time series and was proposed by Box *et al* (1976) is made up of four stages namely; identification, estimation, diagnostic checking and forecasting. It is an iterative process such that as new information is gained during diagnostics, one can circle back to the first stage and incorporate that into new model classes. Forecasting is done when the model adequacy has been verified. Shittu and Yaya (2016).

The ARMA (p, q) series $\{X_t\}$is given by;

$$[\phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p}] + [e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots + \theta_q e_{t-q}] \quad (1)$$

Using the backward-shift operator;

$$\phi_p(B) X_t = \theta_q(B) e_t \quad (2)$$

where: $\phi_p(B) = 1 - \phi_1 B - \ldots - \phi_p B^p$

and $\theta_q(B) = 1 - \theta_1 B - \ldots - \theta_q B^q$

Equation 1 requires that the roots of $\phi_p(B) = 0$ lie outside the unit circle for the process to be stationary and the roots of $\theta_q(B) = 0$ lie outside the unit circle for invertibility.

The stationary and invertible ARMA process can then be written in a pure autoregressive representation as;

$$\pi(B) X_t = e_t \quad (3)$$

where;

$$\pi(B) = \frac{\phi_p(B)}{\theta_q(B)} = (1 - \pi_1 B - \pi_2 B^2 - \ldots)$$

and in a pure moving average representation as;

representation as;

$$X_t = \psi(B) e_t$$

where;

$$\psi(B) = \frac{\theta_q(B)}{\phi_p(B)} = (1 + \psi_1 B + \psi_2 B^2 + \ldots)$$

To derive the Autocorrelation function (ACF) of the ARMA (p, q) process, we rewrite as;

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \ldots - \theta_q e_{t-q} \quad (4)$$

Multiplying both sides by X_{t-k}, we have;

$$X_{t-k} X_t = \phi_1 X_{t-k} X_{t-1} + \phi_2 X_{t-k} X_{t-2} + \ldots + \phi_p X_{t-k} X_{t-p} + e_t - \theta_1 X_{t-k} e_{t-1} - \theta_2 X_{t-k} e_{t-2} - \ldots - \theta_q X_{t-k} e_{t-q} \quad (5)$$

Taking expectation on both sides, we have;

$$\gamma_k = \phi_1\gamma_{k-1} + \ldots + \phi_p\gamma_{k-p} + E(X_{t-k}e_t) - \theta_1 E(X_{t-k}e_{t-1}) - \ldots - \theta_q E(X_{t-k}e_{t-q}) \quad (6)$$

Since $E(X_{t-k}e_{t-i}) = 0$ for all $k > i$, then;

$\gamma_k = \phi_1\gamma_{k-1} + \ldots + \phi_p\gamma_{k-p}$ and

$\rho_k = \phi_1\rho_{k-1} + \ldots + \phi_p\rho_{k-p}$ for all $k \geq (q+1)$

The autocorrelation function of an ARMA (p,q) model tails off after lag q as for the AR(p) process, which depends only on the autoregressive parameters in the model. But the first q autocorrelations $\rho_q, \rho_{q-1}, \ldots, \rho_1$ depend on both autoregressive and moving average parameters in the model and serve as initial values for the pattern.

Since the Moving Average (MA) process is an integral part of the ARMA process, the Partial Autocorrelation Function (PACF) of the ARMA process is also a mixture of exponential decays and/or damped sine waves depending on the roots of $\phi(B) = 0$ and $\theta(B) = 0$.

The mixed ARMA model is obtained as a result of following theorem.

Theorem: Given two independent ARMA series, X_t, and Y_t are with

$$X_t = ARMA(p_1, q_1)\,,\ Y_t = ARMA(p_2, q_2) \quad (7)$$

Then their sum is

$$Z_t = X_t + Y_t = ARMA(P, Q) \quad 7$$

where;

$P = p_1 + p_2$, and

$Q = max(p_1 + q_2, p_2 + q_1)$

This implies that the sum of two independent ARMA processes is also ARMA which is expected to perform better than the individual AR, MA and independent ARMA processes. Then the observed series will be regarded as Mixed Autoregressive Moving Average, MARMA(P, Q) model.

