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Modelling persistence in the conditional mean of inflation using the ARFIMA process with GARCH and GJR-GARCH innovations: The case of Ghana and South Africa

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Abstract

This paper contributes to the debate on inflation persistence by extending an ARFIMA process with GARCH and GJR-GARCH models to describe the time-dependent heteroskedasticity and persistence in the conditional mean of Consumer Price Index (CPI) inflation series of Ghana and South Africa, under three distributional assumptions (i.e., Normal, Student-t and Generalised Error Distributions). ARFIMA-GJR-GARCH under Generalised Error Distribution and Student-t Distribution respectively, provided the best fit for modelling the time-dependent heteroskedasticity and persistence in the conditional mean of CPI inflation rate of Ghana and South Africa. The results from the study provided evidence of persistence, mean reverting though, and asymmetric effect of economic shocks on the conditional mean of CPI inflation rate of the two countries. These results would, therefore, be useful to both countries in making good portfolio decisions, assessing the efficacy of a monetary policy or programme meant to control inflation persistence and also serving as a tool for detecting volatility and its impact, for the Ghanaian and South African inflation rates and their economies at large.

Keywords: CPI Inflation; Fractional integration; Persistence; Conditional mean; ARFIMA; GARCH; GJR-GARCH models.

1. Introduction

Inflation is an important determinant of economic growth, the cost of living, and the total well-being of a population. As a result, it has been one of the most researched topics in macroeconomics, especially from empirical and theoretical points of view. The principal objective of monetary policy is to stabilise prices and increase output. Therefore, in order to optimise the efficacy of monetary policy, monetary authorities monitor inflation persistence.

To contribute to this debate and clarify the issue of inflation persistence, various definitions and approaches have been proposed. The first body of literature is based on whether inflation follows a stationary process, *I(0)* or a non-stationary process, *I(1)*. As an example, Ball and Cecchetti (1990), Kim (1993), Nelson and Schwert (1997), Banerjee *et al*. (2001), and Boateng *et al*. (2016), among others, have all established evidence of unit root, *I(1)* in inflation rate, whereas Rose (1988), and Grier and Perry (1998) claim that the inflation rate is *I(0)* process. However, Kirchgässner and Wolters (1993), and Bos *et al*. (1999) found mixed results.

Another line of argument put forward by Narayan (2014), and Narayan and Lui (2015) is the use of unit root test in ascertaining unit root or stationarity of an inflationary process. They argue that most of the unit root tests are based on the standard linear models and that they it consider to be inappropriate since most of these financial data sets, including inflation rate, occasionally exhibit level shifts or structural breaks with noteworthy Autoregressive Conditional Heteroskedasticity (ARCH) effects. They then went ahead to propose a unit root test with two endogenous structural breaks that explicitly include a Generalised ARCH (GARCH)(1,1) specification for volatility. Their findings brought clarity and answered the question on unit root in inflation rate.

Again, in literature, several opinions on the characterisation of inflation have been debated. A number of these studies contend that inflation is better characterised by a long memory Autoregressive Fractionally Integrated Moving Average (ARFIMA) process in order to capture persistence, for instance, Hassler and Wolters (1995), Baillie *et al*. (1996), Baum *et al*. (1999), Gadea and Mayoral (2005), Morana and Bagliano (2007), and Bos *et al*. (2014). Another stream of modelling inflation persistence with reference to structural breaks across switching regimes has also been proposed and applied by researchers such as Osborn and Sensier (2009), Gonzalez *et al*. (2009), Boero *et al*. (2009), and Narayan (2014), who provided a regular evidence across time periods.

Diebold and Inoue (2001), Granger and Hyung (2004), and Hyung *et al*. (2006) gave another strand of studies on inflation rate that combines the concept of a long memory process and structural breaks. In fact, they argued and gave evidence to the effect that neglecting structural breaks in the time series data including inflation, can introduce biasedness in the estimation of long memory, which induces persistence.

Cecchetti and Debelle (2006) undertook a study on inflation persistence for major industrial economies and found inflation to be less persistent than ignoring regime shifts in the mean, conditional on the break in the intercept. Furthermore, in their study, Cecchetti *et al*. (2007) gave evidence regarding the use and implementation of models that take changes in the mean, volatility and persistence of inflation. In their research, Belkhouja and Boutahar (2009) employed the ARFIMA model to the US inflation rate and found a decrease in long memory persistence estimate after taking regime shifts in the level together with persistence. Kang *et al*. (2009) investigated the existence and timing of changes in US inflation persistence. Using an unobserved components model of inflation with Markov-switching parameters, they measured persistence using impulse response functions based on the model. Indeed, an important feature of their model allowed for multiple regime shifts in parameters related to the size and propagation of shocks. They found inflation persistence to be dependent on the configuration of these parameters, although there was no need for change, even if the parameters changed. Again, using the GDP deflator for the sample period of 1959-2006, they found two sudden permanent regime shifts in US inflation, both of which corresponded to changes in persistence. In their study, the first regime shift occurred around the collapse of the Bretton Woods system at the beginning of the 1970's and produced an increase in inflation persistence, while the second regime shift occurred immediately after the Volcker disinflation in the early 1980's and produced a decrease in inflation persistence. Their results, to a large extent, were consistent with the New Keynesian Phillips Curve. The gap between inflation and its long-run trend displayed little or no persistence throughout the entire sample period.

In another study, Baillie and Morana (2012) proposed an Adaptive ARFIMA Adaptive Fractionally Integrated GARCH (FIGARCH) model permitting the baseline volatility to be time-varying in accordance with Gallants' (1984) specification of a flexible Fourier form (FFF). They then applied this novel methodology on the G7 inflation rates and found the estimates of the fractional integration parameter to be smaller in magnitude than implied by conventional ARFIMA models without any adjustments for the unconditional mean varying.

Another aspect of literature looked at the changes in the long memory persistence in the inflation process. For example, during an estimation of a fractional integration parameter, Kumar and Okimoto (2007) identified a structural break in the US inflation to be coinciding with 1982. Hassler and Meller (2014) extended this method by testing for multiple structural breaks in the mark of long memory. Recently, Belkhouja and Mootamri (2016) examined the long memory and structural change in the G7 inflation dynamics. They employed an extended ARFIMA-GARCH model by allowing the baseline mean and volatility to be time-dependent, using logistic functions for G7 countries from 1995 to 2014. The main finding of their study is that neglecting structural changes in the inflation level and volatility seems to overestimate the long-run and GARCH persistence.

Conrad *et al.* (2011) analysed the applicability of a multivariate constant conditional correlation version of a proposed model initiated by Tse (1998), which combines the fractionally integrated GARCH formulation of Baillie *et al*. (1996), with the asymmetric power ARCH specification of Ding *et al*. (1993), to stock market returns for eight countries. They found multivariate specification to be generally applicable once power, leverage and long-memory effects are taken into consideration. In addition, they discovered that both the optimal fractional differencing parameter and power transformation are remarkably similar across countries. Out-of-sample evidence for the superior forecasting ability of the multivariate Fractionally Integrated Asymmetric Autoregressive Conditional Heteroskedasticity (FIAPARCH) framework is provided in terms of forecast error statistics and tests for equal forecast accuracy of the various models. Ahmed (2013) applied an extended standard GARCH (s-GARCH) and exponential GARCH (e-GARCH) models with an ARFIMA process to model the monthly inflation series of Oman under skewed generalised error distribution. His results suggested that e-GARCH(1,1)-ARFIMA model with skewed generalised error distribution of residuals should be preferred for short term forecasting for Oman inflation rates. In all of these empirical work, it is clear that structural breaks and their ramification in the estimation of persistence have several economic implications, and therefore, failure to incorporate or account for structural breaks in a model building process could have serious repercussions in the parameter estimates.

The bedrock of this paper is the ARFIMA-GARCH model introduced by Baillie *et al*. (1996) to describe inflation dynamics of ten countries and the proposed methodology by Belkhouja and Mootamri (2016), where they employed an extended ARFIMA-GARCH model by allowing the baseline mean and volatility to be time-dependent with logistic functions. In this paper, we attempt to investigate inflation persistence in the conditional mean in another way by extending the ARFIMA model with GARCH (Bollerslev, 1986) and GJR-GARCH (Glosten *et al.*, 1993) processes to describe the time-dependent heteroskedasticity under Student-*t* and Generalised Error distributions together with the Normal distribution, under different regime shifts and compare the results[1]. Indeed, this paper contributes to inflation studies in these two countries along two lines: (1) to examine the relationship between the conditional mean and variance of inflation displaying long memory in its level, but time-varying and (2) to assess the impact of inflationary or economic shocks in the conditional mean of inflation. These extensions have been proposed in order to investigate the impact or effect of economic shocks (be it symmetric or asymmetric) on the inflation rates of Ghana and South Africa, the only two countries in sub-Saharan Africa with IT policy.

