COMPARISON OF MATHEMATICAL MODELS DESCRIBING THE GROWTH OF TROPICALLY ADAPTED ROSS 308 COMMERCIAL BROILER CHICKENS

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ABSTRACT

Mathematical growth models are useful in describing the growth of livestock. The study was done to assess the predictive ability and accuracy of four three-parameter nonlinear mathematical models (namely: Gompertz, Gompertz-Laird, Logistic, and von Bertalanffy) and one four-parameter (namely: Richards) nonlinear mathematical model. Models were used to predict the body weight (BW) of commercial Ross broiler chickens adapted to tropical conditions (n = 1,286). Age-weight data were collected once every week for 6 weeks. The Gauss-Newton iterative process of the nonlinear procedure in SAS was employed to obtain the parameters for each model. In addition, each model's goodness-of-fit, residuals, and computational difficulty were estimated. Model parameters were evaluated using Akaike's information criterion (AIC), Bayesian information criterion (BIC), adjusted coefficient of determination (AdjR^2) and root mean square error (RMSE). The AdjR^2 value for all five models was high; however, the highest value was observed in the Gompertz and Gompertz-Laird models. Furthermore, the lowest AIC, BIC and RMSE values were observed in the Gompertz models. Using a complimentary method (involving a subjective pairwise comparison of the observed and predicted BWs), the Logistic, Gompertz-Laird, von Bertalanffy, and Richards models fitted well for the data used. However, the best fitting was obtained in the Gompertz model. Some similarities were observed between the Logistic and Richards models. In conclusion, all five nonlinear mathematical models fitted the age weight data used in this study well, with the Gompertz model being the best.

Keywords: Body weight, Growth curves, Mathematical models, Nonlinear regression

INTRODUCTION

Today's broiler chickens are the result of selective breeding programmes, aimed at achieving early maturity, fast growth, and better body conformation (Zuidhof *et al.*, 2014). The prediction of broiler performance using mathematical growth models and its resulting curves played a crucial role in the breeding of early-maturing broiler chickens with rapid growth. These models, when properly fitted and interpreted correctly, can provide very accurate estimates of body weight (BW) which is useful in the selection of young and highly producing birds (Freitas, 2005).

Mathematical models are very important in describing the growth patterns in poultry (Aggrey, 2002; Akinsola *et al*., 2021; Pinzón *et al*., 2022). Data such as BW, growth rate, and carcass weight are time series data, models are particularly useful because they facilitate the creation of a delineation scale for comparison (Nahashon *et al*., 2006). A graphical representation of the model parameters efficiently visualizes asymmetrical vacillations in BW attributed to random temporary or permanent environmental effects. Thus, geneticists can reduce these random environmental effects to the barest minimum by using breeding strategies.

One of the benefits of using mathematical growth models to fit chicken growth data is its use for the prognostication of bird BW at different stages of development, as well as exactly when BW experiences a descent (Yakupoglu and Atil, 2001). Furthermore, analysis of growth using mathematical models is particularly useful to animal breeders and farmers in developing countries; again, its prognostic utility is important in achieving genetic gain (Abbas *et al*., 2014).

In animal production, growth modelling is a highly sophisticated process involving the elucidation of longitudinal measurements using mostly 3 to 4 parameters that have biological meaning. Most species possess a growth trajectory that results in a sigmoidal curve, which can be explained using nonlinear models (NLM) (Aggrey, 2002; 2009). The biological meanings of model parameters must be clearly defined, as the (direct or inverse) relationship among the said parameters provides the bedrock for selection and breeding strategies designed to modify the growth pattern. Similarly, Manjula *et al.* (2018) opined that the asymptotic weight, age, and maximum weight gained at the apogee of inflection of the growth curve serve as useful tools for breeders to achieve genetic progress.

Logistic, Brody, Gompertz, and von Bertalanffy are models that have been widely used to describe chicken growth patterns (Thornley and France, 2007) – and these models are variants of the Richards model (Nahashon *et al*., 2006; Mohammed, 2015). The reliability of the estimates produced by the different mathematical growth models is contingent on the fulfilment of statistical assumptions (Mazucheli *et al*., 2011).