Linearity Test

The Lagrange multiplier test of linearity test, put together by Luukkonen *et al.* (1988) was employed to establish the linearity of this process. The test is aimed at assessing the null hypothesis of linearity against the alternative hypothesis of specific non-linear models. The use of either parametric or non-parametric methods can be used to achieve the task. The model hypothesized is;

$$X_t = \sum_{i=1}^{p}\phi_i X_{t-i} + \sum_{i=1}^{q}\theta_i\varepsilon_{t-i} + f(\beta, X_{t-i} + \ldots + X_{t-p}, \varepsilon_{t-1}, \ldots, \varepsilon_{t-q}) + \varepsilon_t \quad (8)$$

The model is made up of a linear (ARMA) part and a non-linear part *f* which depends on an unknown parameter β. The essence of the test for linearity is to assess whether β=0 and the Lagrange multiplier will be used to achieve this.

Let L be the likelihood function which depends on a univariate parameter θ and let x be the data.

The score U (θ) is defined as;

$$\mathrm{U}\,(\theta) = \frac{\partial \mathrm{Log}\mathrm{L}(\theta|x)}{\partial\theta} \quad (9)$$

The Fisher information is;

$$\mathrm{I}(\theta) = -\mathrm{E}\left[\frac{\partial^2}{\partial\theta^2}\log L(X;\theta) : \theta\right] \quad (10)$$

With the hypotheses;

$\mathrm{H}_0 : \theta = \theta_0$

$\mathrm{H}_1 : \theta \neq \theta_0$

And test statistic: $S(\theta_0) = \frac{U(\theta_0)^2}{\mathrm{I}(\theta_0)}$

Which has an asymptotic distribution of χ^2 when H_0 is true.

MAXIMUM LIKELIHOOD METHOD

This study made use of Maximum likelihood Estimation method because of its strength over the Jacknife and the Least square methods. Dickey and Fuller (1979). For the ARMA (p, q) model and n effective number of observations; the log-likelihood function is;

$$\ln L = \frac{-n}{2}\ln 2\pi\sigma_{\varepsilon}^{2} - \frac{1}{2\sigma_{\varepsilon}^{2}} S(\phi, \mu, \theta) \quad (11)$$

Maximizing (3) with respect to ϕ, μ, θ and we have,

$$\ln L = \frac{-n}{2}\ln \sigma_{\varepsilon}^{2} - \frac{n}{2}(1 + \ln 2\pi) \quad (12)$$

The optimal order is determined by the value for which (AIC) is minimum. Akaike (1974).

The likelihood function is the joint density function of the data, but treated as a function of the unknown parameters, given the observed data $X_1,\ldots,X_n$

For the models under study, the likelihood L is a function of the $\emptyset's$, $\theta's$, and σ_e^2 given the observed $X_1,\ldots X_n$

$$C_k = \frac{1}{N}\sum_{i=1}^{N-k}(X_t - \bar{x})(X_{t-k} - \bar{x})k = 0,1,2,\ldots,n-1 \quad (13)$$

Or

$$C'_k = \frac{1}{N-k}\sum_{i=1}^{N-k}(X_t - \bar{x})(X_{t-k} - \bar{x}) \quad (14)$$

It can be shown that both of these estimators are biased. Thus:

$$_N C_k = \sum_{i=1}^{N-k}(X_t - \bar{x})(X_{t-k} - \bar{x})k$$

$$= \sum_{i=1}^{N-k}[(X_t - \mu)(\bar{x} - \mu)][(X_{t-k} - \mu) - (\bar{x} - \mu)$$

$$= \sum_{i=1}^{N-k}[(X_t - \mu)(\bar{x} - \mu)][(X_{t-k} - \mu) - (\bar{x} - \mu)$$