In Ghana, not much has been done as far as estimation of inflation persistence is concerned. However, Boateng *et al*. (2016) examined the CPI inflation rate of Ghana from a different perspective, by allowing for fractional degrees of integration and employing techniques based on Whittle parametric and semi-parametric methods and an ARFIMA model together with standard I(0)/I(1) methods. Their findings indicated the presence of long memory, which induces persistence in CPI inflation rate of Ghana. Alagidede *et al*. (2014) examined the crucial issue of inflation persistence in Ghana in order to gain a better understanding regarding welfare and policy implications. Specifically, their study investigated the existence of persistence at both aggregate (national) and regional levels. Moreover, their study included investigation of persistence across 13 sectors, covering both core and headline inflation rates. Employing fractional integration methods, their study provided some important additions to the literature. Their results showed evidence suggesting i) asymmetries in the degrees of inflation persistence both regionally and sectorally; and ii) high potential for significantly different conclusions about inflation persistence, depending on whether month-on-month inflation or year-on-year inflation was being assessed.

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[1] This study employs the recently extended models in the Rugarch package in 2015 by Alexios Ghalanos. In this package, sGARCH and gjrGARCH allows for exogenous variables and are equivalent to GARCH and GJR-GARCH models. In this paper we chose to apply the GARCH and GJR-GARCH notations.

In South Africa, there has been a surge in research on estimation of inflation persistence, with little emphasis on time-varying volatility. For example, Gil-Alana (2011) analysed South African inflation for the period 1970-2008 using long range dependence techniques and found it to be a covariance stationary process with long range dependence, with an order of integration ranging in the interval (0, 0.5). Burger and Marinkov (2008) conducted a study which sort to obtain recursive estimates for inflation persistence before and after IT policy. They found that inflation persistence had not decreased since the introduction of the IT policy (though mean reverting), but expressed concern about the South African Reserve Bank's ability to decrease inflation persistence at all, with the new monetary policy regime. In another study, Rangasamy (2009) employed an eminent univariate autoregressive equation to measure inflation persistence at aggregate and disaggregate levels of inflation. Balcilar *et al*. (2016) used a fractionally integrated model in the context of regime switching model set up to investigate inflation persistence, and concluded that inflation persistence is much stronger in high inflation regime compared to low inflation regime, while controlling for volatility, confirming the results of Rangasamy (2009).

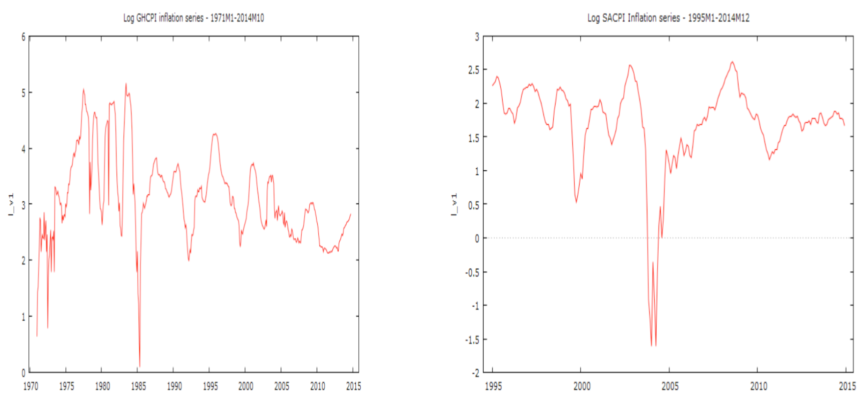
We are motivated to examine inflation persistence in the conditional mean by incorporating the time-dependent heteroskedastic component in the ARFIMA model, mainly because of superficiality and gaps in the literature. These gaps include the exclusion of structural breaks in model specifications and the choice of normal distribution of the error term in the ARFIMA model. To the best of our knowledge, there is a limited number of studies in the two countries that have addressed the issue of persistence in the conditional mean of inflation using the ARFIMA model extended with GARCH and GJR-GARCH innovations, which are mainly meant to capture the impact of economic shocks, be it symmetric or asymmetric; hence the need to fill this gap. Indeed, the results from this study suggest that Ghana Consumer Price Index (GHCPI) inflation series and South Africa Consumer Price Index (SACPI) inflation series may be persistent but mean reverting, though with an asymmetric effect of negative economic shocks impacting heavily on the conditional mean than positive economic shocks.

The outline of the paper is as follows: Section 2 provides characteristics of the data and descriptive statistics. In Section 3, we present ARFIMA-GARCH and ARFIMA-GJR-GARCH processes under three distributional assumptions and different regime shifts or structural breaks. Empirical results are presented in Section 4, while Section 5 provides concluding remarks.

2. The data and descriptive statistics

The data used in this study comprise log monthly Ghana Consumer Price Index (GHCPI) and South Africa Consumer Price Index (SACPI) inflation series, covering the period from January 1971 to October 2014 and from January 1995 to December 2014, respectively. Both data sets were obtained from the Bank of Ghana and Statistics South Africa. Figure 1 describes the behaviour of GHCPI and SACPI inflation series. The descriptive and summary statistics of GHCPI and SACPI inflation series are presented in Table 1. From Table 1, it is evident that the distribution of GHCPI and SACPI inflation is fat-tailed since kurtosis is greater than 3. The coefficient of skewness for GHCPI is 0.29 which is rightly skewed, whereas that of South Africa is skewed to the left with a coefficient of -2.41. These show that the distributions of both countries are non-Gaussian (non-normal) and leptokurtic. The *J-B* test confirms these findings since it rejects the normality assumption.

Figure 1: The Behaviour of Log Monthly Inflation Series of GHCPI (left) and SACPI (right)



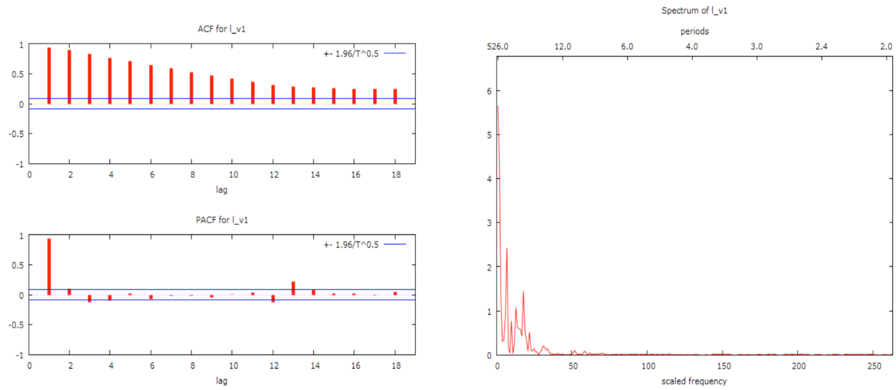
Results from the ARCH-LM test for the conditional heteroskedasticity up to lag 18, provide strong evidence of ARCH effects in the inflation series of both countries. Note, however, that these results may be taken with caution owing to the potential presence of breaks in the data. The presence of a significant non-zero autocorrelation can also be seen in Table 1, with the Box-Pierce *Q*-statistic coefficients of 2985.64 and 916.25 for Ghana and South Africa, respectively.

Table 1: Descriptive and Summary Statistics of GHCPI and SACPI Inflation Series

	GHCPI Inflation		**SACPI Inflation**	
Mean	3.13		1.68	
Standard Deviation	3.03		0.65	
Skewness	0.29		-2.41	
Kurtosis	3.49		11.27	
Min.	0.09		-1.60	
Max.	5.16		2.61	
J-B test	12.94 [0.00]*		921.84 [0.00]*	
ARCH-LM test	357.00 [0.00]*		164.92 [0.00]*	
Box-Pierce Q(18) test	2985.64 [0.00]*		916.25 [0.00]*	
Test type	**Constant**	**Constant + trend**	**Constant**	**Constant + trend**
ADF	-3.29 [-2.86]*	-4.17 [-3.41]*	-3.57 [-2.86]*	-3.57 [-3.42]*
ADF+GLS (Perron and Qu test)	-	-1.32 [-2.89]*	-	-2.41 [-2.89]*
PP	-4.92 [-2.86]*	-5.55 [-3.41]*	-3.37 [-2.87]*	-3.40 [-3.42]*
KPSS	0.76 [0.46]*	0.16 [0.14]*	0.14 [0.46]	0.12 [0.14]
ZA	-5.70 [-4.80]*	-5.81 [-5.08]*	-5.70 [-4.80]*	-5.81 [-5.08]*

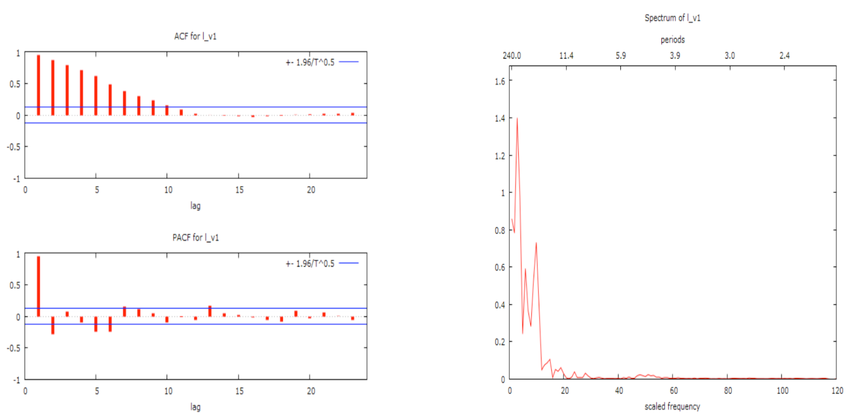
Note: The table describes several descriptive and summary statistics including the mean, median, standard deviation, skewness, kurtosis, Jarque-Bera (J-B), ARCH-LM and Box-Pierce Q(18) tests for GHCPI and SACPI inflation series. In all cases (constant, as well as both constant and trend), the hypothesis of stationarity is rejected at 5% significant level for ADF, PP, KPSS (for GHCPI), ADF-GLS (Perron and Qu) and ZA tests, hence the GHCPI and SACPI inflation series are non-stationary. In parenthesis are the critical values, where * indicates significance at the 5% level.

