Forecasting Financial Crisis using Topological Data Analysis Approach

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ABSTRACT

Traditional financial forecasting methods often struggle to capture the complex interactions and emerging patterns that precede financial crises. By leveraging on TDA, this research aims to uncover potential topological features that might serve as early warning signals for impending financial crises. The study adopts the utilization of Topological Data Analysis, an initiative mathematical framework to explore and analyze the inherent topological structures within financial data set, using secondary data from "Yahoo Finance API". The results of the analysis conducted using Python indicate that persistence homology in TDA successfully identifies key topology features associated with financial crises, implying its potential for developing early warning systems in the financial sector. The insights gained from this analysis could significantly enhance the early detection and proactive management of system risks in financial market, thereby contributing to more robust risk assessment and policy formulation strategies.

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1 Introduction

Financial crises are pivotal events characterized by a sudden disruption in the stability of financial markets or systems. Clearly, a financial crisis refers to a situation where the value of financial institutions or assets rapidly declines, leading to severe disruptions in the financial systems which is characterized by stock markets crashes, disruptions in currency values, etc. Financial crises have significant impacts on economies globally, necessitating robust methodologies for early detection and mitigation.

Financial history is punctuated with numerous market crashes like that of 1929, Black Monday in 1987, and the more recent 2008 financial crisis. What's common among these is their apparent unpredictability and the profound impact they've had on global economies. Traditional indicators and models, built on the assumptions of normalcy, often fail to anticipate these 'black swan' events. Here lies the potential of TDA, which, by its very design, acknowledges and works with data's inherent complexities.[11]

In the dynamic landscape of finance, the ability to foresee and navigate impending financial crises is paramount. Managing and preventing financial crises is a key focus of financial institutions and policy makers, hence this research project delves into an innovative approach, employing Topological Data Analysis as a tool to enhance the forecasting of financial crises.

TDA is a recent field emerging from various in applied (algebraic) topology and computational geometry during the first decade of the century [5]. TDA is a new research field that bridges computational methods with the mathematical theory of topology [34]. TDA is mainly motivated by the idea that topology and geometry provide a powerful approach to infer robust qualitative, and sometimes quantitative, information about the structure of data [6].

While traditional data analysis is currently performed by comparing pairwise similarities between objects, commonly using a metric such as the Euclidean distance, it disregards information such as shapes, including loops and holes. For instance, a traditional clustering algorithm that connects geometrically close points will fail to find sets of points that are spatially distant but that present similar structure, or shape.
Historically, topology was the field of mathematics created to study basic properties such as loops and holes. TDA is a relatively novel approach designed to study and measure certain features of discrete multidimensional data sets, commonly treated as point clouds embedded in $\mathbb{R}^n$, using a combination of statistical, computational and topological tools to find shape-like structures in data [5, **?**, **?**, 7, 8].

For TDA to be applied, data is usually encoded as a discrete geometrical sample naturally embedded in a predefined topological space. TDA explores results from studying the persistence of certain homological features, informally k-dimensional holes, that may arise when constructing simplicial complexes born from our original data set. Accordingly, we use persistent homology [9, 10, 12, 19] as an essential tool to our application.

2 Literature Review

The 21^{st} century has proven to be as economically tumultuous as the preceding 2 centuries. This period has seen multiple crises striking nations, region and entire global economy [30]. In a financial crisis, asset prices see a steep decline in value, businesses and consumers are unable to pay their debts, and financial institutions experience liquidity shortages, and examples are stock market crashes, the bursting of financial bubbles, etc [31]. In recent decades many mainstreams economists appear to have believed that the financial crises were a thing of the past. They pay little attention to heterodox to researchers like Hyman Minsky [17] who underlined the intrinsically volatile dynamics of financial markets [17]. Similarly, the more traditional and rich literature on business cycles clearly demonstrates how important it is to include sections of in the history of finance(financial crises over time) [3].