(15)

$$= \sum_{i=1}^{N-k}(X_t - \mu)(X_{t-k} - \mu) - (N-k)(\bar{x} - \mu)^2$$

where $\sum_{i=1}^{N-k}(X_t - \mu)$ and $\sum_{i=1}^{N-k}(X_{t-k} - \mu)$ and are approximated by $(N-k)(\bar{x}-\mu)$

$$C_k \equiv \frac{1}{N}\sum_{i=1}^{N-k}(X_t - \mu)(X_t - k - \mu) - \frac{N-k}{N}\operatorname{var}(\bar{x})$$

$$\therefore E(C_k) = \gamma_k - \left[\frac{N-k}{N}\right]\operatorname{var}(\bar{x}) \quad \equiv \gamma_k - \left[1 - \frac{k}{N}\right]\operatorname{var}(\bar{x}) \quad (16)$$

Forecasting

Predicting future values is one of the major objectives of time series analysis. It is all about looking into past values and making projections into the future based on a model that best describes the evolution of a series. Model parsimony as a forecasting procedure for ARMA model was summarized by Adhikari R. and Agrawal R.K. (2013). Forecasts are usually done based on the available data. Most of the time, forecast values do not always match with the actual values produced by the data.

For ARMA models as usual. For example in the one-step case, we have;

$$X_{n-1} = \phi_1 X_n + \phi_2 X_{n-1} + \ldots + \phi_p X_{n-p+1} + e_{n+1} + \theta_1 e_n + \theta_2 e_{n-1} + \ldots + \theta_q e_{n-q+1}$$

Therefore the optimal forecast is given by;

$$f_{n,1} = \phi_1 X_n + \phi_2 X_{n-1} + \ldots + \phi_p X_{n-p+1} + \theta_1 e_n + \theta_2 e_{n-1} + \ldots + \theta_q e_{n-q+1} \quad (17)$$

and the one-step forecast error is given by;

$$e_{n,1} = X_{n+1} - f_{n,1} = e_{n+1}$$

Therefore for optimal forecast, given $\{e\}_{t=n-q+1}^{n}$ which can be estimated by beginning with $f_{0,1} = 0$ and then recursively forming e_t using;

$$e_t = X_t - f_{t-1,1} \quad \{t = 1,2,\ldots,n\} \quad (18)$$

Table 1: Summary of Statistics

Attribute	var	n	mean	sd	median	Min	Max	Range	skewness	kur	s.e
Value	1	1368	26.91	1.81	26.61	21.9	31.57	9.67	0.38	-0.65	0.05

Mean Absolute Error (MAE)

The Mean Absolute Error (MAE) helps to calculate the rate at which forecast (X_i) and the actual (X) values differs.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|X_i - X| \qquad (19)$$

where $|X_i - X|$ is the absolute error and 'n', is the number of errors.

SOURCE OF DATA

The historical monthly climate data for this research are secondary data obtained from the Nigerian Meteorological Agency (NIMET), Lagos for the period of 1901 to 2015 and was analysed using the R i386 3.6.1 statistical package.

Result and Discussion

Table 1 shows the minimum and maximum values recorded, mean and range. Also, a positive skewness of 0.38 (a value <2) shows that the data is positively skewed and within the scope of normality. That is, the area under the normality curve should be approximately equal to 1. The kurtosis of negative -0.65 suggested it is not heavily tailed from the normal curve as indicated in the modals Density curve in figure 3.

Time and trend plots are also important ingredients of time series analysis. Ojo, (2012). Figure 1 shows the time plot of the monthly temperature between 1901-2014 which suggests that the data is stationary. Also, in figure 2 and figure 3, the ACF and PACF of the original data $\{X_t\}$, t =1, 2, ...1368, are as shown. They both did show a non-seasonal fluctuation of the series. Concentrating on the ACF of the original data, we note a slow decreasing trend in the ACF. This also indicates a stationary behaviour.

Stationarity Test

The Augmented Dickey Fuller (ADF) test was carried out in order to test for the presence of unit root in the series as follows.