Figure 2: An Autocorrelation (ACF) and Partial Autocorrelation (PACF) Functions (left) and the Spectral Density Functions (right), for GHCPI Inflation Series



Again, the distributional characteristics of GHCPI and SACPI inflation series presented in Figures 2 and 3 - the ACF, PACF and spectral density function respectively, can be investigated further by analysing the behaviour of the autocorrelation and spectral density functions. The autocorrelation functions of GHCPI and SACPI inflation series appear to decrease slowly at a hyperbolic rate, a clear indication of long memory.

Figure 3: An Autocorrelation (ACF) and Partial Autocorrelation (PACF) Functions (left) and the Spectral Density Functions (right), for SACPI Inflation Series



Testing for long memory is an essential task since evidence of long memory, which induces persistence, supports the use of an ARFIMA model. We, therefore, affirm the presence of long memory in the two IT countries using Local Whittle estimator by Robinson (1995) and spectral regression model by Geweke and Porter-Hudak (1983) (see Table 2).

Table 2: Parameter Estimates for Long Memory for GHCPI and SACPI Inflation Series

Test	**GHCPI Inflation series**	**SACPI Inflation series**
Fractional Integration parameter		
Local Whittle estimator (LW)	0.67 [0.00]*	0.93 [0.00]*
Geweke and Potter-Hudak (GPH)	0.75 [0.00]*	0.92 [0.00]*

Note: *The null hypothesis of no long memory is rejected in each case at 5% significance level. In parenthesis are the p-values

3. Methods

In this section, we present an extensive examination of the methods used in modelling persistence in the conditional mean of GHCPI and SACPI inflation series. This involves an extension of ARFIMA model with the GARCH and GJR-GARCH-type process, by taking different regime shifts into consideration, to describe persistence and time-dependent heteroskedasticity under Generalised Error Distribution (GED), Student-*t* Distribution (STD) and Normal Distribution (Norm) assumptions, and compare the results.

3.1. ARFIMA(p,d,q)-GARCH(P,Q) model

The fractionally integrated ARFIMA*(p,d,q)*-GARCH*(P,Q)* process is given by:

$$(L)(1-L)^d(y_t-\mu-bx_{1t}-\delta\sigma_t)=\theta(L)\varepsilon_t \quad (1)$$

$$\varepsilon_t|\Omega_{t-1}\sim D(0,\sigma_t^2) \quad (2)$$

$$\beta(L)\sigma_t^2=\omega+\alpha(L)\varepsilon_t^2x_{2t}+\gamma' x_{2t} \quad (3)$$

where $y_t = 100\Delta logCPI_t$ denotes the CPI inflation series, x_{1t} and x_{2t} are vectors of predetermined variables, μ is the mean of the process,
$\phi(L) = 1- \phi_1(L) - \cdots - \phi_p(L)^p$, $\theta(L) = 1+ \theta_1(L) + \cdots + (L)^q$,
$\beta(L) = 1- \beta_1(L)- \cdots - \beta_p(L)^p$, $\alpha(L) = 1+ \alpha_1(L)+ \cdots + \alpha_Q(L)^Q$, and
all the roots of $\phi(L)$, $\theta(L)$, $\beta(L)$ and $\alpha(L)$ lie outside the unit circle.

Equation (1) describes the ARFIMA process introduced by Granger (1980) and Granger and Joyeux (1980), when $\delta = 0$ and $b = 0$. With $\delta \neq 0$, the model is extended to allow volatility to influence the mean inflation. The innovations ε_t are assumed to follow a conditional density D, which is either GED or STD or Norm. The time-dependent heteroskedasticity σ_t^2 follows symmetric GARCH model of Bollerslev (1986). Lagged inflation, which is predetermined, is allowed to possibly enter the conditional variance equation (3) through x_{2t}.

3.2. ARFIMA(p,d,q)-GJR-GARCH(P,Q) model

The fractionally integrated ARFIMA*(p,d,q)*-GJR-GARCH*(P,Q)* process is given by:

$$\phi(L)(1-L)^d(y_t - \mu - bx_{1t} - \delta\sigma_t) = \theta(L)\varepsilon_t \tag{4}$$

$$\varepsilon_t|\Omega_{t-1} \sim D(0, \sigma_t^2) \tag{5}$$

$$\sigma_t^2 = \left(\omega + \sum_{j=1}^{m} \varsigma_j\, \upsilon_{jt}\right) + \sum_{j=1}^{q}\left(\alpha_j \varepsilon_{t-1}^2 + \gamma_j I_{t-1}\varepsilon_{t-j}^2\right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{6}$$

where $y_t = 100\Delta logCPI_t$ denotes the CPI inflation series, x_{1t} and x_2 are vectors of predetermined variables, μ is the mean of the process,
$\phi(L) = 1 - \phi_1(L) - \cdots - \phi_p(\mathrm{L})^p$, $\theta(L) = 1 + \theta_1(L) + \cdots + (\mathrm{L})^q$,
$\beta(L) = 1 - \beta_1(L) - \cdots - \beta_p(\mathrm{L})^p$, $\alpha(L) = 1 + \alpha_1(L) + \cdots + \alpha_Q(\mathrm{L})^Q$, and all the roots of $\phi(L)$, $\theta(L)$, $\beta(L)$ and $\alpha(L)$ lie outside the unit circle.

Similarly, equation (4) describes the ARFIMA process proposed by Granger (1980) and Granger and Joyeux (1980), with $\delta = 0$ and $b = 0$. With $\delta \neq 0$, the model is extended to allow volatility to influence the mean inflation. Again, the innovations ε_t are assumed to follow a conditional density D, which is either GED or STD or Norm. The time-dependent heteroskedasticity σ_t^2 follows an asymmetric GJR-GARCH model by Glosten *et al*. (1993).

The GJR-GARCH, presented in equation (6), models the positive and the negative inflationary shocks on the conditional variance or inflation uncertainty asymmetrically through the use of an indicator function *I*, where y_j denotes the leverage effect. The indicator function *I* takes on the value 1 for $\varepsilon \leq 0$ and 0 otherwise. Because of the presence of the indicator function, the persistence of the model $\hat{P}$ in equation (7) critically depends on the asymmetry of the conditional distribution used. This is given by:

$$\hat{P} = \sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j + \sum_{j=1}^{q} \gamma_j \kappa \tag{7}$$

where k is the expected value of the standardised residuals z_t below zero (effectively the probability of being below zero):

$$\kappa = E\left(I_{t-j} z_{t-j}^2\right) = \int_{-\infty}^{0} f(z, 0, 1, \ldots,) dz \tag{8}$$

where f is the standardised conditional density with any additional skew and shape parameters.

3.2.1. Maximum likelihood estimation

The Gaussian ARFIMA*(p,d,q)* model is defined as:

$$\phi(L)(1-L)^d (y_t - \mu - b x_{1t} - \delta \sigma_t) = \theta(L) \varepsilon_t$$
$$\varepsilon_t | \Omega_{t-1} \sim NID(0, \sigma_t^2) \tag{9}$$

where d is the fractional integration parameter, y_t denotes the CPI inflation rates, x_{1t} and x_{2t} are vectors of predetermined variables, μ is the mean of the process with, and $\phi(L) = 1 - \phi_1(L) - \cdots - \phi_p(\mathrm{L})^p$, and $\theta(L) = 1 + \theta_1(L) + \cdots + \theta_q(\mathrm{L})^q$ denote the AR and MA lag-polynomial, respectively. Now, for a stationary ARMA process with mean μ and an autocovariance function, $y_i = E[(y_t - \mu)(y_{t-i} - \mu)]$ defines the variance matrix of the joint distribution of $y = (y_1, \ldots, y_T)'$ given by:

$$V(y) = \begin{pmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{T-1} \\ \gamma_1 & \gamma_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \gamma_1 \\ \gamma_{T-1} & \cdots & \gamma_1 & \gamma_0 \end{pmatrix} = \Sigma \tag{10}$$

which is a symmetric Toeplitz matrix, defined by $r\,[y_0, \ldots, y_{T-1}]$.

