Development of a User's-Friendly Algorithm for Gravimetric Data Processing Using Visushrink Thresholds Technique in a Standalone Tool

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Abstract

Gravity is an efficient means of determining the shape and size of the Earth as well as the exploration of natural resources beneath the Earth's surface by examining the sudden variations in the fields (gravity anomalies) caused by the heterogeneous nature of Earth's materials. To satisfactorily and optimally separate the gravity anomalies into regional (deep sources) and residual (shallow sources) components has constrained the use of gravimetric data for geodetic and geophysical exploration. In this paper, an attempt was made to optimally separate gravity anomalies into regional and residual components for geodetic and geophysical surveying using the Visushrink thresholds technique in the Discrete Wavelet domain. Gravity anomalies over an area in the Gongola Basin of Nigeria were obtained and used. The method was found to yield a good Regional Residual Ratio of 22.3113db and a regional anomalies variance of 6.2545 which is smoother and better than the Regional and Residual Ratio of 19.4197dB and a regional anomalies variance of 6.6488 obtained from the least square collocation technique. In separating gravity anomalies into regional and residual components, the VisuShrink Thresholding technique of Discrete Wavelet Transformation is recommended.

Keywords: *Gravity, Wavelet Transformation, Standalone Tool, Visushrink and Geophysical Surveying.*

Introduction

The earth's gravity acceleration is the summation of gravitational and centrifugal accelerations of earth masses generated from the center of the earth to pull the external body towards its center.

The variations in the effect of these gravity fields of attraction are called gravity anomalies which is a summation of the regional gravity anomalies (data needed for geodetic surveying which includes geoid determination) and residual gravity anomalies (data used for geophysical surveying such as mineral exploration). Over the years, a series of sensitive gravimeters have been developed to measure the changes in the earth's gravity fields to achieve the goals of geodetic and geophysical surveying (Petho and Vass, 2009).

The seismic, electrical, and resistance methods of geophysical exploration are practically preferred to the gravimetric method because the complete separation of gravity anomalies into the regional and residual anomalies without interfering with each other is difficult (seems impossible). Methods such as graphical, polynomial fitting, and filtering methods, among others, have been attempted to solve this problem without probable success. Idowu (2005) used an optimum polynomial fitting to extract residual gravity anomalies from gravity anomalies but the technique was limited to profile data and the need for a-prior information of the gravity anomalies (like all other polynomial fitting methods); hence, the need to improve on the separation technique in other to have satisfactory results.

Further review of the literature showed that Albora and Ucan (2001), Eshaghzadeh (2016), and Mousavi et al (2013) carried out studies on gravity anomaly separation using the Wavelet Transformation method. The application of their results was considered good in the fields of Physics, Mathematics, Statistics, Geophysics, image denoising, radiometric signal analysis for medical purposes as well as solving some boundary value problems in Physical Geodesy.

However, more work has to be done to determine the possibility of this method to generate reliable data for geodetic and geophysical usage. Therefore, it is the aim of the paper to optimally separate gravity anomalies into regional and residual components reliable for geodetic and geophysical usage. In light of conserving the processing time, an algorithm with a userfriendly interface was developed to perform the task. The study area (Figure 1) is located at Gongola Basin of Nigeria from latitudes 10 14 22.5 N to 10 59 18.8 N and longitudes 10 08 50.1 E to 11 07 15.3 E.

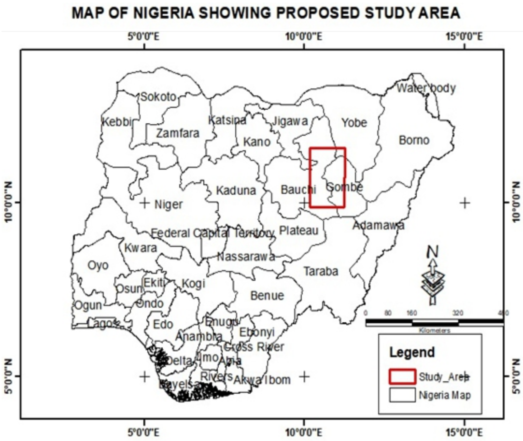

Figure 1: Map of Nigeria Showing the Proposed Study Area

Theoretical Analysis

Wavelet Transformation is a mathematical algorithm designed for image processing by decomposing the image signals into bands of low and high frequencies by scaling and dilating the signals. It gains applications in image compression, image smoothing, edge detection, and other electrical and medical applications.

