Safety Determination of Structural Components Made With Metakaolin

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Abstract

Structural safety estimation is a task of paramount importance, especially in recent situations where adding additives to improve the properties of concrete has become the order of the day. In this paper, the stochastic variables were identified; the variables obtained were obtained under laboratory experiments and assumed to be stochastic. Python programming language was used in Jupyter Notebook (code translator), and math was imported as a Python library for the evaluation of safety indices and failure probabilities of reinforced concrete beams produced with metakaolin. The obtained geometric indices were found to be 3.2967, 3.6428, 3.219, and 3.0176, and the failure probabilities corresponding to the estimated geometric indices were 1.65E-3, 1.83E-3, 1.62E-3, and 1.51E-3 for reinforced concrete beams produced with 0%, 10%, 20%, and 30% replacements of cement with metakaolin, respectively. The values are less than the target safety index of 4.7 for beams in bending or flexure and 3.7 for beams in shear, with tolerable risk levels (10–3) for structural elements. The results showed that the structural elements are unsafe and can lead to a severe accident compared to the prediction of structural performance according to the specifications of the Joint Committee on Structural Safety (JCSS) (2001).

Keywords: *- Structural Safety, Building Process, Stochastic Analysis, Reliability Evaluation, Metakaolin, Python programming*

Introduction

Reliability appraisal of structures becomes a necessity, especially in Nigeria, where several building collapses were reported due to the use of quacks and substandard materials in the construction industry (Faremi and Ajayi 2020). One of the major construction materials used in the construction industry is cement, and the use of cement is associated with the emission of Co2.

Shan, Zhou, Meng, Mi, Liu and Guan (2019) research was conducted to carve out the amount of CO2 emissions due to the production of cement, which brought in the concept of using metakaolin as a partial replacement for cement as an additive (Barbosa, dos Anjos, Cabral and Dias, 2022). Yet, some professionals and quacks still abuse the concept of the use of additives in the construction industry and use them beyond the recommendable value specified in research publications and standard documents. This exposes some of the structures produced with additives to the risk of failure, and such structures' safety is not guaranteed.

The best way to assess the safety of a structure or structural element is by calculating the probability of failure (Yang, Teng and Frangopol, 2017). Given that it is not possible to predict structural loading and intensities with absolute certainty, the probabilistic concept has become a crucial tool for any realistic, quantitative, and logical investigation, and every conceivable circumstance must be accompanied by a numerical estimate of the likelihood that it will occur. (Onwuka and Sule, 2014). This metric can only determine the structural importance of a given condition.

Since absolute reliability is impossible in an uncertain world, evaluating structural safety using a probabilistic approach makes sense, according to Onwuka and Sule (2014). The main goal of probabilistic thinking has been to evaluate uncertainty's effects on structural performance systematically. Although it may not have all the answers, the probabilistic idea has been essential in establishing the viability of many engineering structures. (Ross, Booker, Parkinson, 2002). This paper highlights the use of probabilistic concepts to assess the structural integrity of reinforced concrete beams produced with metakaolin. The probabilistic model is simple and straightforward and can be manually achieved.

The reliability analysis of structures or structural elements produced with cement, sand, aggregate, and steel reinforcements was investigated by Renjian *et al.* (1994), Mohammed *et al.* (2014), Onwuka and Sule (2014), Kigha *et al.* (2014), Farsani and Keshtegar (2015), Aboshio, Uche and Ogork (2016), Mahdi (2017), Ya'u, Okoli, Dahiru and Mohammed (2021).

Mix Design of Concrete and sample preparation

The British Method of Concrete Mix Design (DoE Method 1988) of concrete mix design was used to determine the quantities of the materials for concrete samples of different grades. The mix proportions are presented in Table 1.

Table 1: Mix Proportion of Concrete Samples

Concrete Grades	Cement (Kg/m^3)	Sand (Kg/m^3)	Coarse Aggregate (Kg/m^3)	Water content (Kg/m^3)	W/C ratio	Metakaolin replacement (%)
C25	360	693.89	1207.35	190	0.53	10-30

Experimental Design

In preparing specimens, grade 25 was prepared, including control, 10%, 20%, and 30% cement replacement with metakaolin. Each category contained nine (9) specimens, which sum up to thirty-six (36) cubes and thirty-six (36) beams.