Under normality, $y \sim N_T(\mu, \Sigma)$, and combined with a procedure to compute the autocovariance in equation (9), the log-likelihood is (i.e. writing $z = y - \mu$)[3] :

$$\log L\,(d, \phi, \theta, \beta, \sigma_\varepsilon^2) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|\Sigma| - \frac{1}{2} z' \Sigma^{-1} z \tag{11}$$

At this point, we shall employ the autocovariance function scaled by the error variance denoted by $r_i = y_i / \sigma_\varepsilon^2$, expressed as in equation (11):

$$R = \tau\left[\frac{\gamma_0}{\sigma_\varepsilon^2}, \ldots, \frac{\gamma_{T-1}}{\sigma_\varepsilon^2}\right] \tag{12}$$

Concentrating on σ_ε^2 out of the log-likelihood saves one parameter for estimation. Given $\Sigma = R\,\sigma_\varepsilon^2$ in equation (11) we obtain:

[3] Additional regression parameter in μ is represented by β, but can be ignored at the initial stage.

$$\log L\,(d,\phi,\theta,\beta,\sigma_\varepsilon^2) \propto -\frac{T}{2}|\mathrm{R}| - \frac{1}{2}\log\sigma_\varepsilon^2 - \frac{1}{2\sigma_\varepsilon^2} z'\mathrm{R}^{-1}z \tag{13}$$

Differentiating with respect to σ_ε^2 and solving equation (5) yields $\sigma_\varepsilon^2 = T^{1}z'R^{-1}z$ with concentrated likelihood function given by[4]:

$$l_c(d,\phi,\theta,\beta) = -\frac{T}{2}(2\pi) - \frac{T}{2} - \frac{1}{2}\log|R| - \frac{T}{2}\log[T^{-1}z'R^{-1}z] \tag{14}$$

Indeed, the maximum likelihood estimator (MLE) of d, ϕ, θ, β maximises the likelihood function quantity presented in equation (14).

3.3.2. Normal distribution

The normal distribution probability function is given by:

$$f(x) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \tag{15}$$

where μ and σ denote the location and scale parameters, respectively. The case where $\mu = 0$ and $\sigma = 1$, reduces equation (15) to the standard normal distribution:

$$f(x) = \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}} \tag{16}$$

3.3.3. Student-t Distribution (STD)

If $z \sim N(0,1)$ and $\mu \sim X^2(r)$ are independent, then the random variable:

$$T = \frac{z}{\sqrt{\frac{v}{r}}}$$

follows a student-t distribution (STD) with r degrees of freedom. The probability density function of STD is given by:

$$f(t) = \frac{\Gamma(\frac{r+1}{2})}{\sqrt{\pi r}\,\Gamma(\frac{r}{2})} \times \frac{1}{(1+\frac{t^2}{r})^{\frac{r+1}{2}}} \text{ , for } -\infty < t < \infty \tag{17}$$

3.3.4. Generalised Error Distribution (GED)

The GED is a three-parameter distribution belonging to the exponential family with the conditional probability density function given by:

$$f(x) = \frac{\kappa e^{-0.5\left|\frac{x-\alpha}{\beta}\right|^{\kappa}}}{2^{1+\kappa^{-1}}\beta\Gamma(\kappa^{-1})} \tag{18}$$

where α, β and κ denote the location, scale and shape parameters, respectively.

4. Details of this discussion on the estimation of an AFRIMA model with maximum likelihood can be found in the notes prepared by Doornik and Ooms (2004).

4. Results

In this section, we present different ARFIMA(*p,d,q*) models specified for the conditional mean under Gaussian distribution assumption, using different orders of (p,q). We then compare the performance of these ARFIMA(*p,d,q*) models to determine the orders of *p* and *q* appropriate for the detection of persistence in the full sample of GHCPI and SACPI inflation series. Each series proceeds as follows: First, is the estimation of the different ARFIMA(*p,d,q*) models where both *p* and *q* are less than or equal to 3 (the idea enacted by Gil-Alana and Toro, 2002). Second, for each of the series, a number of tests were performed on the residuals to ensure that they are white noise. These include tests for normality and heteroskedasticity or ARCH-LM and Box-Pierce tests. The Log-likelihood (LL) and Akaike Information Criterion (AIC) were applied to select the correct model specification for each series. This paper considers all possible combinations for the ARMA(*p,q*) part of the model with *p* = 0, 1, 2 and 3, and *q* = 0, 1, 2 and 3. The results of the parameter estimates of ARFIMA(*p,d,q*) for GHCPI and SACPI inflation series are presented in Table 3.

Table 3: Parameter Estimates for Long Memory for ARFIMA(p,d,q) Models for GHCPI and SACPI Inflation Series

GHCPI Inflation series: 1971M1 - 2014M10 (Full sample)										
ARMA	*LL*	*A1C*	*d*	*Cont.*	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
(1,1)	430.96	-851.93	0.32	1.26	0.88	-	-	-0.30	-	-
SE	-	-	0.10	0.26	0.03	-	-	0.08	-	-
p-val.	-	-	[0.00]	[0.00]	[0.00]	-	-	[0.00]	-	-
(1,2)	435.30	-858.60	0.27	1.28	0.87	-	-	-0.27	0.13	-
SE	-	-	0.14	0.18	0.05	-	-	0.11	0.04	-
p-val.	-	-	[0.05]	[0.00]	[0.00]	-	-	[0.01]	[0.00]	-
(1,3)	435.39	-856.78	0.24	1.29	0.87	-	-	-0.24	0.13	0.02
SE	-	-	0.16	0.16	0.05	-	-	0.12	0.04	0.16
p-val.	-	-	[0.11]	[0.00]	[0.00]	-	-	[0.04]	[0.00]	[0.11]
(2,2)	436.67	-859.35	0.31	1.29	1.74	-0.78	-	-1.17	0.35	-
SE	-	-	0.20	0.17	0.10	0.10	-	0.28	0.15	-
p-val.	-	-	[0.13]	[0.00]	[0.00]	[0.00]	-	[0.00]	[0.02]	-
(3,1)	436.81	-859.63	0.38	1.25	-0.37	0.70	0.31	0.87	-	-
SE	-	-	0.08	0.35	0.09	0.07	0.04	0.06	-	-
p-value	-	-	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	-	

SACPI Inflation series: 1995M1 – 201M12 (Full sample)										
(1,1)	250.68	-491.36	0.06	0.74	0.89	-	-	0.29	-	-
SE	-	-	0.11	0.09	0.04	-	-	0.09	-	-
p-val.	-	-	[0.57]	[0.00]	[0.00]	-	-	[0.00]	-	-
(1,2)	250.76	-489.52	0.13	0.74	0.88	-	-	0.22	-0.05	-
SE	-	-	0.20	0.11	0.05	-	-	0.18	0.12	-
p-val.	-	-	[0.51]	[0.00]	[0.00]	-	-	[0.21]	[0.66]	-
(2,2)	250.77	-487.54	0.13	0.74	0.96	-0.06	-	0.15	-0.07	-
SE	-	-	0.19	0.11	0.50	0.44	-	0.51	0.14	-
p-val.	-	-	[0.50]	[0.00]	[0.06]	[0.88]	-	[0.76]	[0.63]	-
(3,2)	264.38	-512.77	0.43	0.76	1.40	-1.36	0.76	-0.67	0.78	-
SE	-	-	0.06	0.52	0.07	0.07	0.06	0.09	0.61	-
p-val.	-	-	[0.00]	[0.14]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	-
(1,3)	250.82	-487.64	0.13	0.74	0.89	-	-	0.23	-0.04	-0.02
SE	-	-	0.18	0.11	0.05	-	-	0.17	0.11	0.07
p-val.	-	-	[0.48]	[0.00]	[0.00]	-	-	[0.07]	[0.70]	[0.70]

Notes: Model selection was based on the log likelihood (LL) and Akaike Information Criterion (AIC); SE denotes the standard error.

Using the significance of the parameter estimates, LL and AIC model selection criteria, ARFIMA (3,0.38,1) and ARFIMA (3,0.43,2) models provided the best fit for GHCPI and SACPI inflation series, respectively. The residual diagnostics test on these selected models is provided in Table 4.