In recent years, the sigmoid and polynomial models (among other models) have been fitted to the growth data of chickens (Narushin and Takma, 2003; Michalczuk *et al*., 2016; Narinç *et al*., 2017). Litanies of studies have undoubtedly proven that Gompertz models – either the modified form (Laird *et al*., 1965) or the original form (Gompertz, 1825) - are the most suitable model for chicken growth data (Freitas, 2005; Tompić *et al*., 2011; Mazucheli *et al*., 2011; Drumond *et al*., 2013; Mohammed, 2015; Mota *et al*., 2015).

It is paramount to be careful when selecting which model to use. Every model produces varying parameter estimates, consequently, affecting the point of inflection of the growth curve (Gbangboche *et al*., 2008), and the inflection point is reflective of the economic merit or value of animals. For instance, the Logistic and Gompertz models have inflection points close to the asymptote around 50 and 37%, respectively (Teleken *et al.,* 2017), while that of the von Bertalanffy model is 26.9% (Demuner *et al*., 2017). However, for the Richards model (Richards, 1959), the inflection point vacillates based on the shape parameter (m).

The Gompertz model (Gompertz, 1825) modified by Laird *et al*. (1965) has been used by many researchers to analyse chicken growth data (Aggrey, 2002; Koncagul and Cadirci, 2009; Tjørve and Tjørve, 2017a; Quintana-Ospina *et al*., 2023) because, one, the model successfully passes several goodness of fits tests, making the model possess an overall fitness; and two, the biological interpretability of the model components. Meat-type birds, commonly known as broiler chickens, are usually slaughtered at 42 days of age, which means that these birds less than often reach mature BW and it’s difficult to know which bird will attain close to a mature BW. Here lies the advantage of the Gompertz-Laird model: using the inflection point and initial BW, the exponential decay rate of the bird can be calculated (Aggrey, 2002).

Gompertz and Logistic growth models fall under the 3-parameter sigmoid growth models of the Richards family (Tjørve and Tjørve, 2017a). Some of the prevailing conclusions in literature are: Richards and Gompertz models are suitable for broiler growth data (Tompić *et al*., 2011); the Logistic model works well with slow-growing chickens (Eleroğlu *et al*., 2014); and for Jinghai mixed-sex yellow chicken (Yang *et al*., 2006) and Japanese quails (Adedeji *et al*., 2017), the von Bertalanffy model has the best fit.

The Richards model, unlike Gompertz and Logistic models, does not have a fixed inflection point. Rather the point of inflection – that is, the transition from the accelerated growth phase to the retardation phase – for the Richards model is determined by the shape parameter (Richards, 1959). Even though the Richards model is deemed flexible and attractive by biostatisticians, it is not widely used in describing the growth patterns of chickens, because the Richards model is difficult to fit and the shape parameter (m) has no biological interpretation (Aggrey, 2002). Tjørve and Tjørve (2017b), on the other hand, held a favourable view towards Richards model. The shape parameter, according to Tjørve and Tjørve (2017b), is more inclined to respond to external fluctuations (such as environmental changes), unlike the asymptotic weight or growth or maturing rate. Thus, they concluded that the shape parameter could be used to assess the effects of environmental stressors on chicken growth.

Several studies have compared the growth curves of indigenous and commercial chickens in Africa. According to Yapi-Gnaore *et al*. (2011), in Cote d'Ivoire, indigenous chickens approached the point of inflection in 51.22 days, slower than commercial chickens (46.91 – 50.68 days for two different strains). Concomitantly, the function of the ratio of maximum growth to mature size growth (K) from chickens in Kabyle, Algeria, reported by Ait Kaki and Moula (2013) was 0.0260 and 0.0294 gd^{-1}; the K values were higher than those of Yapi-Gnaore *et al*. (2011). However, there is sparse literature on mathematical models describing the growth pattern of commercial broilers in tropical climates, especially in Sub-Saharan Africa; hence, the aim of this study.

The study was designed to compare the predictive power of different mathematical three- and four-parameter mathematical growth models, namely Gompertz, Gompertz-Laird, von Bertalanffy, Logistic, and Richards, to describe the growth pattern of commercial broilers at 0 to 6 weeks.