In the recent past the macro prudential authorities have developed early warning systems to try and predict crisis. By constructing such an early warning system, Coudert and Idier [15] showed notably that indicators associated with property prices, bank credit or debit are good predictors of banking crises, which varies from other regions. But there are limitations of this tool, adopting this, economists can only forecast crises whose warning signals are similar to those of past crises and it must contain the greatest number of relevant indicators in order to cover all aspects of risk, but it greatly limits the choice since it is only possible to use indicators that are available for a long time periods. For all these reasons, a statistical system will never be sufficient to automatically predict the crises; it can only contribute to the assessment, use of TDA independently or in combination with data analysis and statistical techniques will be of great help [4].

Understanding the patterns in financial data is crucial for predicting or forecasting financial crises. For the following reasons:

- **Insight into Relationships:** Identifying patterns helps reveal relationships and correlations within the data. This understanding can be instrumental in making predictions or drawing conclusions about cause-and-effect.
- **Effective Problem Solving:** Recognizing patterns allows you to better understand the nature of a problem or situation. This, in turn, enables more effective problem-solving and decision-making.
- **Risk Mitigation:** Understanding patterns in historical or current data can help in assessing risks. By recognizing trends or anomalies, you can anticipate potential challenges and take proactive measures to mitigate risks.
- **Optimizing Processes:** Patterns in data often highlight inefficiencies or opportunities for optimization. Whether in business processes, manufacturing, or any other field, identifying patterns can guide improvements and increase efficiency
- **Data driven decision making:** In a data driven approach, understanding patterns ensures that decisions are based on evidence and analysis rather than intuition alone. This enhances the reliability and validity of decisions.
- **Performance Monitoring:** Monitoring patterns over time allows for the assessment of performance and the effectiveness of strategies. This continuous evaluation helps in adapting and refining decision-making processes.

- **Predictive Modeling:** Recognizing patterns lays the foundation for predictive modeling. By understanding how variables interact and influence outcomes, you can build models to forecast future trends and make decisions with a forward-looking perspective.

Financial data often exhibits various patterns that, when analyzed, can provide insights into the health and stability of the financial system. This empowers decision-makers to navigate complexities, antincipate future scenarios, and optimize strategies. For best results, employing TDA will serve as a powerful tool to uncover intricate topological patterns within complex datasets, providing a unique perspective on the global structure and relationships within the data. This specialized approach can enhance your understanding, particularly in interdisciplinary fields and scenarios where traditional analytical methods may fall short in capturing the nuanced features of the data.

TDA is a recent field that emerged from various works in algebraic topology and computational geometry during the first decade of the century. Although one can trace back geometric approaches to data analysis quite far into the past, TDA really started as a field with pioneering works of Edelsbrunner et.al [12] and Zomorodian and Carlsoson [34] in persistent homology and was popularized in a landmark article in Carlsson [16]. TDA is mainly motivated by the idea that topology and geometry provide a powerful approach to infer robust qualitative, and sometimes quantitative, information about the structure of data.

Topological Data Analysis TDA [5, 14] refers to a combination of statistical, computational, and topological methods allowing to find shape-like structures in data. The TDA has proven to be a powerful exploratory approach for complex multi-dimensional and noisy data-sets. For TDA to be applied, a dataset is encoded as a finite set of points in some metric space. The general and intuitive principle underlying TDA is based on persistence of k-dimensional holes, e.g., connected components $(k = 0)$, loops $(k = 1)$, etc., in a topological space that is inferred from random samples for a wide range of scales (resolutions) at which data is looked at. Accordingly, persistent homology is the key topological property under consideration [18, 13].

With the introduction of sensors in everything and online systems with click by click data on all user activity, data science now touches nearly every field of study. However the traditional techniques of data analysis have not always kept up with the exploding quantity and complexity of data since they often rely on overly simplistic assumptions. The field of TDA has attempted to fill this void by producing a collection of techniques stemming from the idea that data has shape that can be rigorously quantified in order to investigate data. TDA focuses on the arrangement and proximity of data points rather than relying solely on numeric attributes. This offers a fresh perspective on data analysis, allowing us to capture and understand the qualitative relationships that shape our data [32].