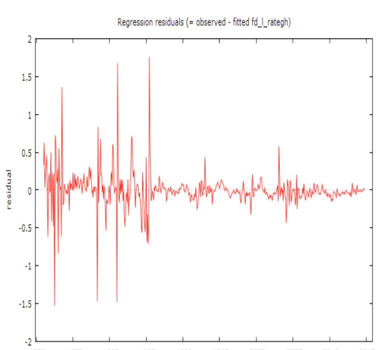
Table 4: Residual Diagnostics Test for Long Memory for ARFIMA(p,d,q) Models selected for GHCPI and SACPI Inflation Series

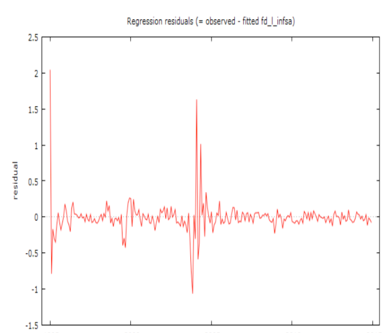
Type of test	ARFIMA (3,0.38,1) for GHCPI (Full sample)		ARFIMA (3,0.43,2) for SACPI (Full sample)	
	Test statistic	P-val.	Test statistic	P-val.
Ljung-Box test	7.82	[0.25]	8.15	[0.14]
ARCH-LM test	60.18	[0.00]*	112.02	[0.00]*
J-B test	8302.04	[0.00]*	12260.80	[0.00]*

Notes: The Ljung-Box tests the null hypothesis of no autocorrelation against the alternative of the existence of autocorrelation; The ARCH-LM tests the hypothesis of no ARCH effect against an alternative of the presence of ARCH effect; and Jarque-Bera statistics tests the null hypothesis that errors are normally distributed versus an alternative of errors not normally distributed. *Indicates the significance at all levels; The presence of ARCH effects demonstrated in Table 4 and Figure 4, necessitated the extension of ARFIMA(p,d,q) with GARCH (s-GARCH) and GJR-GARCH (gjr-GARCH) models. All the analyses were implemented in the Rugarch package in R by Alexios Ghalanos (2015).

From Table 4, the ARFIMA(3,0.38,1) model selected with a fractional integration parameter *d=0.38* is found to be significant and statistically significant at 5% level. The results strongly support the presence of persistence induced by long memory in the conditional mean of GHCPI inflation series. Similarly, ARFIMA (3,0.43,2) is found to be appropriate for modelling the conditional mean of SACPI inflation series, based on the significance of the parameter estimates, AIC and LL model selection criteria (see Table 4). The estimate of the fractional integration *d=0.43* is significant and also supports the existence of persistence in the conditional mean of SACPI inflation series. These results appear to be satisfactory, but the presence of ARCH effects and a neglect of regime shifts or structural breaks can bias the estimation of persistence in the conditional mean, measured by the fraction integration parameter, *d* (see Diebold and Inoue, 2001; Granger and Hyung, 2004 and Hyung *et al*., 2006) (Figure 4). We, therefore, incorporate regime shifts or structural breaks based on the results from Bai and Perron (2003) test, (see Appendices 2 and 3) and the remaining ARCH effects by extending the full sample ARFIMA(*p,d,q*) models for GHCPI and SACPI inflation series in Table 4, with standard generalised conditional autoregressive heteroskedastic model (GARCH) and Glosten-Jagannathan-Runkle generalised conditional autoregressive heteroskedastic model (GJR-GARCH) model, under three distributional assumptions. These extensions will enable us to ascertain the influence of regime shifts on persistence in the conditional mean with time-varying volatility. It will also assist in measuring the effect or impact of economic shocks on the conditional mean, while controlling volatility.

Figure 4: Residual Analysis for GHCPI (left) and SACPI (right) Inflation Series for ARFIMA(3,0.38,1) and ARFIMA (3,0.43,2)





We present the extension of the ARFIMA model with GARCH and GJR-GARCH volatility components under Generalised error distribution (GED), Student-*t* distribution (STD) and normal distribution (Norm), across the identified regime shifts in the GHCPI and SACPI inflations series. The models across these different regimes were estimated using the MLE and the results are presented in Appendices 1A and 1B with the selected models presented in Table 5.

TABLE 5: SELECTED MODELS FOR GHCPI AND SACPI INFLATION SERIES

	GHCPI: 1971M1–2014M10			**SACPI: 1995M1–2014M12**		
Models/ Criteria	**ARFIMA(3,d,1)-GARCH(1,1)**			**ARFIMA(3,d,2)-GARCH(1,1)**		
	Norm	**STD**	**GED**	**Norm**	**STD**	**GED**
d	0.50	0.19	0.38	0.50	0.05	0.50
LL	255.02	429.73	431.51	196.14	208.59	206.79
AIC	-0.95	-1.63	-1.64	-1.63	-1.72	-1.71
	ARFIMA(3,d,1)-GJR-GARCH(1,1)			**ARFIMA(3,d,2)-GJR-GARCH(1,1)**		
	Norm	**STD**	**GED**	**Norm**	**STD**	**GED**
d	0.11	0.00	0.26	0.50	0.50	0.50
LL	282.21	429.12	434.48	204.56	213.25	210.46
AIC	-1.06	-1.62	-1.64	-1.67	-1.76	-1.72

Notes: Model selection is based on LL and AIC

In Table 5 ARFIMA(3,0.26,1)-GJR-GARCH(1,1) and ARFIMA(3,0.50,2)-GJR-GARCH(1,1) under GED and STD provided the best fit for GHCPI and SACPI inflation series, respectively. The selection of these models was based on LL and AIC. To ascertain the influence of structural breaks or regime change on persistence, we employ the selected models to the two respective subsamples for GHCPI and SACPI inflation series obtained through Bai and Perron test (2003) (see Appendices 2 and 3). The results are presented in Table 6 for Ghana and South Africa.

TABLE 6: ESTIMATION OF *d* FOR GHCPI AND SACPI INFLATION SERIES ACROSS REGIMES USING ARFIMA (P,D,Q) -GJR-GARCH(P,Q)

	GHCPI Subsample 1: 1971M1-1984M12			**SACPI Subsample 1: 1995M1-2003M12**		
ARMA (p,q)	***d***	**LL**	**AIC**	***d***	**LL**	**AIC**
(1,1)	0.41	10.28	-0.03	0.47	100.15	-1.82
(1,2)	0.48	15.89	-0.11	0.29	102.12	-1.84
(1,3)	0.30	14.13	-0.10	0.41	105.14	-1.88
(2,2)	0.33	13.11	-0.04	0.04	109.19	-1.94
(3,1)	0.30	11.75	-0.02	0.00	105.13	-1.88
	Subsample 2: 1985M1-2014M12			**Subsample 2: 2004M1-2014M12**		
ARMA (p,q)	***d***	**LL**	**AIC**	***d***	**LL**	**AIC**
(1,1)	0.39	116.38	-2.14	0.20	101.83	-1.85
(1,2)	0.42	115.02	-2.10	0.50	102.64	-1.85
(1,3)	0.44	117.97	-2.13	0.50	102.09	-1.82
(2,2)	0.34	112.07	-2.02	0.35	116.82	-2.09
(3,1)	0.22	116.81	-2.11	0.50	105.38	-1.88

Notes: Selected model fitted across detected structural breaks in GHCPI and SACPI inflation series. Default sample size used is 100 in the estimation process in the Rugarch package by Alexios Ghalanos (2015).

The disparities in the fractional integration parameter *d*, which measures persistence, show the influence of structural breaks or regime shift on persistence levels in GHCPI and SACPI inflation series, confirming studies conducted by Diebold and Inoue (2001), Granger and Hyung (2004) and Hyung *et al*. (2006), among others.

This study also investigated the impact of IT in controlling persistence in the Ghanaian and South African inflation series. Preliminary results displayed in Table 7 show evidence of the influence of IT monetary policy in the degree of integration of the series, especially in the case of South Africa where we observe a substantial increase in the estimated value of d for the post-IT subsample. However, for Ghana, the results are similar in the two subsamples. The table below displays the estimates of *d* (and its corresponding 95% intervals) computed for the subsamples, before and after the IT in the two countries.

Table 7: Determination of Fractional Intergration *d* for GHCPI and SACPI Inflation Series

Ghana			
	No regressors	**An intercept**	**A linear trend**
Pre - IT (2000M1-2006M12)	1.19 (0.91, 1.54)	1.20 (0.87, 1.59)	1.20 (0.86, 1.61)
Post - IT (2007M1-2014M10)	1.17 (1.00, 1.45)	1.28 (1.09, 1.53)	1.28 (1.09, 1.53)
South Africa			
Pre - IT (1995M1-1999M12)	0.53 (-0.11, 1.26)	0.60 (-0.68, 1.23)	0.72 (-0.56, 1.23)
Post - IT (2000M1-2014M12)	1.27 (1.03, 1.52)	1.18 (0.94, 1.42)	1.18 (0.94, 1.44)

NB: Estimation of *d* for GHCPI and SACPI inflation series for pre and post-IT together with confidence intervals in the parenthesis.