The wavelet transformation is divided into two (the Continuous and Discrete Wavelet Transformation) based on its applications in the real sense based on how the scale parameter is discretized. Continuous Wavelet Transform (CWT) of a signal f(x) is defined as the inner product of the signal sequence with the specified analysing function, the wavelet family ($\Psi_{(x)}$), as expressed in Equation 1 by El Habiby, (2007).

The Discrete Wavelet Transform (DWT) is a versatile signal processing tool that decomposes signals into frequencies of varying amplitude over time domain. The discrete (Equation 3) uses exponential scales of base equal to 2 while the continuous uses the same exponential scale but less than 2. (The Mathworks Inc., 2015).

The Continuous Wavelet Transform can be mathematically expressed as,

$$Q_W(m, x_n) = \frac{1}{\sqrt{m}} \int_{-\infty}^{\infty} q(x)\Psi^*\left(\frac{x - x_n}{m}\right) dx \qquad (1)$$

The inverse is presented as;

$$q(x) = \frac{1}{C_\Psi} \iint_{-\infty}^{\infty} \frac{1}{\sqrt{m}} Q_W(m, x_n) \Psi\left(\frac{x - x_n}{m}\right) dx_n \frac{dm}{m^2} \tag{2}$$

Where m is the scaling that determines the oscillating behaviour of the particular analytic function, x_n is the shifting, C_Ψ is the admissibility constant, and Ψ^* is the complex conjugate of Ψ. The analyzing function Ψ(x) is not limited to the complex exponential as is the case of the Fourier transform.

The Discrete Wavelet Transform is mathematically expressed (El Habiby, 2007),

$$q(x) = \sum_{m \in Z} \sum_{n \in Z} d_n^m \Psi_{m,n}^{(\lambda_0, x_0)}(x) \tag{3}$$

Where,

$$d_n^m = \langle q(x), \Psi_{m,n}^{(\lambda_0, x_0)} \rangle = \sum_n \mathrm{q(x)} \Psi_{m,n}^{(\lambda_0, x_0)}(x) \tag{4}$$

$$\Psi_{m,n}^{(\lambda_0, x_0)}(x) = \lambda_0^{-m/2} \Psi(\lambda_0^{-m} x - n x_0) \tag{5}$$

d_n^m are detailing coefficients, $\Psi_{m,n}$ is the wavelet function generated from the original mother wavelet function $\Psi \in L^2(\Re)$, λ_0 is the scale space parameter, x_0 is the translation space parameter, m is the scale or level of decomposition integer, and n is the shifting or translation integer.

The Discrete Wavelet Transformation (DWT) was used to separate the field into the distinguished components belonging to different scales and horizontal positions, that is, into regional and residual components. DWT is used as a powerful tool in filtering and de-noising and for gravity anomalies separation. The continuous Wavelet Transformation (CWT) is used to characterize the sources of the separated field effects (Mousavi and Ardestani., 2016); hence the use of the DWT for gravity anomalies separation.

Methodology

Transformation method was used to separate gravity anomalies by adopting the Daubechies3 wavelet function and decomposition level 3. This process involves the arrangement of the gravity anomalies in the square matrix, transformation of the matrix into spectrums of approximate and detailed coefficients, estimation of a threshold value for the separation of approximate coefficients from the detailed coefficients, reconstruction of the coefficients 2-D Inverse Discrete Wavelet Transformation (2-D IDWT) at decomposition level 3 to obtain the regional and residual gravity anomalies.

Data Acquisition

Secondary data set of gravity anomalies over a mineral prospecting site at the Gongola Basin of Nigeria as obtained. This consists of rectangular coordinates (Easting and Northing) and gravity anomalies of observed gravity stations within the study area.

In addition, the results of the separations of acquired gravity anomalies into the regional and residual anomalies using the least squares collocation technique were collected to be used as standard values to determine the reliability of the method used in the research. Table 1 below shows the sample of acquired gravity anomalies data.