Hypotheses: H_o: $\alpha_0=0$

H_1: $\alpha_0<0$ ($|\alpha_0|<0$)

α =0.05%

$$t_{\phi-1} = \frac{\hat{\phi}-1}{SE(\hat{\phi})}$$

We can test H_o with a t-test:
Calculated value=-16.716
p value = 0.01
Conclusion: It is 100%stationary

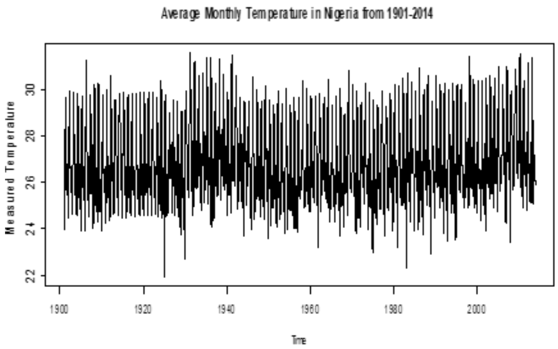

Fig. 1: Time plot of temperature in Nigeria from 1801-2014

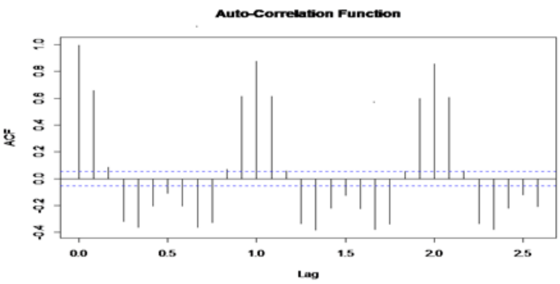

Fig. 2: Autocorrelation function (ACF) plot

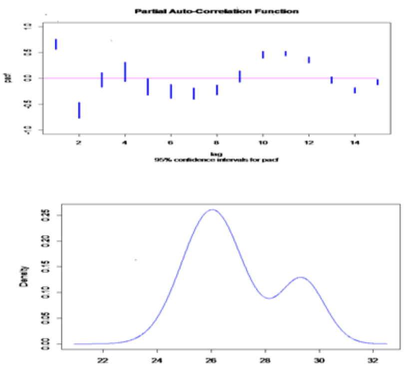

Fig. 3: Partial Autocorrelation function (PACF) plot and the Modal Density curve

Fitted ARMA models of different order (p, q) Independent ARMA Models of different order of p and q are fitted and their corresponding AIC, BIC as well as residual variances are recorded as shown in table 2. ARMA (5, 1) and ARMA (5,2) models have the least AIC and BIC and residual variance values. Hence they are selected to obtain the mixed ARMA (10, 7) given by:

where;

$P = p_1 + p_2, \; Q = MAX\,(p_1 + q_2, p_2 + q_1)$

$$\hat{X}_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + \phi_4 X_{t-4} + \phi_5 X_{t-5} + \phi_6 X_{t-6} + \phi_7 X_{t-7} + \phi_8 X_{t-8} + \phi_9 X_{t-9} + \phi_{10} X_{t-10} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \theta_4 \varepsilon_{t-4} + \theta_5 \varepsilon_{t-5} + \theta_6 \varepsilon_{t-6} + \theta_7 \varepsilon_{t-7}$$

where: ϕ_1 =0.1856, $\phi_2 = -$ 0.6663, $\phi_3 = -$ 0.1506, ϕ_4 =0.1353, $\phi_5 = -$ 0.0742, ϕ_6 =0.0509, = $\phi_7 = -$ 0.2587, $\phi_8 = -$ 0.8801 $\phi_9 = -$ 0.10577, ϕ_{10} =0.2273, θ_1 =0.2569, θ_2 =0.0575, θ_3 =0.5042, θ_4 =0.6149, $\theta_5 = -$ 0.6974, θ_6 =0.0469, $\theta_7 = -$ 0.8439.

Simulation Study

To be able to further establish the performance of our optimal model, simulated data of sample sizes n=250, 500 and 1000 are generated using the Monte Carlo technique. The results are as follows.