The versatility of TDA is evident through its application across diverse range of fields. In biology, TDA helps researchers understand complex protein structures and gene interactions. In neuroscience, it aids in mapping the brain's functional connectivity. In social sciences, it unveils hidden patterns in networks of relationships. From materials science to economics, TDA is proving to be a valuable tool for gleaning insights from complex datasets that were previously untapped. Hence TDA, combining it with other methods, empowers us to see beyond the surface of data. As we navigate the complex world of modern data analysis, Topological Data Analysis stands as a powerful tool, ready to reveal new insights and enhance our understanding of the intricate patterns that permeate our data-rich world [32].

A remarkable property of persistence homology is that both persistence diagrams and persistence landscapes are robust under perturbations of the underlying data. That is, if the data set changes only little, the persistence diagrams and/or persistence landscapes move only by a small distance. This feature is a key ingredient for mathematically 2 well-founded statistical developments using persistence homology.

A standard procedure to compute the persistent homology associated to a point cloud data set relies on the construction of a filtration of simplicial complexes; though there exists various work considering different types of assemblies of complexes [5], a valid approach, both theoretical and computationally, is given by the Vietoris-Rips scheme, which contrives complexes by setting a minimum distance parameter ε for an edge of a simplex to form, e.g., $\sigma = [p_0, ..., p_k]$ forms a k-simplex iff $d(p_i|, p_j) < \varepsilon$ for all i, j. The basic principle underlying this procedure relies on the fact that altering this distance parameter ε results in modifying the construction and thus, homological attributes characterizing the simplicial complex are intrinsically dependent on it.

We say a feature is more significant if it persists for a longer range of parameters, thus considering it relevant qualitatively towards interpreting an underlying geometry; on the other hand, as features tend to persist less they are considered to be of minor importance to determine any objective shape and hence, usually referred to as $noise$. As features appear and disappear,associated parameters encode a $birth - death$ pair for every k-dimensional hole. This information is captured in a concise form using the means of a *persistence diagram*. Assertively, every point in the diagram records as coordinates the birth and death of every k-dimensional feature from the corresponding simplicial complex. The geometry of the natural embedding space of persistence diagrams can be sometimes hard to work with. This is reflected mostly when we wish to compare persistence diagrams between diverging data sets.

In today's data-driven world, the amount of information in our fingertips has grown exponentially. As we grapple with increasingly complex data-sets, traditional data analysis techniques often fall short in uncovering the complex relationships and the hidden structures within the data but TDA rooted in mathematical topology, offers a unique perspective. TDA works sufficiently when its applied to data with a higher dimension. This is done by representing some aspect of the structure of the data in a simplified topological structure, i.e. the persistence homology (persistence diagram represents loops and holes in the space by considering connectivity of the data points for a continuum of values).

3 Data Collection

In this study, we collected adjusted closed price from stock in Nairobi Security Exchange **NSE!** (**NSE!**) using Python YFinance, extracting historical data from Yahoo Finance **API!** (**API!**). The historical data fetched was between January 1, 2015 and January 1, 2022 from the Yahoo Finance database. The python code to do so is shown in listing 1, and this code can be modified to other markets.

Listing 1: A Python code that implements the data collection procedure.

```
def fetch_data(ticker_name, start_date, end_date):
"""Fetch stock data from Yahoo Finance."""
```

```
raw_data = yf.download(ticker_name, start=start_date, end=end_date)
adjusted_close = raw_data['Adj Close'].dropna()
prices = adjusted_close.values
log_returns = np.log(prices[1:] / prices[:-1])

return adjusted_close, log_returns
```

4 Data Cleaning and Preprocessing

The elegant art of data preprocessing not only enhances the overall analytical process but also ensures that the results are both accurate and reliable, paving the way for innovation and success. In our research project, after collecting the raw data, the data needed to be cleaned. It appeared that there was a data point with a value of 0.20499999821186066, which might be an incorrect value or an outlier, so we removed it from the list. After cleaning, we ended up with the times series data for 1821 distinct stocks. We used the python code below for cleaning;

Listing 2: A Python code that implements cleaning. label

```
stock_data = stock_data[stock_data['Adj Close'] !=0.20499999821186066]
```

Figure 2 shows the time series after cleaning.