MATERIALS AND METHODS

Data used for this study was collected on 1,286 unsexed day-old Ross 308 broiler chicks. The birds were housed in a floor pen and provided with 22 hours of continuous light per day. Standard biosecurity procedures and vaccination routines were followed (AVIAGEN, 2018). Commercial chick mash (Topfeeds, Premier Feed Mills Company Limited) comprising of 2,993 kcal ME/kg and 22.3% CP was given for 0 to 42 days *ad libitum*, as well as clean drinking water. Birds were weighed at fixed intervals of one week for six weeks.

Growth Models: The models used for fitting the growth data are presented in Table 1.

Statistical Analysis: The data obtained from the repeated BW measurements of the birds at different ages were fitted to three- and four-parameter mathematical growth models. The growth curves and their resulting parameters were calculated using PROC NLIN (Gauss-Newton iterative process) of SAS 9.4 (SAS, 2002).

The BW measurements employed in this study were left unadjusted; primarily because these values are indicative of the bird's growth under an intensive system of poultry production. In this study, Computational Difficulty (CD) depended on whether or not the mathematical models used converged and the number of iterations it took to achieve convergence. Also, the amount of time taken to estimate the (unknown) parameter for each model was considered.

Goodness-of-fit criteria adopted for this study were adjusted coefficient of determination ($AdjR^2$), root means square error (RMSE), Bayesian information criterion (BIC), and Akaike's information criterion (AIC) based on Aggrey (2002), Demuner *et al*. (2017) and Akinsola *et al*. (2021).

Table 1: Growth models used for the analyses

Model	Equation	Age at point of inflection	Weight at point of inflection
Logistic	$\frac{a}{1 + b \cdot \exp(-k \cdot t)}$	$\frac{-ln(\frac{1}{b})}{k}$	$a \times 0.5$
Gompertz	$a \cdot \exp(-b \cdot \exp(-k \cdot t))$	$\ln(\frac{b}{k})$	$\frac{a}{e}$
Gompertz-Laird	$a_0 \cdot \exp((\frac{b_*}{k_*}) \cdot (1 - \exp(-k_* \cdot t)))$	$(\frac{1}{k_0}) \cdot \log(\frac{b_*}{k_*})$	$a_0 \cdot \exp((\frac{b_*}{k_*})^{-1})$
von Bertalanffy	$a \cdot (1 - b \cdot \exp(-k \cdot t))^3$	$\frac{\ln(3 \cdot b)}{k}$	$(\frac{8}{27}) \cdot a$
Richards	$\frac{a}{1 + b \cdot exp(-k \cdot t)^{1/m}}$	$\frac{-\ln(\frac{m}{b})}{k}$	$\frac{a}{(m+1)^{1/m}}$

Note: *W_t is the calculated weight at age t; a is the asymptotic (final or mature) weight; b is the scale parameter or integration constant, measuring the rate of gain in BW between hatch and maturity; k is the maturity rate; a_0 is the initial BW after hatching; b_* is the initial growth rate; k_* is the rate of decay; m is the shape parameter; and t, is the age (days). Gompertz, von Bertalanffy and Logistic models were adapted from FREITAS (2005); Gompertz-Laird model was adapted from LAIRD (1965); and Richards model was adapted from TOMPIĆ et al. (2011)*

A model with the highest R^2, and lowest RMSE and AIC for growth data is considered the best (Keskin and Dag, 2006; Keskin and Daskiran, 2007; Sahin *et al*., 2014). Similarly, Kaps and Lamberson (2004) posited that the model with the largest $AdjR^2$ *or* R^2 value is objectively the best, while a model with the lowermost BIC, MSE or AIC is considered the most superior model. Other values, such as age at the point of inflection, weight at the point of inflection and correlation coefficients between model parameters, were assessed.