Indeed, it is evident from Figures 1 and 4 that GHCPI inflation series had been subjected to three significant breaks corresponding to 1977M07 (i.e. July 1977), 1984M06 (i.e. June 1984) and 2004M02 (i.e. February 2004), respectively. These break dates coincided with some difficulties the Ghanaian economy experienced as a result of inflation. For example, after experiencing some price stability over a five-year period after independence (i.e., 1957-1961) owing to inflation hovering around a single digit in July 1977, the annual inflation rate exceeded 100% (Sowah and Kwakye, 1993). In the period 1983-1984, Ghana continued to experience high inflation as a result of external shocks, unsustainable macroeconomic policies and exchange rate depreciation. The last significant break of January 2004 is associated with the period when Ghana started to experience a disinflation from 27.1% to about 10% (Sowah and Kwakye, 1993).

Similarly, a close inspection of Figures 1 and 4, show that SACPI inflation series has also been subjected to four significant breaks corresponding to 1999M07, 2003M06, 2006M06 and 2009M08. These break dates also concurred with significant events in the South African economy as a result of inflation. It is also evident that SACPI inflation has undergone severe fluctuations, especially during 2000Q3 to 2009Q3, with two big shocks in 2004Q4 (as a result of a substantial depreciation of the South African rand) and 2008Q3 (as a result of global increase in food price together with a rise in oil prices, and a positive shock which took place in 2004Q1 (as a result of South African rand appreciation), before stabilising near the upper bound of 6% during the financial crisis (Kabundi *et al*., 2015)[5].

In our quest to model persistence in the conditional mean of the Ghanaian and South African inflation series, ARFIMA(3,0.26,1)-GJR-GARCH(1,1) and ARFIMA(3,0.50,2)-GJR-GARCH(1,1) models respectively, under GED and STD, have been identified to provide a good a fit with the estimation of parameters presented in Tables 8 and 9, for Ghana and South Africa, respectively.

Table 8: ARFIMA(3,0.26,1)-GJR-GARCH(1,1) under GED for GHCPI Inflation Series

Parameter	**Estimates**	**Std. Err**	**t-val.**	**P-val.**	**Nyblom stats.**
μ	2.47	0.00	1702.97	[0.00]*	0.71
ϕ_1	1.76	0.00	1642.50	[0.00]*	0.14
ϕ_2	-0.73	0.00	-972.41	[0.00]*	0.07
ϕ_3	-0.04	0.00	-365.86	[0.00]*	0.24
θ_1	-0.74	0.00	-467.52	[0.00]*	0.84
d	0.26	0.00	267.71	[0.00]*	1.18
ω	0.00	0.00	1.42	[0.15]	1.24
α	0.15	0.10	1.51	[0.13]	1.90
y	0.63	0.13	4.73	[0.00]*	1.24
β	0.41	0.15	2.68	[0.01]*	3.05
κ	0.63	0.04	14.19	[0.00]*	0.14
LL	434.48	-	-	-	-
AIC	-1.64	-	-	-	-
$Q^2(.)$	3.00	-	-	[0.69]	-
ARCH-LM	2.99	-	-	[0.98]	-
JSNS Test	6.95	-	-	[0.47]	-
			Sign Bias Test		
Sign Bias			1.13	0.25	-
Negative Sign Bias			0.25	0.80	-
Positive Sign Bias			0.07	0.93	-
Joint Effect			181	0.61	-

Notes: *Denotes the significance of all parameters; Box-Pierce Q^2 (.) statistics all failed to reject the null hypothesis of no serial correlation; ARCH-LM test also failed to reject the null hypothesis of no ARCH at 5% after effects; *y* denotes the leverage effect; JSNS denotes Joint Statistic Nyblom Stability test.

5. Q_i, i = 1; 2; 3; 4 denotes the quarters in a year, e.g. Q_i refers to January to March.

Table 9: ARFIMA(3,0.26,1)-GJR-GARCH(1,1) under STD for SACPI Inflation Series

Parameter	Estimates	Std. Err	t-val.	p-val.	Nyblom stats.
μ	2.03	0.18	11.07	[0.00]*	0.12
ϕ_1	0.89	0.13	6.82	[0.00]*	0.11
ϕ_2	0.77	0.09	8.39	[0.00]*	0.10
ϕ_3	-0.69	0.08	-8.21	[0.00]*	0.09
θ_1	-0.01	0.02	-0.70	0.48	0.13
θ_2	-0.94	0.03	-32.34	[0.00]*	0.09
d	0.50	0.16	3.01	[0.00]*	0.05
ω	0.00	0.00	2.46	[0.01]*	0.33
α	0.03	0.07	0.48	0.62	0.22
β	0.60	0.09	6.30	[0.00]*	0.25
y	0.47	0.19	2.47	[0.01]*	0.29
κ	5.94	2.26	2.61	[0.00]*	0.11
LL	213.24	-	-	-	-
AIC	-1.76	-	-	-	-
$Q^2(.)$	0.38	-	-	[0.99]	-
ARCH-LM Test	0.58	-	-	[1.00]	-
JSNS Test	2.78	-	-	[0.47]	-
Sign Bias Test					
Sign Bias			0.56	0.57	-
Negative Sign Bias			0.52	0.60	-
Positive Sign Bias			0.50	0.61	-
Joint Effect			0.95	0.81	-

Notes: * Denotes the significance of all parameters; Box-Pierce statistics all failed to reject the null hypothesis of no serial correlation; ARCH-LM test also failed to reject the null hypothesis of no ARCH at 5% after effects; *y* denotes the leverage effect; JSNS denotes Joint Statistic Nyblom Stability test.

A close inspection of Tables 8 and 9 reveal the significance of all parameter estimates, except the intercept ω in the variance equation of GHCPI inflation series and ARCH component α of both models. All parameters estimates have been found to be stable (according to the Nyblom stability statistics) and the sign bias test presented also demonstrates a non-rejection of no sign bias effect. There also appears to be an asymmetric effect of economic or inflationary shocks (see Figures 5 and 6), which induces persistence on the conditional mean.

The results further show a considerable amount of persistence in the variance equation measured by $(\alpha + \beta + \gamma)$ with both GHCPI and SACPI inflation series being less than or equal to 1.

FIGURE 5: ACF OF STANDARDISED & SQUARED STANDARDISED RESIDUALS (LEFT), AND NEWS IMPACT CURVE & Q-Q PLOT (RIGHT) FOR GHCPI INFLATION SERIES

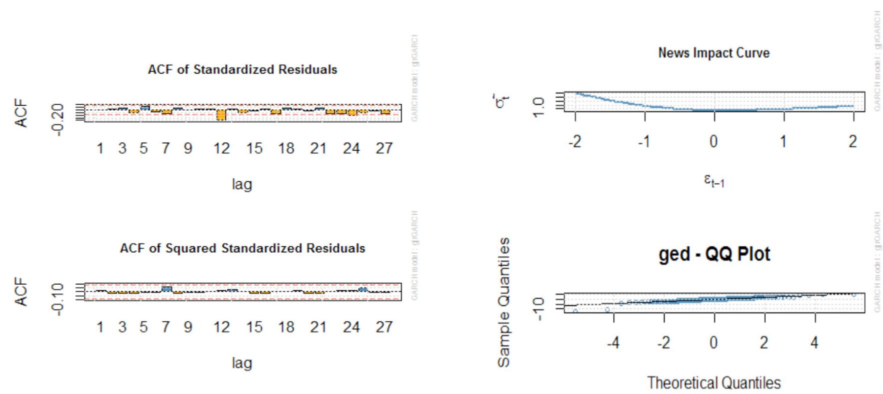
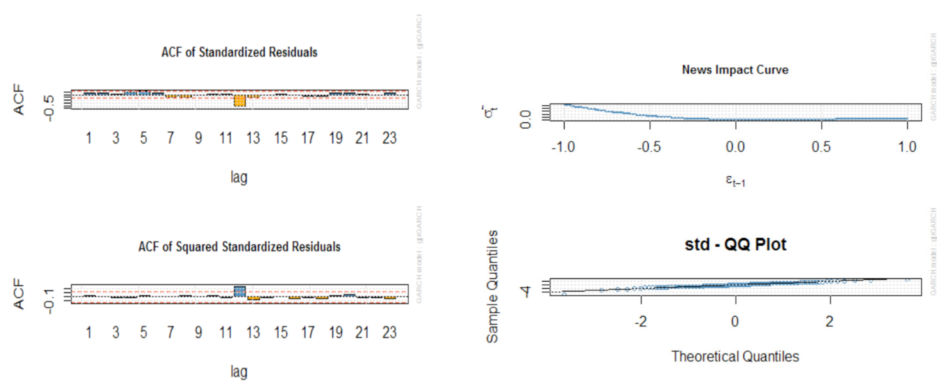


FIGURE 6: ACF OF STANDARDISED & SQUARED STANDARDISED RESIDUALS (LEFT), AND NEWS IMPACT CURVE &Q-Q PLOT (RIGHT) FOR SACPI INFLATION SERIES



The ARCH-LM test statistics from both models indicate no ARCH effects up to lag 27 in the standardised residuals. Box-Pierce Q-statistics test of standardised squared residuals have been found to be serially uncorrelated. Having taken the overall model assessment displayed in Table 10 into consideration, ARFIMA(3,0.26,1)-GJR-GARCH(1,1) under GED and ARFIMA(3,0.50,2)-GJR-GARCH(1,1) models under STD, provided the best fit for the Ghanaian and the South African inflation series, respectively. For the purposes of observing the trend behaviour of GHCPI and SACPI inflation series, these models have been used to forecast a 10-month inflation rate (see Appendices 4 and 6) together with the residual analysis, where the forecast or predicted values appear to be increasing into the future for both IT countries disaggregated into private and public components of health expenditure.