5 Topological Model

In our research project, we present an approach to data analysis that leverages the rich mathematical framework of the topological model. Traditional methods often struggle to capture the underlying structures and relationships within complex datasets, particularly those exhibiting high dimensionality or noisy characteristics. Recognizing these challenges, we turn to the topological model, a powerful paradigm that offers a fresh perspective on data analysis.

By embracing principles from algebraic topology, the topological model allows us to explore the inherent geometric properties of our dataset in a manner that transcends traditional statistical techniques. In this project, we harness the capabilities of the topological model to unravel the hidden complexities within our data and extract meaningful insights that might otherwise remain elusive.

In this section, we outline the procedure we've used within the topological model, detailing the steps involved and the rationale behind our approach. Through this innovative method, we aim to push the boundaries of data analysis and unlock new avenues for understanding and interpretation. The procedures are shown in a flowchart below.

i. Embedding of the time series into a point cloud and construction of point cloud sliding windows.

ii. To create a filtration on each window to provide a developing structure encoding each window's geometrical shape.

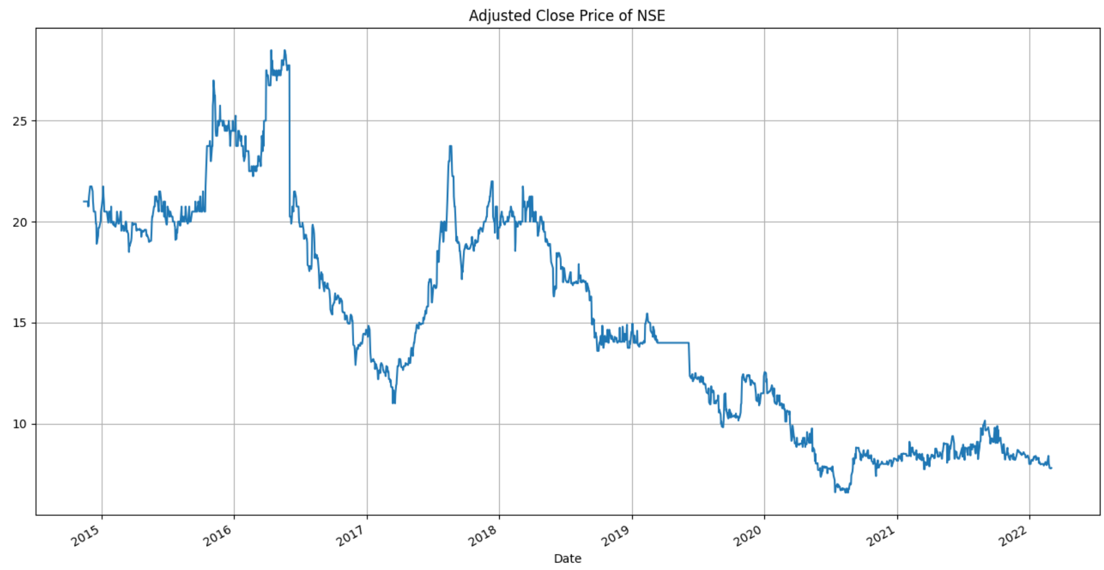
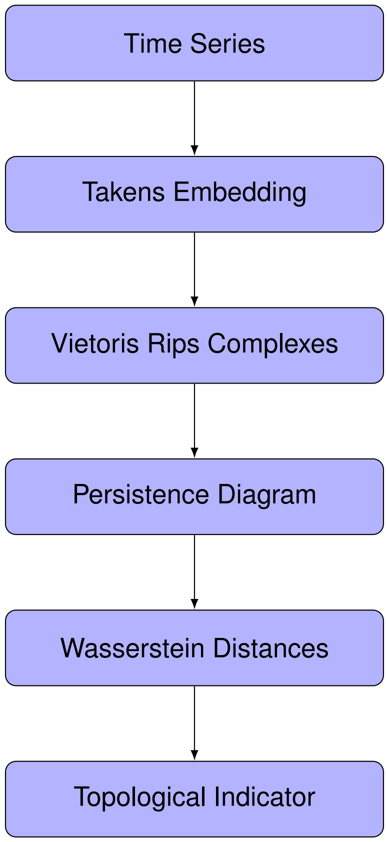