The sizes of the specimen cast were 150 mm x 150 mm x 150 mm cubes and 150 mm x 750 mm x 150 mm beams. Details of the experimental design are shown in Tables 2 and 3.

Table 2: Percent Content of Metakaolin in Cube Specimens for Grade 25

MK (%) / Curing days	0 %	10%	20%	30%
7 days	Cube	Cube	Cube	Beam
14 days	Cube	Cube	Cube	Beam
28 days	Cube	Cube	Cube	Beam

Table 3: Percent Content of Metakaolin in Beam Specimens for Grade 25

MK (%) / Curing days	0 %	10%	20%	30%
7 days	Beam	Beam	Beam	Beam
14 days	Beam	Beam	Beam	Beam
28 days	Beam	Beam	Beam	Beam

Formulation of Stochastic Model

Let X, Y and be the applied stress random variable and allowable stress random with statistical properties described by the first and second moment (μ_x, σ_x) and (μ_y, σ_y), respectively.

The limit state function is given by: *YXZ*–= (1)

Violation of limit state occurs when: 0>*z* (2)

Again, using equation (1), the probability of failure is given by:

The probability of failure is given by:

$$P_f = \int_0^\infty g(z)dz \quad (3)$$

The capacity demand is assumed to be statistically independent.

$$f(x)f(x-z)dxdz \quad (4)$$

Using equation (1) and applying the convolution theorem, the probability density function of *z*

$$g(z) = \int_a^b f(x)f(x-z)dxdz \quad (5)$$

where represent the structural stress limits. From Figure 1, they are assumed to be normally distributed. Therefore, the probability density functions are given by equations (6) and (7) respectively [13]. b and a Y and X

$$f(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\exp\left[-\frac{1}{2}\right]\left(\frac{x-\mu_x}{\sigma_x}\right)^2 \qquad (-\infty,\infty) \quad (6)$$

$$f(y) = \frac{1}{\sigma_y\sqrt{2\pi}}\exp\left[-\frac{1}{2}\right]\left(\frac{x-\mu_y}{\sigma_y}\right)^2 \quad (-\infty,\infty) \quad (7)$$

Substituting for using equation f(x) and f(x-z)

$$g(x) = \int_{-\infty}^{\infty}\frac{1}{2\pi\sigma_x\sigma_z}\exp\left[-\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{x-z-\mu^2}{2\sigma^2}\right]dx \quad (8)$$

Let the expression in the bracket be denoted by λ

Therefore,

$$\lambda = -\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(x-z-\mu_x)^2}{2\sigma_y^2} \quad (9)$$

Multiplication of the top and bottom of equation (9) by $\sigma^2{}_x + \sigma^2{}_y$ gives:

$$\lambda = \frac{\sigma^2{}_x + \sigma^2{}_y}{2\sigma_x 2\sigma_y}\left(\frac{-2\sigma^2{}_y(x-\mu_x)^2 - 2\sigma^2{}_x(x-z-\mu_y)^2}{\sigma^2{}_x + \sigma^2{}_y}\right) \quad (10)$$

$$\lambda = \frac{\sigma^2{}_x + \sigma^2{}_y}{2\sigma^2{}_x 2\sigma_y}\left[\frac{x^2 - 2x\left(\mu_x\sigma^2{}_z + z\sigma^2{}_x + \mu_y\sigma^2{}_x\right)}{\sigma^2{}_x + \sigma^2{}_y} + \mu^2{}_x\sigma^2{}_y + \frac{\left(z^2 + \mu^2{}_y + 2z\mu_z\right)\sigma^2{}_x}{\sigma^2{}_x + \sigma^2{}_y}\right] \tag{11}$$

Multiplying the top and bottom of the last term of equation (11) by $\sigma^2{}_x + \sigma^2{}_y$ gives:

$$\lambda = \frac{\sigma^2{}_x + \sigma^2{}_y}{2\sigma_x{}^2\sigma^2{}_y}\left\{\begin{array}{l}\dfrac{x^2 2x\left[\mu^2{}_x\sigma^2{}_y + \left(z + \mu_y\right)\sigma^2{}_x\right]}{\sigma^2{}_x + \sigma^2{}_y} \\ + \dfrac{\mu^2{}_x\sigma^2{}_x\sigma_y\left(z^2 + 2z\mu_y + \mu^2{}_y\right)\sigma^4{}_x + \mu^2{}_x\sigma^4{}_y + \left(z^2 + 2z\mu_y + \mu^2{}_x\right)\sigma_x{}^2\sigma_y{}^2}{\left(\sigma_x{}^2 + \sigma_y{}^2\right)^2}\end{array}\right\} \tag{12}$$