Table I: Sample of Acquired Gravity Anomalies Data

Sta.	Eastings (m)	Northings (m)	Gravity Anomalies (mGal)
1	658003.033	1129134.467	-9.9086
2	660709.700	1129134.467	-12.5655
3	663416.367	1129134.467	-14.8181
4	666123.033	1129134.467	-14.8616
5	668829.700	1129134.467	-13.1694
6	671536.367	1129134.467	-12.0838
7	674243.033	1129134.467	-11.9359
8	676949.700	1129134.467	-11.9013
9	679656.367	1129134.467	-11.7009
10	682363.033	1129134.467	-11.525

Data Quality

Gravity data over the study area was acquired using LaCoste and Romberg model G gravity meter that measured up to 0.01mGal with a drift error of 0.006mGal was used to carry out the observations. Also, on the average of 10 repeated observations taken at each gravity station, the standard deviation was less than 0.013 mGal at each station.

The average standard error of the gravity base stations was 0.015mGal while that of other gravity stations was better than 0.016mGal. Based on the above, the data acquired for this research is valid, reliable, and of good quality based on the specifications of the instrument manufacturer.

Data Processing

Generally, there are three main steps involved in the data processing to obtain the regional and residual gravity anomalies. These are; gravity data reduction to obtain reduced gravity values; computation of gravity anomalies from reduced gravity values by subtracting the normal gravity values at each gravity station, as shown in Equation (6) (Idowu, 2005); and the separation of gravity anomalies into regional and residual gravity anomalies.

$$\Delta g_a = g_{obs} + \left(-\delta g_D \pm \delta g_{ET} \pm \delta g_{EC} + \delta g_F - \delta g_B + \delta g_{TC} + \delta g_L \pm \delta g_I \right) - \gamma \qquad (6)$$

$$\gamma = g_e (1 + \underline{k} \sin^2 \phi) / \sqrt{1 - e^2 \sin^2 \phi}$$

$$\underline{k} = (b_p g_p - a_e g_e) / a_e g_e$$

Δg_a is the gravity anomalies at the observed point; g_{obs} is the observed gravity; δg_D is the drift correction; δg_{ET} is the earth tide correction; δg_{EC} is the Eotvos correction; δg_F is the Free-air correction; δg_B is the Bouguer correction; δg_{TC} is the terrain correction; δg_L is the Latitude correction; δg_I is the Isostatic Correction; γ is the normal gravity at the place of observation; g_e is the gravity at the equator; g_p is the gravity at the pole; e is the first eccentricity of the ellipsoid; ϕ is the latitude at the place of observation;a is the equatorial earth's radius, semi-major axis of the ellipsoid; and b is the polar earth's radius, semi-minor axis of the ellipsoid.

By subtracting the Normal gravity γ from the corrected/reduced gravity, the gravity anomalies at points of observation are obtained. At the successful reduction of the gravity anomalies, an interpolated surface was developed using the Kriging method, a Geostatistical analysis tool. The Kriging Geostatistical tool is mathematically presented (Abubakar & Idowu, 2013; Abubakar & Idowu, 2014).

$$\begin{aligned} \sigma_k^2(x_0) &= Var[Z(x_0) - Z(x)] \\ &= \sum_{i-1}^{n} \sum_{i-1}^{n} \omega_i(x_0)\omega_j(x_0)c(x_i x_j) + Var[Z(x)] \\ &\quad - 2\sum_{i-1}^{n} \omega_j(x_0)c(x_i x_j) \end{aligned} \qquad (7)$$

The Kriging variance given as;

$$Var\hat{Z}(x_o) = Var\left(\sum_{i-1}^{n} \omega_i Z(x_i)\right) = \sum_{i-1}^{n}\sum_{i-1}^{n} \omega_i \omega_j c(x_i x_j) \quad (8)$$

Where, $\hat{Z}(x_o)$ is the Kriging predictor (Abubakar and Idowu, 2014).