Figure 1: Time Series Plot of NSE Adjusted Close Prices after Data Cleaning.

iii. Using persistence homology, extracting the related features of those windows.

iv. By measuring the difference between these features from one window to the next, comparing each window.

v. Constructing an indicator of crash based on this difference.



6 Analysis and Results

We want to analyze the trend of a stock, we can view it as a node in a topological space, with prices changes represented by edges. By analyzing these edges using topology, we determine the volatility, complexity, and stability of the stock price. If the stock price is highly volatile, then its topological space may be more complex and require more edges to describe. If the stock price is more stable, then its topological space is may be simpler and require fewer edges to describe. The application of topology in financial market analysis is not limited to stock markets. It can also be applied to other data analysis in financial markets, such as interests rates and foreign exchange rates.

At its core, TDA asks a simple yet profound question: What is the intrinsic shape or structure of my data? It doesn't care about the distances or specific locations as much as how data point are connected and how the cluster together.

6.1 Persistence Homology

Persistence Homology is one of the key tools in TDA. At a high level, it helps us understand the 'holes' in our data at various scales. Mathematically, persistence homology is captures by a diagram called persistence diagram. It's a collection of intervals, where each interval represents a feature (like a hole) in the data. The start and end of an interval tells us when a feature appears and when it disappears as we change scale.

$[PH(D) = \{[\text{b,d}] \mid b \text{ is birth time, } d \text{ is death time of a feature in data } D\}]$

6.1.1 Generating Persistence Diagrams

First we slice the time series data by taking segments of our log-return data, we aim to compare consecutive periods, understanding how the market's structure evolves. Persistence diagrams are topological constructs capturing the birth and death of "features" represent patterns or structures in price changes. The birth and death of such features corresponds to the emergencies or disappearance of these patterns.

The persistence diagram is a way of summarizing the topology of this complex in a concise and interpretable way. Each point in the diagram represents a "homology class", which roughly corresponds to a loop or higher-dimensional hole in the complex. The coordinates of the point correspond to the "birth" and "death" times of the homology class, which represent the scales at which the loop or hole exists.

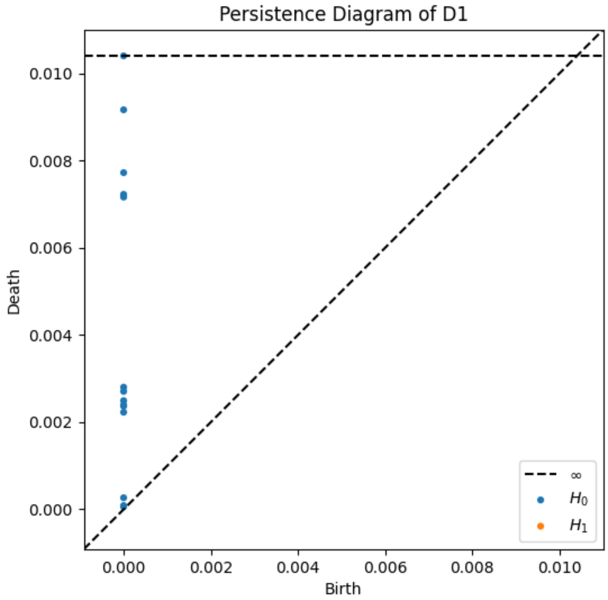


Figure 2: Persistence Diagram of segment one.