The separation of the two middle terms of the last fraction of equation (12) from the other two terms followed by addition and subtraction of expression

$$\frac{2\mu_x\left(z + \mu_y\right)\sigma^2{}_x\sigma^2{}_y}{\left(\sigma^2{}_x + \sigma^2{}_y\right)^2}$$

Transforms equation (12) to:

$$\lambda = \frac{\sigma_x{}^2 + \sigma_y{}^2}{2\sigma_x{}^2\sigma_y{}^2}\left[\begin{array}{l}x^2 - 2x\dfrac{\mu_x\sigma_y{}^2 + \left(z + \mu_y\right)\sigma_x{}^2}{\sigma_x{}^2 + \sigma_y{}^2} \\ + \dfrac{\mu_x{}^2\sigma_y{}^4 + 2\mu_x\left(z + \mu_y\right)\sigma_x{}^2\sigma^2{}_y + \left(z + \mu_y\right)\sigma_x{}^4}{\left(\sigma_x{}^2 + \sigma_x{}^2\right)} \\ + \dfrac{\mu_x - 2\mu_x\left(z + \mu_y\right) + \left(z + \mu_y\right)^2\sigma_x{}^2\sigma_y{}^2}{\left(\sigma_x{}^2 + \sigma_y{}^2\right)^2}\end{array}\right] \tag{13}$$

Also, multiply the last term of equation (13) by $-\dfrac{\sigma^2{}_x + \sigma^2{}_y}{2\sigma_x{}^2\sigma_y{}^2}$

$$\lambda = -\frac{\sigma^2{}_x + \sigma^2{}_y}{2\sigma_x{}^2\sigma_y{}^2}\left(x - \frac{\mu_x\sigma_y{}^2 + \left(z + \mu_y\right)\sigma_x{}^2}{\sigma_x{}^2{}_x + \sigma_y{}^2}\right) - \frac{\left(z + \mu_x - \mu_y\right)^2}{2\left(\sigma^2{}_x + \sigma_y{}^2\right)} \tag{14}$$

$$g(z)=\frac{1}{\sqrt{2\pi\left(\sigma^2{}_x+\sigma^2{}_y\right)}}\exp\left\{\begin{matrix}\frac{\sigma_x{}^2+\sigma^2{}_y}{\sigma_y{}^2y^2}\left[x-\frac{\mu_x\sigma^2{}_y+\left(z+\mu_y\right)\sigma^2{}_x}{\sigma^2{}_x+\sigma^2{}_y}\right] \\ -\frac{\left(-z-\mu_y-\mu_x\right)}{2\left(\sigma^2{}_x+\sigma^2{}_y\right)}\end{matrix}\right\}dx \tag{15}$$

$$\alpha=\int_{-\infty}^{\infty}\frac{\sqrt{\sigma^2{}_z+\sigma^2{}_y}}{\sqrt{2\pi\sigma_x\sigma_z}}\exp\left\{\frac{-\sigma^2{}_x+\sigma^2{}_y}{2\sigma^2{}_x\sigma^2{}_z}\left[\frac{x-\mu_x\sigma^2{}_y+\left(z+\mu_y\right)^2}{}\right]^2\right\}dx \tag{16}$$

Equation (15) now becomes:

$$g(x)=\frac{1}{\sqrt{2\pi\left(\sigma^2{}_x+\sigma^2{}_y\right)}}\exp\left[\frac{\left(z-\mu_y-\mu_x\right)}{2\left(\sigma^2{}_x+\sigma^2{}_y\right)}\right]\alpha \tag{17}$$

From equation (14), let

$$t=\frac{\sqrt{\sigma^2{}_x+\sigma^2{}_y}}{\sqrt{\sigma_x\sigma_y}}\left[x-\frac{\mu_x\sigma_y+\left(z+\mu_y\right)\sigma^2{}_x}{\sigma^2{}_x+\sigma^2{}_y}\right] \tag{18}$$