Separation of Gravity Anomalies into Regional and Residual Anomalies

The acquired gravity anomalies were arranged in a squared matrix. The anomalies were separated using the 2D Discrete Wavelet Transform. The Two-dimensional Discrete Wavelet Transform (2D-DWT) is similar to the multi-resolution approximation expression. In practice, multi-resolution analysis is done by using four-channel filter banks (for each level of decomposition) which are composed of low-pass and high-pass filter banks, and each filter bank is then sampled at a half rate (1/2 down sampling) of the previous frequency (Mastriani, 2009).

The decomposition in DWT consists of four frequency channels for each level of decomposition (Mastriani, 2009). For example, for the decomposition level, the Discrete Wavelet Transformation gives Approximation Coefficients (LL3), Vertical Detail Coefficients (LH3), Horizontal Detail Coefficients (HL3), and Diagonal Detail Coefficients (HH3) as illustrated in Figure 2.

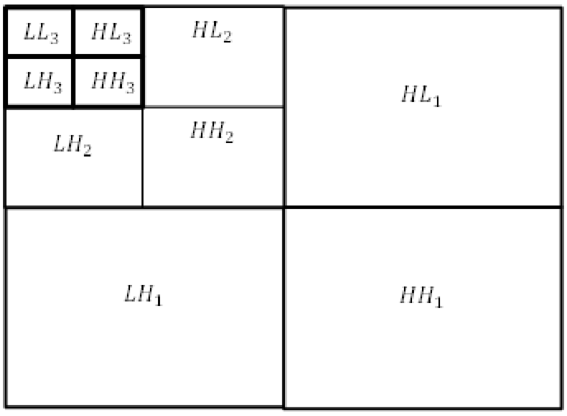

Figure 2: Discrete Wavelet Transformation Decomposition Structure at Level 3

The step-by-step procedure adopted for the separation is as follows,

1. Arrange gravity anomalies data in a squared matrix.
2. Decompose the gravity anomalies to level 3 by using the Daubechies (db3) 2-D Discrete Wavelet Transform (2D DWT). The result of this decomposition is a column vector matrix of coefficients, which consists of Approximate (LL) Coefficients and Detail Coefficients (LH, HL and HH) (equation 3).
3. Calculate the median absolute deviation (σ) on the diagonal detail coefficients (HH). The median is calculated from the absolute value of the diagonal detail

coefficients. The equation for the median absolute deviation as discussed by Siegel (1999) is given below:

$$\sigma = \frac{Absolute\ Median\ (HH_3)}{0.6745} \quad (9)$$

This estimated result is a suitable value that best represents the standard deviation of Gaussian white noise in the gravity anomalies.

4. Compute the threshold value (λ) using the estimated median absolute deviation (σ). The modified version of the equation as discussed by Siegel (1999) was used. The equation is:

$$Threshold\ Value\ (\lambda) = \sigma\sqrt{2log_e(N)} \quad (10)$$

Where, N is the length of the column vector matrix (i.e., the length combination of LL, LH, HL and HH) and σ is the median absolute deviation. The result of this is a value that represents the boundary of separation of the residual gravity anomalies from the regional gravity anomalies.

5. Application of the threshold value to the detail coefficients (i.e. LH_3, HL_3 and HH_3). The soft thresholding application technique (equation 11) was used because it better reduces the gravity anomalies variance with respect to the estimated threshold value. It sets any coefficient less than or equal to the threshold value to zero (Siegel, 1999).

$$\delta_\lambda = \begin{cases} x(t) - \lambda, & if\ x(t) > \lambda \\ 0, & if\ |x(t)| \leq \lambda \\ x(t) + \lambda, & if\ x(t) < -\lambda \end{cases} \quad (11)$$

Where, $x(t)$is the transformation coefficient and λ is the threshold value. The threshold value is subtracted from any coefficient that is greater than it. This therefore reduces the high frequencies to zero. In other words, the residual gravity anomalies coefficients were reduced to zero, leaving only the regional gravity anomalies coefficients.

However, the regional gravity anomalies coefficients need to be converted back to gravity values. This is done by reconstructing the separated regional gravity anomalies coefficients using the inverse 2-D Discrete Wavelet Transform (2D IDWT) at level 3 (the same level as the initial decomposition).