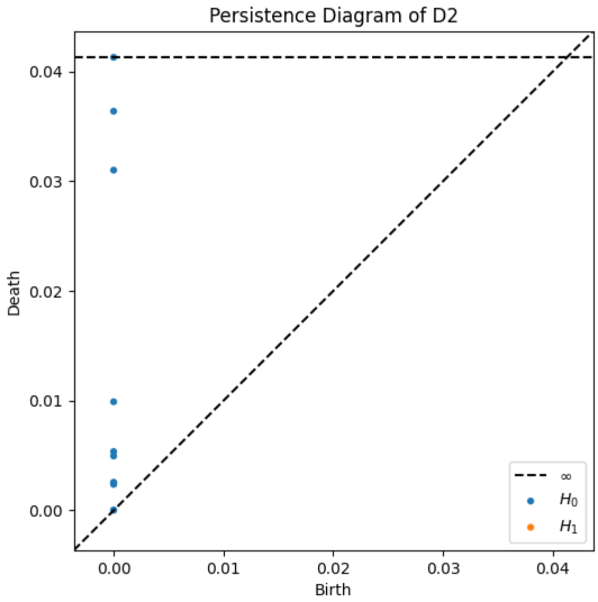


Figure 3: Persistence Diagram of segment two.

Each point in a persistence diagram represents a topological feature that appears and disappears as the scale of observation changes. These points are usually denoted by (birth, death) pairs, where birth represents the scale at which the feature is born or first appears, and death represents the scale at which it disappears. The diagonal line y = x often serves as a reference, where points above this line represent features that persist for a longer duration.

The birth of a feature refers to the scale at which it emerges or becomes noticeable in the dataset, while death refers to the scale at which it ceases to exist or merges with another feature. The distance between the birth and death points indicates the lifespan or persistence of the topological feature. Persistence measures the significance or robustness of a topological feature. Features with longer persistence are considered more stable or significant as they persist across a wider range of scales.
The comprehensive understanding of how the data structure evolves over time can be summarized as follows:

- Persistence diagrams derived from the provided code offer a insightful information for analyzing the structure and dynamics of the log returns data across two segments. By analyzing these diagrams, we gain insights into how the topological features within the data change, persist, emerge, and disappear across different segments.
- The first diagram provides a snapshot of the initial segment's structure, highlighting its connectivity, dominant trends, transient fluctuations, and overall complexity. Comparatively, the second diagram enables us to track the evolution of these features over time, showcasing how they either persist or transform across subsequent segments. Points that align closely between the two diagrams indicate consistent trends, while new points in the second diagram reveal emerging features.
- Conversely, the absence of points in the second diagram suggests the disappearance of certain features. This dynamic perspective allows for a deeper understanding of how the underlying structure of the log returns data unfolds over time, offering valuable insights for analyzing trends, identifying anomalies, and informing decision-making processes in the context of stock market dynamics.

6.2 Wasserstein Distance

Think of it as a measure to compare two Persistence Homologies. Mathematicaly, given two data points, the Wasserstein Distance calculates the "minimal moving cost" to match the points from one set to the other. Formally, given two persistence diagrams D1 and D2, the Wasserstein distance measures the minimal effort to match features from D1 to D2.

$W_p(D_1, D_2) = (inf_{\gamma \in \Gamma(D_1, D_2)} \sum_{x \in D_1} \| x - \gamma(x) \|_p^p)^{1/p}$ (minimal effort to match points from D_1 to D_2)

We employ the Wasserstein distance on financial time series data. The essence is to compare and contrast two sets of points, segments of log returns in this case, to qualify the divergence in their structural patterns. This metric, while being inherently dynamic, offers insights into shifts in market dynamics, potentially hinting at unforeseen market movements.