Table 10: Model Diagnostics for GHCPI and SACPI Inflation Series

Criteria	GHCPI		SACPI	
	ARFIMA(3,0.26,1)-GJR-GARCH(1,1) for GED		ARFIMA (3,0.50,2)-GJR-GARCH(1,1) for STD	
	Test statistic	P-val.	Test statistic	P-val.
LL	434.48	-	213.24	-
AIC	-1.64	-	-1.76	-
$Q^2(.)$	3.00	[0.69]	0.38	[0.99]
ARCH-LM	2.99	[0.98]	0.58	[1.00]
JSNS	6.95	[0.47]	2.78	[0.47]

Notes: The null hypothesis of no ARCH effects measured by the ARCH-LM test, no serial correlation in the residuals by Box-Pierce test and no parameter stability by Nyblom stability tests, all failed to be rejected 5%, an indication of a good fit.

5. Summary, conclusion and implications

This paper has explored an extension of the ARFIMA process, which has a fractionally integrated conditional mean with GARCH and GJR-GARCH innovations, to model persistence in the CPI inflation series of Ghana and South Africa under GED, STD and Norm. ARFIMA(3,0.26,1)-GJR-GARCH(1,1) under GED and ARFIMA(3,0.50,2)-GJR-GARCH(1,1) under STD have been found to be appropriate for modelling persistence in the conditional mean of GHCPI and SACPI inflation series, respectively. The results depicted varied degree of persistence emanating from structural breaks or regime change and time-dependent volatility, coinciding with major economic events in the two IT countries. The fact that the two orders of integration are in the interval (0,1) implying evidence of long memory behaviour, with the effects of the shocks dying away in the long run, though rather slowly, especially in case of the SACPI series. Additionally, the asymmetric model results reveal that negative inflationary or economic shocks have a larger effect or impact on the conditional mean with time-dependent volatility than positive inflationary or economic shocks, confirming the study conducted by Caporale and Caporale (1998). These results would, therefore, be useful to both countries in making good portfolio decisions, accessing the efficacy of an intervention policy or programme meant to control persistence and serving as a tool for detecting volatility in GHCPI and SACPI inflation series.

The implication of these results is that governments and central banks should be mindful of the actions and decisions because unguarded decisions and unnecessary alarms could create and raise inflation uncertainties in the Ghanaian and South African economy, which could in turn affect the future trajectory of inflation. Hence, policymakers could also design measures to control inflation due to adverse effects on market volatility. Again, the empirical consistencies of the persistence measured by the fractional integration parameter, d for GHCPI and SACPI inflation series, raise interesting queries with respect to monetary policy guidelines and price-transmission that would be reliable with this form of behaviour. Nevertheless, when conducting standard $I(d)$ methods on different subsamples according to the implementation of the IT, we observe substantial differences in the degree of integration of the series, especially in case of SACPI, with a significant increase in the estimated value of d in the post-IT subsample. Along with this, it is known that non-linear specifications often lead to an improvement in forecasting financial time series data such as inflation rates over conventional linear models (Parker and Rothman, 1997; Montgomery *et al*., 1998; Rothman, 1998). Thus, we recommend that future research should consider the issue of non-linearities and structural breaks in the context of fractional integration in order to ascertain the true persistence in the conditional mean.

Biographical Notes

Alexander Boateng is a Senior Lecturer in the Department of Statistics and Operations Research, University of Limpopo, South Africa. He obtained his B.Ed., MPhil and PhD from the University of Cape Coast (Ghana), Kwame Nkrumah University of Science and Technology (Ghana) and the University of Limpopo (South Africa), in that order. His research interests are in Time Series Analysis and State Space Modelling with Applications to Finance and Economics, Statistical Modelling with Applications to Epidemiology and Concepts of Bayesian Inference with Applications in Epidemiology. He is also a member of International Statistical Institute (ISI) and South Africa Statistics Association (SASA).

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References

Ahmad, M. I. (2013). GARCH Models for Inflation Volatility in Oman. *Review of Integrative Business and Economics Research*, vol 2, no 2, pp 1.

Alagidede, P., Coleman, S. and Adu, G. (2014). A regional analysis of inflation dynamics in Ghana. *Working Paper*, pp 1-32.

Bai, J. and Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, vol 18, no 1, pp 1-22.

Baillie, R. T., Chung, C. F. and Tieslau, M. A. (1996). Analysing inflation by the fractionally integrated ARFIMA-GARCH model. *Journal of Applied Econometrics*, vol 11, no 1, pp 23-40.

Baillie, R.T. and Morana, C. (2012). Adaptive ARFIMA models with applications to inflation. *Economic Modelling*, vol 29 no 6, pp 2451-2459.

Ball, L. and Cecchetti, S. G. (1990). Inflation and uncertainty at short and long horizons. *Brooking Papers on Economic Activity*, pp 215–254.

Balcilar, M., Gupta, R. and Jooste, C. (2016). Analysing South Africa's inflation persistence using and ARFIMA model with Markov-switching differencing parameter *Journal of Developing Areas*, vol 50, no 1, pp 47-57.

Banerjee, A., Cockerell, L. and Russell, B. (2001). An I(2) analysis and the mark-up. *Journal of Applied Economics*, vol 16, pp 215-254.

Bank of Ghana: www.bog.gov.gh, Accessed on 09 October, 2014.

Baum, C. F., Barkoulas, J. T. and Caglayan, M. (1999). Persistence in international inflation rates. *Southern Economic Journal*, vol 65, pp 900-913.

Belkhouja, M. and Boutahar, M. (2009). Structural change and long memory in the dynamic of US inflation process. *Computational Economics*, vol 34, no 2, pp 195-216.

Belkhouja, M. and Mootamri, I. (2016). Long memory and structural change in the G7 inflation dynamics. *Economic Modelling*, vol 54, pp 450-462.

Boateng, A., Gil-Alana, L. A., Lesaoana, M., Siweya, H. and Belete, A. (2016). Long memory and ARFIMA modelling: The case of CPI inflation rate in Ghana. *The Journal of Developing Areas*, vol 50, no 3, pp 287-304.

Boero, G., Smith, J. and Wallis, K.F., 2009. Modelling UK inflation uncertainty, 1958– 2006. Volatility and Time Series Econometrics: Essays in Honour of Robert F. Engle (M. Watson, T. Bollerslev and J. Russell, eds). Oxford: Oxford University Press, forthcoming.

Bollerslev, T. (1986). Generalized autoregressive conditional heterokcedasticity, .*Journal of Econometrics*, vol 31, no 3, pp 307-327.

Bos, C. S., Franses, P. H. and Ooms, M. (1999). Long memory and level shifts: Re-analysing inflation rates. *Empirical Economics*, vol 24, pp 427-449.

Bos, C.S., Koopman, S.J. and Ooms, M. (2014). Long memory with stochastic variance model: A recursive analysis for US inflation. *Computational Statistics and Data Analysis*, vol 76, pp 44-157.

Burger, P. and Marinkov, M. (2008). Inflation targeting and inflation performance in South Africa. *Annual Forum, Department of Economics*. University of Free State.

Caporale, B. and Caporale, T. (1998). Asymmetric effects of inflation shocks on inflation uncertainty. *Atlantic Economic Journal*, vol 30, no 4, pp 385-388.

Cecchetti, S.G. and Debelle, G. (2006). Has the inflation process changed? *Economic Policy*, vol 21, no 46, pp 312-352.

Cecchetti, S.G., Hooper, P., Kasman, B.C., Schoenholtz, K.L. and Watson, M.W. (2007). Understanding the evolving inflation process. In *US Monetary Policy Forum*, March, vol. 8.

Conrad, C., Karanasos, M. and Zeng, N. (2011). Multivariate fractionally integrated APARCH modeling of stock market volatility: A multi-country study. *Journal of Empirical Finance*, vol 18, no 1, pp 147-159.

Diebold, F.X. and Inoue, A. (2001). Long memory and regime switching. *Journal of Econometrics*, vol 105, no 1, pp 131-159.

Ding, Z., Granger, C. W. J., and Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance,* vol 1, pp 83–106.

Doornik, J.A. and Ooms, M. (2004). Inference and forecasting for ARFIMA models with an application to US and UK inflation. *Studies in Nonlinear Dynamics and Econometrics*, vol 8, no 2, pp 1208-1218.

Gadea, M. and Mayoral, L. (2005). The persistence of inflation in OECD countries: a fractionally integrated approach.