6. Subtract the converted regional gravity anomalies from the gravity anomalies to give the residual gravity anomalies.

Standalone Application Package

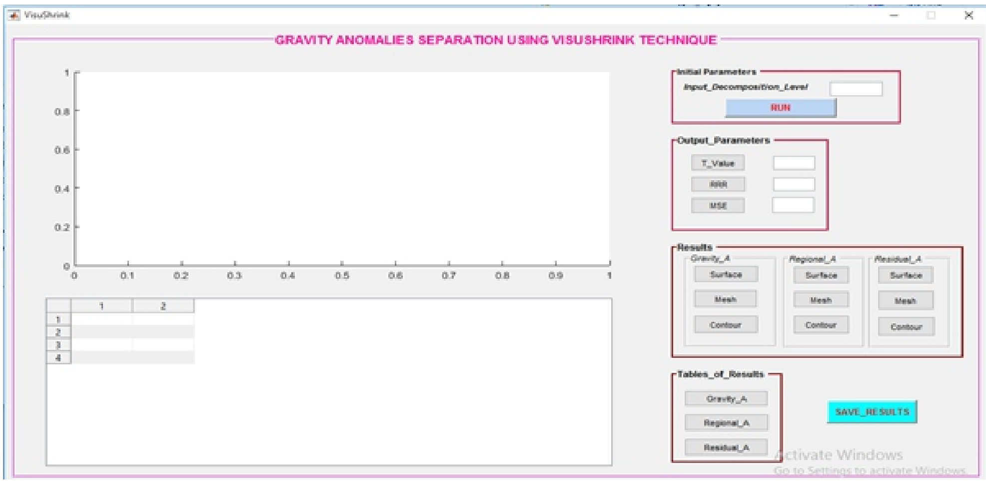

Figure 3: GUI Standalone Computational Tool

In this research, a computer program algorithm written in MATLAB software environment was developed into a Graphical User Interface to perform the gravity anomalies separation task (Figure 3). The sample Graphical User Interface (GUI) of the standalone application program is shown in Figure 3. In the standalone application, the gravity anomalies are arranged in a square matrix and saved with the 'txt' files extension in a desired folder.

To run the application program, the level of decomposition is typed into the "Input_Decomposition_Level" space box. For gravity anomalies separation, a decomposition level of 3 was chosen. After, running the program will display a dialogue box that requests the saved gravity anomalies 'txt' file extension after which the results will be displayed in tables. Also, the results will automatically be saved in the folder earlier opened for that purpose. The two-dimensional surface plots (i.e. mesh and contour plots) of gravity, regional, and residual anomalies will be displayed.

Results and Discussion

The separation results of the gravity anomalies into regional and residual anomalies using the discrete wavelet transform separation technique developed into the GUI Standalone Computational Tool are shown below. Table 2 shows the rectangular coordinates, gravity anomalies, regional anomalies, and residual anomalies of gravity stations. Also, Figures 4, 5, and 6 show the graphical plots of the results.

Table 2: Separation Results of VisuShrink Threshold Estimating Model

Sta.	Eastings (m)	Northings (m)	Gravity Anomalies (mGal)	Regional Anomalies (mGal)	Residual Anomalies (mGal)
1	658003.033	1129134.467	-9.9086	-8.8671	-1.0415
2	660709.700	1129134.467	-12.5655	-11.3126	-1.2529
3	663416.366	1129134.467	-14.8181	-13.4484	-1.3697
4	666123.033	1129134.467	-14.8616	-13.3289	-1.5327
5	668829.700	1129134.467	-13.1694	-11.8452	-1.3242
6	671536.367	1129134.467	-12.0838	-11.1146	-0.9692
7	674243.033	1129134.467	-11.9359	-11.2587	-0.6772
8	676949.700	1129134.467	-11.9013	-11.597	-0.3043
9	679656.367	1129134.467	-11.7009	-11.4277	-0.2732
10	682363.033	1129134.467	-11.525	-11.1687	-0.3563