The Wasserstein distance measures the 'cost' of transforming one set of points to another - in this case, segments of log returns. A spike in this distance can hint at significant market shifts. Taken together, both segments underscore the transformative power of topological method in interpreting complex datasets.

6.2.1 Computing Wasserstein Distance

The computation of Wasserstein Distance serves as a pivotal analytical tool for understanding the dynamics within financial time series data. This metric, originating from the field of TDA, facilitates the comparison of two Persistence Homologies, each representing the topological features inherent in consecutive segments of log returns. By quantifying the "minimal moving cost" required to match features between these homologies, the Wasserstein Distance offers a quantitative measure of structural divergence or similarity in the market's behavior over time. Through its application, we gain insights into the evolution of market patterns, allowing us to identify significant shifts or anomalies in market dynamics. Moreover, the incorporation of visualizations depicting the Wasserstein Distance over time enhances our ability to interpret and analyze these structural changes.

7 Results

Figure 3 and 4 shows the temporal analysis of NSE through topology.
Figure 3 showcases the NSE stock prices over time. The puntunctuated red markers highlights significant topological shifts in the market structure. These markers signify periods where consecutive segments of log returns have substantial topological differences, potentially pointing to key market events or anomalies. To be more clear, red dots indicates points where the Wasserstein distance exceeds the threshold. This plot helps visualize price movements and significant changes.

Figure 4 is a plot of Wasserstein Distances between consecutive segments of log-returns, providing a quatitative measure of these topologival changes. Peaks in this graph may correspond to structural shifts in market dynamics. The green dashed line represents the threshold value. Any points above this line indicate significant changes in the log returns.

Together, these visualizations offer a unique, topological perspective on market behaviours, revealing hidden intricacies not immediately discernible through conventional analyses.

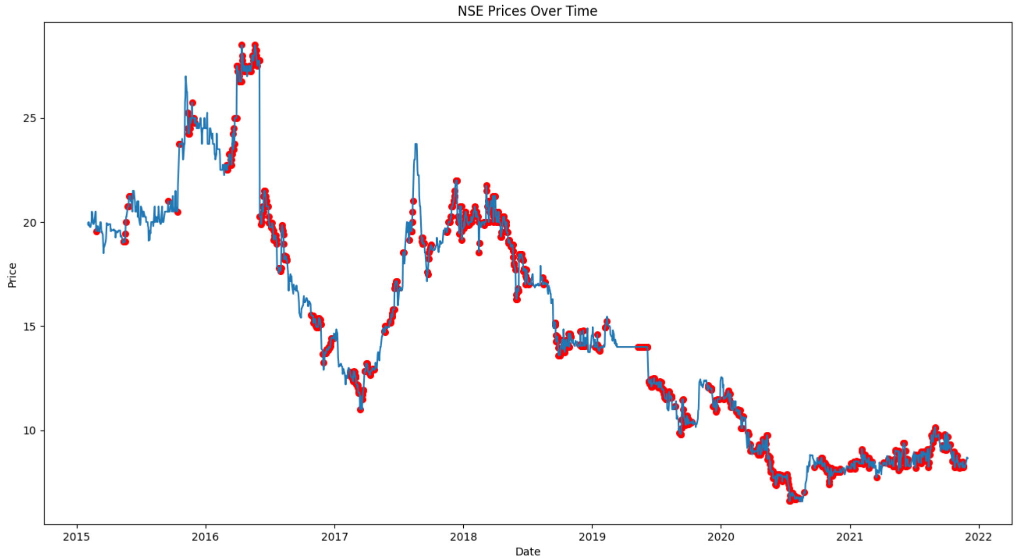


Figure 4: NSE Adjusted Close prices over time.