Differentiating t with respect to x in equation (18) yields:

$$dt=\frac{\sqrt{\sigma_x{}^2+\sigma_x{}^2}}{\sigma_x\sigma_y}dx \tag{19}$$

Substituting for in equation (16), we have dx and t

$$\alpha=\int_{-\infty}^{\infty}\frac{1}{2\pi}e^{-\left(t^2/2\right)dt} \tag{20}$$

$$\int_{-\infty}^{\infty}pdfdx=1 \tag{21}$$

Therefore, equation (15) now transforms to:

$$g(z)=\frac{1}{\sqrt{2\pi\left(\sigma^2{}_x+\sigma^2{}_y\right)}}\exp\left[-\frac{1}{2}\frac{\left(z-\mu_y-\mu_x\right)^2}{\left(\sigma^2{}_x+\sigma^2{}_y\right)}\right] \tag{22}$$

From Figure 1, Z is a normally distributed random variable. The mean and standard deviation are therefore $\mu_z = \mu_x - \mu_y$ (23)

$$\sigma_z = \sqrt{\sigma^2{}_x + \sigma^2{}_y} \tag{24}$$

The probability that the structure fulfils the intended purpose is structural reliability defined by:

$$\text{Reliability} = \int_0^\infty g(z)dz \tag{25}$$

Again, let

$$t = \frac{z + \mu_y - \mu_x}{\sqrt{\sigma_x{}^2 + \sigma_x{}^2}} \tag{26}$$

Differentiation of equation (26) with respect to x yields:

$$dt\frac{dz}{\sqrt{\sigma_x{}^2 + \sigma_y{}^2}} \tag{27}$$

Using equation (27), equation (25) now transforms to:

$$\mathrm{Re}\,liability = \int_0^\infty g(x)dz = \int_0^\infty \frac{\mu_y - \mu_x}{\sqrt{\sigma^2{}_x + \sigma^2{}_y}} - \frac{1}{2\pi}e^{-(t/2)dt} \tag{28}$$

Using equation (28), the transformation which relates μ_x, μ_y to the standard normalised variable is given by z

$$z = \frac{\mu_y - \mu_x}{\sqrt{\sigma_x{}^2 + \sigma_y{}^2}} \tag{29}$$

Applied stress = 0

Therefore,

$$y = \mu_y = \sigma_y = 0 \tag{30}$$

$$z = \frac{|\mu_x|}{\sigma_x} \tag{31}$$

σ_{cu} and x represent the concrete cube strength of control and strength of concrete containing metakolin

According to BS 8110 [11], the mean design strength is given by: $\mu_x = 0.67\sigma_{cu}$ (32)

The resulting coefficient of variation of concrete strength is supplied as follows: error in the defined reliability mode, error resulting from test procedures, error resulting from in-batch variability of concrete strength reinforcement, and error resulting from dimensional variability.

$$COV_{\text{Re}\,sul\tan t} = \left(COV^2{}_y + COV^2{}_{testing} + COV^2{}_{in-batch}\right)^{1/2} \quad (33)$$

σ_x μ_x Mean value and standard deviation of structural capacity, respectively.

COV_y is a function of the mix design

According to Ranganathan $COV_{testing} = COV_{in-batch} = 0.10$

$X < \sigma_{all}$ Therefore, the probability of failure for a particular structural (P_{fi}) for a particular structural member is given as:

$$(P_f) = P(x_i < \sigma_{all}) \quad (34)$$

Where:

P, σ_{all} Represents probability operator and allowable concrete stress in axial compression, respectively.

$$P_{fi} = P(x_i < \sigma_{all}) \quad (35)$$

According to BS8110

$$\sigma_{all} = 0.33\sigma_{cu} \quad (36)$$

Assuming X to be normally distributed, the probability of failure in the structure is given by:

$$P_f = \phi\left(\frac{\sigma_{all} - \sigma_x}{\sigma_x}\right) \quad (35)$$

Using equations (32) and (35), equation (36) can be written as:

$$P_f = \phi\left(\frac{0.33\sigma_{cu} - 0.67\sigma_{cu}}{COV_{\text{Re}\,sul\tan t}(0.67\sigma_{cu})}\right), \qquad \sigma_x = COV_{\text{Re}\,sul\tan t} \times 0.67\sigma_{cu} \quad (37)$$

According to Ranganathan [2], the probability of structural failure can be approximated as:

$$P_f \approx \phi(-\beta) \tag{39}$$

Where:

$\phi(.)$ is the standard Gaussian cumulated function and

$$\beta = \min\|u\| = \left(\sum_{i=1}(X_i)^2\right) \tag{40}$$

Results and Discussion

Parameter Identified for Reliability Analysis

Table 4 : Parameter Identified for Reliability Analysis

Parameter	Variable	Distribution	Mean	STD	Cov.	JSCC (2000)
X1	Fcu_{25}	Normal	(25.21-15.11) N/mm^2	2.30886	0.09	Lognormal or Normal
X2	*Qk (Variable load$_{25}$)*	Gumbel	(234.82-156.38) kN/m	7.84793	0.03	Gumbel or Frechet
X3	*Gk25,*	Log-normal	0.65156-0.57393) kN/m	0.021194	0.04	Lognormal or Normal
X4	*B*	Normal	153.46 mm	4.22	0.027	Lognormal or Normal
X5	d_{eff}	Normal	124.93 mm	5.13	0.040	Lognormal or Normal
X6	l_{eff}	Log-normal	453.7 mm	2.983	0.00657482	Lognormal or Normal
X7	fy	Log-normal	727.24 N/mm^2	13.14	0.0189	Lognormal or Normal

Table 4 shows the parameters identified for reliability analysis, and it can be seen from the table that the concrete compressive strength and the steel yield strength have a normal distribution. These findings are not in line with those of Yusuf (2002), Farsani and Keshtegar (2015), and Kioumarsi *et al*. (2017). It is also observed from the table that the self-weight of the beams has a normal distribution; this finding agrees with the finding of Kioumarsi *et al*. (2017), who noted that the self-weight of beams has a normal distribution. Furthermore, distributions of section width and height are obtained as normal, and these findings are also in agreement with the findings of Farsani and Keshtegar (2015) and Kioumarsi *et al.* (2017) and disagree with Yusuf (2002), who noted the section width is lognormal. The live load was obtained to have a normal distribution, which disagrees with Mohammed *et al*. (2014), Kioumarsi *et al*. (2017), and Yusuf (2002), who identified the live load as having a Gumbel distribution. This may be due to the fact that in the assignment of distribution models to a random variable, two or more distributions may appear to be plausible probability distribution models, according to Mohammed. (2014) Ya'u, Okoli, Dahiru, and J. M. Kaura (2021)

Table 5: Variation of safety indices and probability of failure as against variation of strength of concrete with higher percentage of metakaolin

Grade of concrete With percentage of Metakaolin	Mean (N/mm2) (μ_y)	STD/ (N/mm2) (σ_y)	COV	Probability of failure P_f	Safety index (-β)
25 with 0% of MK replacement	25.10N/mm2	2.797147	0.111	1.65E-3	3.2967
25 with 10% of MK replacement	25.21N/mm2	2.30886	0.091	1.85E-3	3.6428
25 with 20% of MK replacement	18.763N/mm2	2.16583	0.115	1.62E-3	3.2319
25 with 30% of MK replacement	15.11N/mm2	1.95275	0.129	1.51E-3	3.0176

Table 5 shows the results of safety indices and probabilities of failure of reinforced concrete beams produced with metakaolin at 0% MK replacement, 10% MK replacement, 20% MK replacement, and 30% replacement of cement. From the results obtained, reinforced concrete beams produced with 10% metakaolin have a safety index of 3.6428, which is above the minimum safety index recommended for shear but below the safety index recommended for bending, while the safety indices for 0%, 20%, and 30% replacement of cement with metakaolin are 3.2967, 3.2319, and 3.0176, respectively. It is also observed that reinforced concrete beams produced with 0% replacement of cement with metakaolin have a standard deviation of 2.797. This could be due to the variability of the material strength of the sample used or samples being affected by environmental factors beyond their control, which leads to a lower safety index than the sample produced with a 10% replacement of cement with metakaolin. It also confirms the sensitivity of the standard deviation to the safety index.

Conclusion

In conclusion, the reinforced concrete beam produced with a 10% replacement of cement with metakaolin has met the minimum value of the safety index recommended by JSCC (2001) for shear failure, while reinforced concrete produced with 20% and 30% metakaolin did not meet the minimum value of the safety index for shear.

A higher coefficient of variation reduces the safety of reinforced concrete beams; therefore, stringent supervision should be carried out while producing concrete samples to minimise the risk of failure.

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