The goodness-of-fit criteria were obtained using the following equations:

RMSE = $\sqrt{\frac{SSE}{(n-z)}}$; $MSE = \frac{SSE}{n-z} - (\frac{(n-1)(1-R^2)}{(n-z)})$

$R^2 = 1 - \left(\frac{SSE}{SST}\right)$ or $\frac{SST-SSE}{SST}$

AIC = $n \cdot \ln(\frac{SSE}{n}) + 2z$

BIC = $n \cdot \ln\left(\frac{SSE}{n}\right) + 2z \cdot \ln(n)$, where: SSE is the sum of the square of error; SST is the sum of the square total; n is the number of observations (weighting done on the live chickens); and *z*, is the number of parameters in the model. RMSE and $AdjR^2$ were adapted from Demuner *et al.* (2017), while AIC and BIC were adapted from Akinsola *et al*. (2021).

RESULTS

Predicted Coefficients for the Growth Parameters: The predicted coefficients for the growth parameters (a, a_0, b, b_*, k, k_*. and m) of each model are presented in Table 2.

Table 2: Predicted coefficients for the Gompertz, Richards, Gompertz-Laird, Logistic and von Bertalanffy growth parameters in commercial chickens

Models	Growth parameters	Predictions
Gompertz	Asymptotic weight (a)	4405.00
	Integration constant (b)	4.90
	Maturity rate (k)	0.04
Richards	Asymptotic weight (a)	1094.30
	Integration constant (b)	36.49
	Maturity rate (k)	0.11
	Shape parameter (m)	0.50
Gompertz-Laird	Asymptotic weight[1] (a)	2948.96
	Hatching weight (a_0)	32.76
	Initial growth rate (b_*)	0.18
	Rate of decay (k_*)	0.04
Logistic	Asymptotic weight (a)	2188.60
	Integration constant (b)	36.49
	Maturity rate (k)	0.11
von Bertalanffy	Asymptotic weight (a)	13901.20
	Integration constant (b)	0.89
	Maturity rate (k)	0.01

[1]Derived parameter; a is the asymptotic (final or mature) weight; b is the scale parameter or integration constant, measuring the rate of gain in BW between hatch and maturity; k is the maturity rate; a_0 is the initial BW after hatching; b_* is the initial growth rate; k_* is the rate of decay, and m is the shape parameter

von Bertalanffy predicted the highest mature or asymptotic weight (a), while Richards model gave the lowest prediction for a. The estimate of scale parameter or integration constant was highest in Richards and Logistic models, followed by the Gompertz, von Bertalanffy and Gompertz-Laird models, respectively. k predicted values ranged from 0.01 in von Bertalanffy to 0.11 in Richards and Logistic models.

Computational Difficulty (CD): The fitting of the five models to the growth data presented no computational difficulty whatsoever. Four out of the five models achieved convergence using a small number of iterations – only the Gompertz-Laird model converged using 41 number of iterations. The computation time (CT) varied from 0.25 to 0.29 seconds. The longest CT was detected in the Logistic model, while the shortest CT was observed in the Richards model. All in all, the models employed in this study met the convergence criterion for describing the body weight-age sigmoidal relationship in commercial broiler chickens (Table 3).

Table 3: The computational difficulty of each model

Model	Convergence criterion	Number of iterations	CT (sec)
Gompertz	Met	15	0.26
Gompertz-Laird	Met	41	0.28
Logistic	Met	12	0.29
von Bertalanffy	Met	13	0.28
Richards	Met	6	0.25

CT: Computation time

Goodness-of-Fit Criteria: The goodness-of-fit estimates for all the models can be found in Table 4. The difference in the $AdjR^2$ values for all the models was very small; however, the $AdjR^2$ values of the Gompertz and Gompertz-Laird models were the highest and similar, while the lowest values were seen in the Richards model.

Table 4: Goodness-of-fit estimates for the Gompertz, Richards, Gompertz-Laird, Logistic and von Bertalanffy growth parameters in commercial chickens

Model	RMSE	$AdjR^2$
Richards	72.21	0.994181
Gompertz	62.88	0.995587
Gompertz-Laird	62.88	0.995587
Logistic	65.92	0.995151
von Bertalanffy	62.98	0.995573
Model	**AIC**	**BIC**
Richards	79.74	89.32
Gompertz	76.89	84.08
Gompertz-Laird	76.89	84.08
Logistic	77.74	84.93
von Bertalanffy	76.92	84.11

Note: *RMSE is the root mean square error; $AdjR^2$ is the adjusted coefficient of determination; AIC is the Akaike information criterion; and BIC is the Bayesian information criterion*

The uppermost RMSE value was observed in the Richards model, while the lowest value was recorded in the Gompertz and Gompertz-Laird models. Similar observations were noted for the BIC and AIC values. The RMSE, AIC and BIC estimates for the von Bertalanffy model were closer to those of the Gompertz and Gompertz-Laird models, followed by the Logistic model.