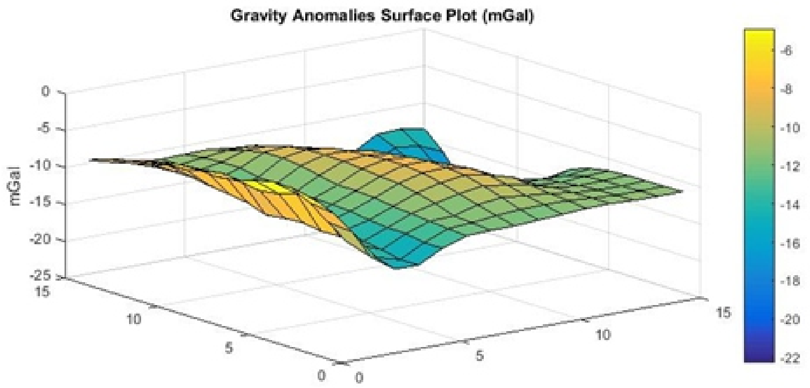

Figure 4: Surface plot of Gravity Anomalies

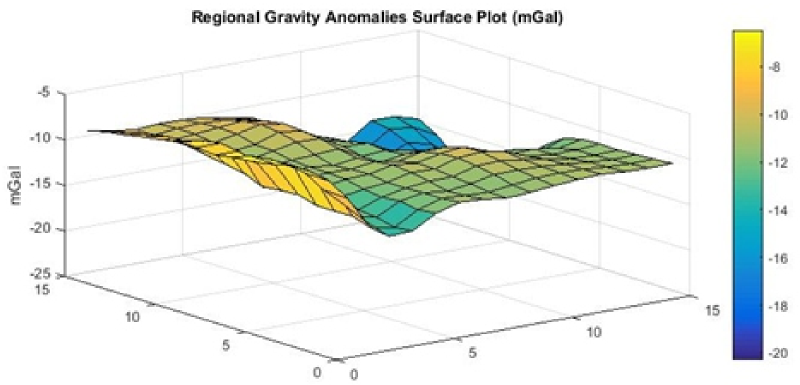

Figure 5: Surface plot of Regional Gravity Anomalies

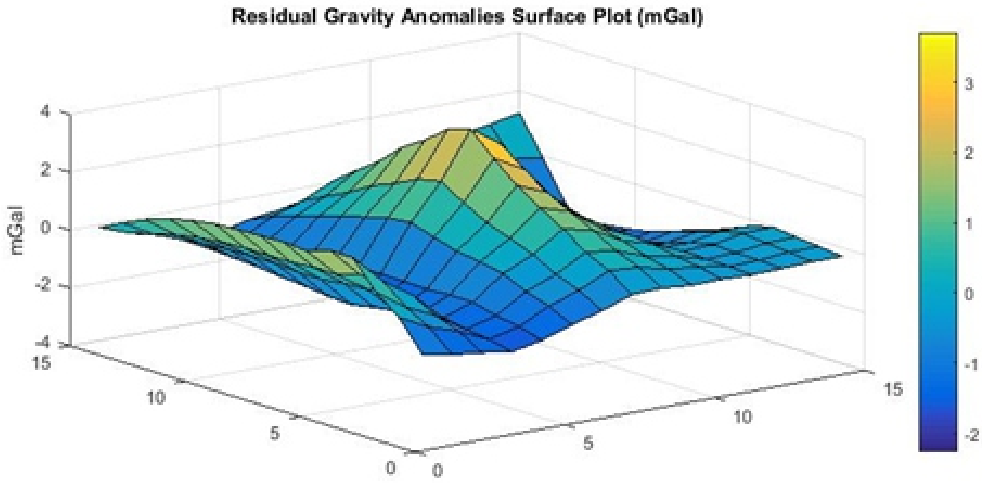

Figure 6: Surface plot of Residual Gravity Anomalies

Analysis of Results

Gravity Anomalies separation result was analyzed using Mean Square Error (MSE), Regional-to-Residual Ratio (RRR), and the anomalies variances of both the regional and residual gravity anomalies, (Mousavi et al, 2013). The variance obtained from the determined regional gravity anomalies was 6.2545 while that of residual gravity anomalies was 1.1037. Also, the Regional-to-Residual Ratio (RRR) is 22.3113 dB while the Mean Square Error is 27.0552.