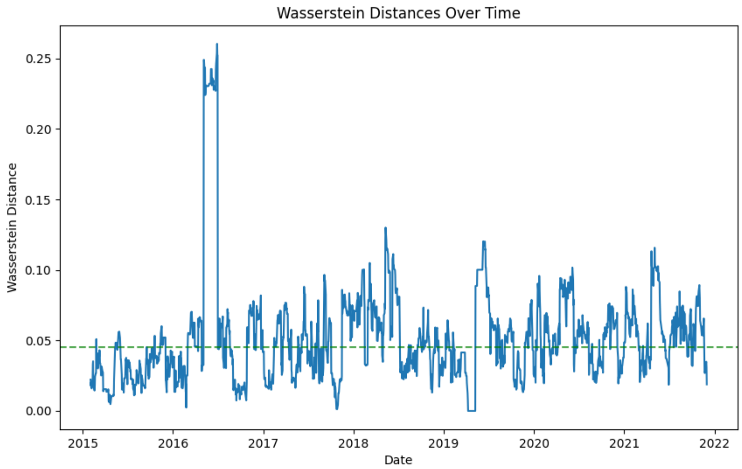


Figure 5

8 Conclusion and Recommendations

Can we predict market crashes with topology? The answer is multifaceted. While the topological method does highlight periods of significant shifts that sometimes precede market downturns, it's not an infallible predictor. Markets are influenced by myriad factors, many of which are unpredictable by nature. Our approach offers a novel angle, but as with all quantitative approaches in finance, it's not a crystal ball.

In this project, we explored the application of TDA in forecasting financial crises. The objective was to leverage the topological properties of financial time series data to identify patterns and signals indicative of impending crises. We began by collecting daily adjusted closed prices from NSE and analyzed it using persistence homology.

Next, we employed techniques from TDA to analyze the topological features of the data. Specifically, we utilized persistent homology to capture the evolution of topological structures across different time periods. By representing the data as a simplicial complex and computing its persistent homology, we were able to extract topological signatures that characterize the underlying dynamics of the financial system.

To enhance the interpretability of our results, we employed dimensionality reduction technique, persistence homology, to visualize the high-dimensional data in a lower-dimensional space. This facilitated the identification of clusters and patterns that corresponds to critical states or transitions in the financial system.

8.1 Findings

Our analysis revealed several noteworthy findings:

(i.) Firstly, our findings indicate that TDA offers a novel and promising approach to financial forecasting. By capturing the topological features of complex financial networks, TDA allows us to gain insights that may not be discernible through traditional statistical methods alone. The ability of TDA to detect subtle changes and disruptions in the underlying structure of financial data provides a valuable tool for anticipating and mitigating systemic risks.

(ii.) Secondly, our results demonstrate the effectiveness of persistent homology—a fundamental concept in TDA—in identifying critical topological features associated with financial crises. By analyzing the persistence diagrams generated from various financial datasets, we were able to identify persistent topological signatures that precede periods of instability and crisis. This suggests that persistent homology has the potential to serve as a powerful tool for early warning system development in the financial sector.

(iii.) Furthermore, our project highlights the importance of interdisciplinary collaboration in addressing complex challenges such as financial crisis forecasting. By bridging the gap between mathematics, finance, and computer science, we have been able to develop innovative methodologies and approaches that have the potential to significantly enhance our understanding of financial systems and improve risk management practices.

8.2 Recommendation

The findings of this study have several implications for policymakers, financial institutions, and risk managers:

i. **Early Warning Systems**: The identification of topological anomalies can inform the development of early warning systems for financial crises, enabling preemptive measures to mitigate systemic risks and safeguard financial stability.

ii. **Risk Assessment**: TDA-based methods can complement existing risk assessment frameworks by providing a novel perspective on the interconnectedness and resilience of financial systems.

iii. **Model Integration**: Future research may explore the integration of TDA techniques with traditional econometric models to enhance the accuracy and robustness of financial crisis forecasting models.

iv. **Further Research**: The project lays the groundwork for future research into TDA's application in financial forecasting. Continued exploration into advanced TDA methods and their integration with machine learning algorithms could yield even more accurate and robust crisis prediction models.

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