Correlation Coefficients for Paired Growth Parameters: The PROC NLIN command in SAS produced an approximate correlation matrix, containing the correlation coefficients for paired growth parameters. That is, the correlation between a/a_0 and b/b_*, a/a_0 and k/k_*, and b/b_* and k/k_*, was estimated. Presented in Table 5 are the correlation coefficients among the growth model parameters. In all the models, excluding von Bertalanffy, a negative correlation was observed between a/a_0 and b/b_*. Similarly, a positive correlation coefficient was produced for the relationship between b/b_* and k/k_* for all the models, except von Bertalanffy. A negative correlation coefficient was observed between a/a_0 and k/k_* for all the models.

Table 5: Correlation among the parameters for each model

Model	a/a_0 and b/b	a/a_0 and k/k_*	b/b_* and k/k_*
Gompertz	-0.3	-0.99	0.44
Gompertz-Laird	-0.99	-0.95	0.99
Logistic	-0.51	-0.93	0.77
von Bertalanffy	0.63	-0.99	-0.58
Richards	-0.51	-0.93	0.77

Note: *a is the asymptotic (final or mature) weight; b is the scale parameter or integration constant, measuring the rate of gain in BW between hatch and maturity; k is the maturity rate; a_0 is the initial BW after hatching; b_* is the initial growth rate; and k_* is the rate of decay*

Body Weight and Age at Inflection Point: The age and BW at inflection for all five nonlinear models are presented in Table 6. The von Bertalanffy model gave the highest predictions for BW and age at the inflection point (4118.87 g and 98.21 days, respectively). The Gompertz-Laird model, on the other hand, had the lowest value prediction for BW at inflection (40.91 g).

Table 6: Body weight and age at the inflection point

Model	T_i (days)	W_i (grams)
Richards	39.00	1459.07
Gompertz	39.73	1620.51
Gompertz-Laird	16.33	40.92
Logistic	32.70	1094.30
von Bertalanffy	98.21	4118.87

Note: *Ti is the age at the point of inflection, while Wi is the BW at the point of inflection*

Juxtaposing the Observed Body Weight with the Predicted Body Weight: The predicted BWs by each model and the observed BWs were juxtaposed and presented as sigmoid growth curves (Figures 1 – 5).

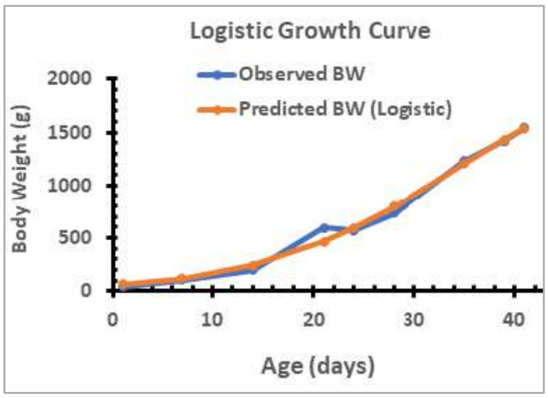

Figure 1: Growth curve of commercial broiler chickens as predicted by the Logistic model

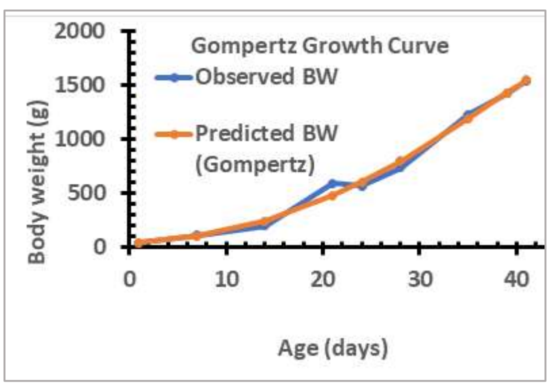

Figure 2: Growth curve of commercial broiler chickens as predicted by the Gompertz model

Furthermore, the residual between the predicted BWs and the observed BWs were compared (Table 7). The Gompertz and Gompertz-Laird models produced BW predictions that were closer to the observed values, followed by the von Bertalanffy model. Compared to the other models, the Logistic and Richards models gave the worst predictions.