From the acquired data, the results obtained by Idowu (2005) using the Least Squares Collocation method shows that the regional anomalies variance obtained is 6.6488 and that of residual gravity anomalies is 1.5042, Regional-to-Residual Ratio (RRR) is19.4197dB and the Mean Square Error is 26.6775. This shows that the application of the VisuShrink method produces better variance and good smoothing of the regional gravity anomalies. Statistical (F-test) analysis was carried out to compare the results obtained in the research with the results obtained by Idowu (2005).

The statistical analysis was carried out by hypothesis testing of the regional gravity anomalies variance to determine whether there is a significant difference between the regional gravity anomalies results obtained from the VisuShrink Thresholding (Discrete Wavelet Transformation) Technique and those obtained from Least Square Collocation Technique (Idowu, 2005) at 5% level of significance. That is:

Null hypothesis: H$_o$: $\sigma_1^2 = \sigma_2^2$ (there is no significant difference between the variance of the VisuShrink DWT results and the Least Square Collocation results)

Alternative hypothesis: H$_1$: $\sigma_1^2 > \sigma_2^2$ (there is a significant difference between the variance of the VisuShrink DWT results and the Least Square Collocation results)

The condition to reject the Null hypothesis is when *F computed* is greater than *F-critical*. That is, if F > F-Critical, the Null hypothesis is rejected.

Performing the hypothesis, the computed P-value is 0.3238, the computed F statistic is 1.0631 and the F-critical is 1.2464. Since the computed F statistic is smaller than the F-critical at the significance level of 0.05, it shows that the null hypothesis is accepted at 0.05 significant levels.

Table 3: *F*-Test for Regional Gravity Anomalies Variance

***F*-Test Two -Sample for Variances**		
	Least Square	*Visushrink*
Mean	-11.5046998	-11.72669022
Variance	6.648810043	6.254475023
Observations	225	225
Df	224	224
F	1.06304846	
P(*F*<=f) one-tail	0.323839439	
***F* Critical one-tail**	1.246413292	

Therefore, it can be inferred that there is no significant difference between the regional gravity anomalies obtained from the two methods at a 5% significance level.

Table 4: *F*-Test for Residual Gravity Anomalies Variance

***F*-Test Two -Sample for Variances**		
	Least Square	*Visushrink*
Mean	-0.212013	0.00997747
Variance	1.504164134	1.10373363
Observations	225	225
Df	224	224
F	1.362796324	
P(*F*<=f) one-tail	0.010466158	
***F* Critical one-tail**	1.246413292	

Similarly, to examine the residual gravity anomalies separation result (Table 4), the computed F statistic is 1.3628 while the F-critical is 1.2464. That is, the computed "F" is greater than the F-critical. This infers that the null hypothesis is rejected while the alternative hypothesis is accepted at a 5% significance level. However, the VisuShrink method produced lower residual gravity anomaly variance.

Therefore, it can be stated that the VisuShrink method is an optimum threshold estimating model for obtaining the regional gravity anomalies (for geodetic surveying) and residual gravity anomalies (for geophysical surveying).

Conclusion

In this work, an attempt was made to create a standalone application to separate gravity anomalies into regional and residual gravity anomalies components based on the VisuShrink threshold method in Discrete Wavelet Transformation. The results obtained proved to be viable when comparing it with the Least Square Collocation result which was considered as a standard.

The separation results from the Least Square Collocation technique have higher regional and residual gravity anomaly variances (6.6488and1.5042) compared with the variances of the VisuShrink threshold estimating model (of 6.2545and 1.1037) with a significant difference of 0.4005 in the residual gravity anomalies.

The VisuShrink threshold estimating model in two-dimensional Discrete Wavelet Transformation is recommended for optimum processing gravity anomalies separation other than processing gravity data in a point/profile. One of the advantages of this technique is that it is immaterial to have a-prior information of the trend of the gravity field which gives it an edge over the trend or polynomial fitting technique. Further work should be carried out to determine the depth, quantities, and types of mineral resources underneath the earth's surface that are achievable using this method.

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