The procedure:

A data set of sample size, n = 250 values are generated for X_t and ε_t . The data set is subjected to the optimum model above and new set of parameters are estimated using the nonlinear least square method. The procedure is repeated 50 times (replicates) with different sets of X_t and ε_t . Averages are then obtained for each estimate and compared with the test values (hypothesized values). The entire procedure is then repeated for sample sizes n=500 and n=1000. From the results, we discover that:

The parameters of the simulated data compared favourably with the parameters of our Mixed ARMA (MARMA) model.

As the sample size increases (from n=250 to n=1000), the model further shows better performance with non-significance in difference for all the parameters.

Forecasting procedure for ARMA

For the purpose of in-sample forecast, we used the monthly data from 1901 to 2014 (amounting to 1368 monthly data points) were used for the analysis and estimation of the parameters our models. As shown in table 3 our optimal independent ARMA (5, 2) model and the MARMA (10, 7) models were used to predict the temperature values for the year 2015 because they have the least AIC and BIC values. The MARMA model performed better than the independent ARMA model by giving closer estimate of the actual value for each month of the year.

Similarly, out-sample forecast for the year 2016 was carried out using the optimal independent ARMA (5, 2) and the MARMA (10, 7) models to predict the temperature values for the year 2016. The results are as shown in table 4. We equally believe that the MARMA model would perform better than the independent ARMA model by giving closer estimate of the actual value for each month of the year.

Conclusion

Optimum independent ARMA models have been identified in this study of temperature pattern using their AIC and BIC. These could not independently model the temperature pattern excellently. Hence, the optimal ARMA (5, 2) and the resulting MARMA (10, 7) models were employed model which performed more excellently in studying the behavior of temperature pattern and forecasting its future values than the independent ARMA models. Hence, it would be a more useful tool for researchers carrying out analysis in the study of temperature and other related phenomena.

Table 2: Fitted ARMA models of different order (p, q)

S/N	MODEL	AIC	BIC	RESIDUAL VARIANCE
1	(1, 1)	469.5	485.1	0.8926
2	(2, 1)	639.7	660.5	1.5783
3	(1, 2)	289.5	310.4	0.7202
4	(3, 1)	162.6	188.7	0.6243
5	(2, 2)	141.4	167.4	0.3037
6	(3, 2)	142.4	173.7	0.4106
7	(3, 3)	108	144.4	0.2039
8	(1, 3)	252.8	278.8	0.6921
9	(2, 3)	112.4	143.7	0.2002
10	(4, 1)	151	182.2	0.5827
11	(4, 2)	144.2	180.7	0.5042
12	(4, 3)	146.1	187.8	0.6551
13	(1, 4)	154.7	186	0.6318
14	(2, 4)	106.4	142.8	0.1988
15	(3, 4)	108.1	149.8	0.4210
16	(4, 4)	367	67.5	0.3456
17	(5, 1)	19.1	55.6	0.0958
18	(5, 2)	-475.9	-434.2	0.00364
19	(5, 3)	963.3	1010.3	7.9036
20	(6, 3)	904.67	894.05	0.23417
21	(6, 2)	88.7	8332.38	0.56739

Table 3: Optimal in-sample forecast of 12 steps ahead prediction for the year 2015

Month	Actual values	MARMA	ARMA
1	24.1771	21.897	21.3965
2	28.1564	30.002	30.732
3	30.0905	28.456	28.0326
4	29.9188	28.6834	28.4902
5	30.0922	29.782	29.003
6	28.1705	26.9956	26.456
7	27.156	25.999	25.839
8	25.8361	23.782	23.498
9	26.1659	27.582	27.099
10	27.7909	26.545	26.290
11	26.8572	27.390	27.107
12	23.8596	21.762	21.673

Table 4: Optimal out-sample forecast of 12 steps ahead prediction for 2016

month	MARMA	ARMA
1	26.3965	26.0838
2	29.6636	29.385
3	30.5994	30.0326
4	29.3664	29.016
5	21.3920	21.114
6	24.2901	24.309
7	31.701	31.940
8	27.001	26.988
9	25.658	25.309
10	19.678	19.302
11	14.902	14.901
12	34.035	33.509

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