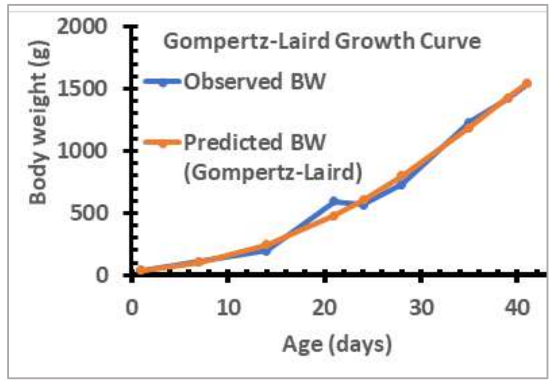

Figure 3: Growth curve of commercial broiler chickens as predicted by Gompertz-Laird curve

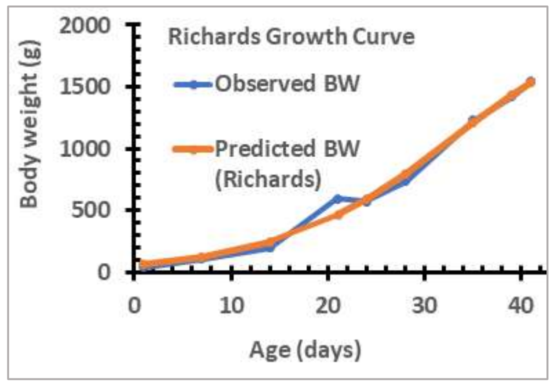

Figure 4: Growth curve of commercial broiler chickens as predicted by Richards model

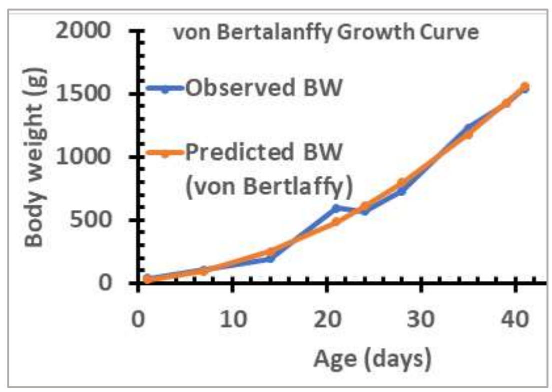

Figure 5: Growth curve of commercial broiler chickens as predicted by von Bertalanffy model

DISCUSSION

Mathematical growth models must have parameters that can be interpreted within a biological context or framework. The interpretability of these growth models is crucial when describing time series data (such as BW measurements) and predicting the expected BW at different ages (Selvaggi *et al*., 2015).

Table 7: Residuals for each growth model at different ages

Age (days)	Gompertz	Gompertz-Laird	Logistic	von Bertalanffy	Richards
1	-0.82	-0.82	-26.47	11.457	-26.47
7	5.43	5.43	-13.77	10.747	-13.77
14	-49.37	-49.37	-49.00	-54.575	-49.00
21	115.86	115.86	130.63	109.624	130.63
24	-37.32	-37.32	-24.48	-40.358	-24.48
28	-67.50	-67.50	-65.80	-64.596	-65.80
35	42.59	42.59	24.35	50.976	24.35
39	-6.34	-6.34	-12.20	-4.662	-12.20
41	-5.67	-5.67	6.51	-11.722	6.51

Aside from the interpretability of growth models, it is important to pay attention to the model parameters themselves, as it allows for pairwise comparisons among the models – by determining which models yielded the closest and most realistic matured or asymptotic weight (*a*). Moreover, the matured weight estimate creates the basal rock for model comparison, seeing as the integration constant (*b*) and maturity rate (*k*), unlike *a*, are known to measure somewhat different aspects of growth among models (Aggrey, 2002). The matured weight of a chicken is the apogee of its growth curve; that is, it is the highest growth response the bird will produce in its lifetime (Narinç *et al*. 2010). As expected, all the five nonlinear mathematical growth models used in this study gave different and similar matured weight predictions. The Gompertz and Gompertz-Laird models gave similar predictions for the matured weight because the Gompertz-Laird model is a modified version of the original Gompertz model. Also, similar estimates were obtained from the Logistic and Richards models. On the other hand, the largest matured weight prediction was obtained from the von Bertalanffy model. The rule of thumb is, that larger matured weight estimates are linked with small maturity rate estimates, as evidenced by the high negative correlation observed between *a* and *k* in this study. The high matured weight (*a*) prediction by the von Bertalanffy model may be indicative of an overestimation, seeing as the standard error for (*a*) value estimated by this model was the highest observed.

Logistic and Richards models produced the highest and similar integration constant (*b*) estimated. Abe *et al*. (2022) also reported that the highest (*b*) value prediction occurred using the Logistic model. This high value suggests that both models forecasted more BW gain after the initial hatching weight of the birds, unlike other models (Durosaro *et al*., 2021). However, the matured weight predictions by these models were not reflective of the excessive BW gain predictions. Furthermore, a high *b* estimate implies that the model will reach its diminishing growth phase quicker than other models. The obtained *b* value for the Gompertz model in this study was similar to the one obtained by Al-Samarai (2015) and higher than the *b* values obtained for Gompertz, Logistic and von Bertalanffy models by Durosaro *et al*. (2021).

The growth model parameter *k* measures the rate of maturity. The Logistic and Richards models produced similar and the highest *k* estimate, followed by the Gompertz and Gompertz-Laird models, while the lowest value was observed in the von Bertalanffy model. Typically, large *k* values indicate that the birds will attain maturity early, while small *k* values suggest the opposite. Thus, Logistic and Richards models, contrasting the other models, predict that the chickens used in this study will attain early maturity. The estimates obtained for *k* were lower than the values reported by Al-Samarai (2015).

The correlation coefficients among the growth parameters obtained in this study bear semblance to the reports of Al-Samarai (2015). Notably, the correlation between *a* and *k* was highly negative across all models. This negative correlation suggests that fast-growing chickens will not attain larger matured weight. Furthermore, it indicates that attaining larger matured weights will require longer growth periods. Similar results were obtained by Durosaro *et al*. (2021) and Abe *et al* (2022). When *a* is negatively correlated with *k*, it poses a risk to genetic improvement programs that aim to increase the growth rate, as such an increment will cause a concomitant decrease in matured weights.

The growth data was not large, so, no computational difficulty was experienced during the analyses. All models met the convergence criterion. However, the Richards model, compared to the other models used in this study, converged using the lowest number of iterations. The moderate to low iteration numbers observed in the converged Gompertz, Logistic, von Bertalanffy and Richards models used in this study infer their usefulness and relevance when describing the growth pattern of commercial broilers.

As much as modelling the growth of commercial broilers helps to reduce the variables involved in describing chickens' growth, its usefulness is tied to the accuracy of its predictions (Durosaro *et al.*, 2021). Consequently, goodness-of-fit criteria are used to judge the predictive accuracy of growth models. In this study, a high adjusted coefficient of determination ($AdjR^2$) was observed in all the models – with the highest value detected in the Gompertz and Gompertz-Laird models. The observed highest $AdjR^2$ indicated that the model(s) fitted the bodyweight data more accurately. Hypothetically, the four-parameter Richards model is supposed to yield an $AdjR^2$ value that is more than the three-parameter Logistic, von Bertalanffy and Gompertz models (Aggrey, 2002). Both the $AdjR^2$ and R^2 values were relatively close, in this study. Similar $AdjR^2$ values were obtained by Moharrery and Mirzaei (2014); they reported that the range for $AdjR^2$ values of the Gompertz, Logistic and Richards models was between 0.979 and 0.995.

Using the $AdjR^2$ value alone, all the models seem like a good fit for the age-weight data used in this study. The difference in the $AdjR^2$ values obtained for each is quite small, making them so numerically close that the difference is almost insignificant. Therefore, other goodness-of-fit criteria were employed to find which model best explained the observed changes in BW. The Akaike's information criterion (AIC), Root Mean Square Error (RMSE), and Bayesian Information Criterion (BIC) were estimated to determine the best-fitting model for the growth data used. BIC and AIC measure how well each growth model fits the live weight data because maximum likelihood ratio tests favour models that offer predictive accuracy using few parameters. Gompertz and Gompertz-Laird models had the lowest RMSE, AIC and BIC values, while the highest values were observed in the Richards model. The RMSE, BIC and AIC values in this study were lower than the values stated by Demuner *et al.* (2017), Akinsola *et al.* (2021) and Abe *et al.* (2022). A plausible explanation for these variations is the experimental location and sample size. Using the $AdjR^2$, RMSE, AIC, and BIC values, the Gompertz and Gompertz-Laird models seem to be the best model for the age-weight data used in this study.

There were observed differences in age and weight at the inflection point among the models. The von Bertalanffy model predicted that it would take a longer time for the bird to reach the inflection point. Similar results were obtained by Abe *et al.* (2022). Since von Bertalanffy gave the highest prediction for the asymptotic weight and weight at inflection, it is only corollary that the time at inflection will be longer than the predictions of other models. However, the predictions obtained from the von Bertalanffy model in this study are regarded as an overestimation because of its high standard error.

The initial hatching or BW (a_0) predicted by the Gompertz-Laird model can be used in forecasting the initial growth rate from hatching till the inflection point. On the other hand, the Gompertz-Laird model seems to underestimate the weight at the point of inflection for the growth used here. This underestimation casts doubt on the predictive accuracy of the Gompertz-Laird model for the data used in this study. Solutions to improve the fitting of the growth data, such as limiting the a_0, using the variance inverse to weight a_0, and restricting a_0 within two standard deviations of the mean, have been proffered by Naimi *et al.* (2014) and Goto and Xu (2015) respectively.

For the growth data used in this study, the values obtained for the *b* and *k* parameters were similar for the Richards and Logistic models. Other similarities are: that the predicted BWs for both models were the same; the weight at inflection for the Logistic model was the same as the asymptotic weight for the Richards model; and, both models showed similar growth trajectories. This observed similarity between the

Logistic and Richards models is in contrast with Nahashon *et al.* (2006), who reported that when the shape parameter is 2.0, the Richards model is equivalent to the Logistic model. However, the similarity was not observed in the age and weight at the inflection of both models. Consequently, the model parameters from different growth models are not exactly analogous.

Aside from using the goodness-of-fit criterion to judge each model, a complimentary method, involving a subjective pairwise comparison of the observed and predicted bodyweights, to judge the accuracy of the model was done. This comparison is useful in detecting whether or not a model overestimated or underestimated BW at any age. The five models showed a good fit for the data; however, the Gompertz model appears to be the best fit. Safari *et al*. (2021) posited that any model that produced a difference between the observed and predicted BW, as well as changes in numerical signs at short breaks is better than a model that changes in signs at longer breaks. Also, the growth curve for each model provides a template for deducing the fitness of a model; that is, the growth curve that is closest to the observed BW is the most suitable model. Another keen observation noted in this study was: that there seems to be a relationship between the four-parameter Richards model and the three-parameter Logistic and (traditional) Gompertz model based on the value of the shape parameter.

Lastly, the results of this study will be particularly useful to commercial broiler farmers, as suitable growth models can be used to detect the matured BW of birds after the first seven days. The calculation is quite simple. The obtainable BW of a chicken can be estimated by inputting the age (in days) of the bird in place of the parameter t in the most suitable model.

Conclusion: Using the complimentary method, involving a subjective pairwise comparison of the observed and predicted BW, the Logistic, Gompertz-Laird, Richards and von Bertalanffy models fitted the growth data very well. However, the Gompertz model produced the best fitting. Using the goodness-of-fit criteria, the Gompertz and Gompertz-Laird models are the best for describing commercial broiler chickens' growth. All in all, the Gompertz models are the most appropriate models for describing the growth pattern of commercial broiler chickens.

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