Geweke, J. and Porter-Hudak, S. (1983). The estimation and application of long memory time series models. *Journal of Time Series Analysis*, vol 4, no 4, pp 221-238.

Ghalanos, A. (2015). Introduction to the rugarch package: (Version 1.3-1).

Gil-Alana, L.A. and Toro, J. (2002). Estimation and testing of ARFIMA models in the real exchange rate. *International Journal of Finance and Economics*, vol 7, no 4, pp 279-292.

Gil-Alana, L.A. (2011). Inflation in South Africa. A long memory approach. *Economics Letters*, vol 111, no 3, pp 207-209.

Glosten, L., Jagannathan, R. and Runkle, D. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, vol 48, no 5, pp1779-1801.

Gonzalez, A., Hubrich, K. and Teräsvirta, T. (2009). Forecasting inflation with gradual regime shifts and exogenous information. *CREATES Research Paper*, No. 2009-03.

Granger, C. W. J. A. (1980). Long-memory relationships and the aggregation of dynamic models. *Journal of Econometrics*, vol 14, pp 227-238.

Granger, C. W. J. A. and. Joyeux, R. (1980). An introduction to long memory time series models and fractional differencing. *Journal of Time Series Analysis*, vol 1, no 1, pp 15-29.

Granger, C.W. and Hyung, N. (2004). Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns. *Journal of Empirical Finance*, vol 11, no 3, pp 399-421.

Grier, K. B. and Perry, M. J. (1998). On inflation and inflation uncertainty in the G7 countries. *Journal of International Money and Finance*, vol 17, no 4, pp 671-689.

Hassler, U. and Meller, B. (2014). Detecting multiple breaks in long memory the case of US inflation. *Empirical Economics*, vol 46, no 2, pp 653-680.

Hassler, U. and Wolters, J. (1995). Long memory in inflation rates: International evidence. *Journal of Business Statistics*, vol 13, pp 37-45.

Hyung, N., Franses, P.H. and Penm, J. (2006). Structural breaks and long memory in US inflation rates: Do they matter for forecasting? *Research in International Business and Finance*, vol 20, no 1, pp 95-110.

Kabundi, A., Schaling, E. and Some, M. (2015). Monetary policy and heterogeneous inflation expectations in South Africa. *Economic Modelling*, vol 45, pp 109-117.

Kang, K.H., Kim, C.J. and Morley, J. (2009). Changes in US inflation persistence. *Studies in Nonlinear Dynamics and Econometrics*, vol 13, no 4, pp 1-21.

Kim, J. C. (1993). Unobserved-component time series models with Markov-switching heteroskedasticity: Changes in regime and the link between inflation rates and inflation uncertainty. *Journal of Business and Economic Statistics*, vol 11, no 3, pp 341-349.

Kirchgässner, G. and Wolters, J. (1993). Are real interest rates stable? An international comparison. *Studies in Applied Econometrics*, pp 214-238.

Kumar, M.S. and Okimoto, T. (2007). Dynamics of persistence in international inflation rates. *Journal of Money, Credit and Banking*, vol 39, no 6, pp 1457-1479.

Montgomery, A. L., Zarnowitz, V., Tsay, R. S. and Tiao, G. (1998). Forecasting the US unemployment rate. *Journal of American Statistical Association*, vol 93, pp 478-493.

Morana, C. and Bagliano, F.C., 2007. Inflation and monetary dynamics in the USA: a quantity-theory approach. *Applied Economics*, vol 39, no 2, pp 229-244.

Narayan, P. K. and Liu, R. (2015). A unit root model for trending time-series energy variables. *Energy Economics*, vol 50, pp 391-440.

Narayan, P. K. (2014). Response of inflation to shocks: New evidence from sub-Saharan African countries. *Economic Modelling*, vol 36, pp 378-382.

Nelson, C. R. and. Schwert, G. J. (1997). Short-term interest rates as predictors of inflation: On testing the hypothesis that the real rate of interest is constant. *The American Economic Review*, vol 67, no 3, pp 478-486.

Osborn, D.R. and Sensier, M. (2009). UK inflation: persistence, seasonality and monetary policy. *Scottish Journal of Political Economy*, vol 56, no 1, pp 24-44.

Parker, R. E. and Rothman, P. (1997). The current depth-of-recession and unemployment-rate forecasts. *Studies in Non-linear Dynamics and Econometrics*, vol 2, pp 151-158.

Rangasamy, L. (2009). Inflation persistence and core inflation: The Case of South Africa. *South African Journal of Economics*, vol 77, no 3, pp 430-444.

Robinson, P.M. (1995). Gaussian semiparametric estimation of long range dependence. *The Annals of Statistics*, pp 1630-1661.

Rose, A. K. (1988). Is the real interest rate stable? *Journal of Finance*, vol 43, no 5, pp 1095-1112.

Rothman, P. (1998). Forcasting asymmetric unemployment rates over business cycle. *Journal of Macroeconomics*, vol 13, pp 164-168.

Sowah, N. K. and Kwakye, J. K. (1993). Inflationary trends and control in Ghana. AERC, *Nairobi Research Paper* No: 22.

Statistics South Africa: StatsSA.gov.za, Accessed on 09 September, 2014.

Tse, Y.K. (1998). The conditional heteroscedasticity of the yen–dollar exchange rate. *Journal of Applied Econometrics*, vol 13, pp 49–55.

Appendix 1(A): Fitted Models for the GHCPI Inflation Series for the Full Sample: 1971M1-2014M10

Fitted Models	d	LL	AIC
Normal Distribution (Norm)			
ARFIMA (1,d,1)-GARCH(1,1)	0.40	234.48	-0.88
ARFIMA (1,d,2)-GARCH(1,1)	0.11	239.87	-0.89
ARFIMA (1,d,3)-GARCH(1,1)	0.50	249.92	-0.93
ARFIMA (2,d,2)-GARCH(1,1)	0.49	251.96	-0.94
ARFIMA (3,d,1)-GARCH(1,1)	0.50	255.02	-0.95
ARFIMA (1,d,1)-GJR-GARCH(1,1)	0.50	258.85	-0.97
ARFIMA (1,d,2)-GJR-GARCH(1,1)	0.27	271.98	-1.02
ARFIMA (1,d,3)-GJR-GARCH(1,1)	0.49	272.18	-1.02
ARFIMA (2,d,2)-GJR-GARCH(1,1)	0.21	282.82	-1.05
ARFIMA (3,d,1)-GJR-GARCH(1,1)	0.11	282.21	-1.06
Student-*t* Distribution (STD)			
ARFIMA (1,d,1)-GARCH(1,1)	0.48	422.28	-1.61
ARFIMA (1,d,2)-GARCH(1,1)	0.50	425.68	-1.62
ARFIMA (1,d,3)-GARCH(1,1)	0.49	427.15	-1.62
ARFIMA (2,d,2)-GARCH(1,1)	0.49	425.92	-1.61
ARFIMA (3,d,1)-GARCH(1,1)	0.19	429.73	-1.63
ARFIMA (1,d,1)-GJR-GARCH(1,1)	0.50	422.81	-1.60
ARFIMA (1,d,2)-GJR-GARCH(1,1)	0.50	425.49	-1.61
ARFIMA (1,d,3)-GJR-GARCH(1,1)	0.50	429.72	-1.62
ARFIMA (2,d,2)-GJR-GARCH(1,1)	0.50	425.45	-1.61
ARFIMA (3,d,1)-GJR-GARCH(1,1)	0.00	429.12	-1.62
Generalised Error Distribution (GED)			
ARFIMA (1,d,1)-GARCH(1,1)	0.37	420.52	-1.59
ARFIMA (1,d,2)-GARCH(1,1)	0.46	425.14	-1.61
ARFIMA (1,d,3)-GARCH(1,1)	0.48	426.53	-1.61
ARFIMA (2,d,2)-GARCH(1,1)	0.27	400.53	-1.51
ARFIMA (3,d,1)-GARCH(1,1)	0.38	431.51	-1.63
ARFIMA (1,d,1)-GJR-GARCH(1,1)	0.49	424.48	-1.61
ARFIMA (1,d,2)-GJR-GARCH(1,1)	0.50	430.07	-1.63
ARFIMA (1,d,3)-GJR-GARCH(1,1)	0.49	432.08	-1.63
ARFIMA (2,d,2)-GJR-GARCH(1,1)	0.44	431.69	-1.63
ARFIMA (3,d,1)-GJR-GARCH(1,1)	0.26	434.48	-1.64

Notes: Competing models fitted to the full sample of GHCPI inflation rates under three distributional assumptions, where LL and AIC denote the log-likelihood and Akaike Information criterion.

Appendix 1(B): Fitted Models for the SACPI Inflation Series for the Full Sample: 1995M1-2014M12

Fitted Models	d	LL	AIC
Normal Distribution (Norm)			
ARFIMA (1,d,1)-GARCH(1,1)	0.30	193.98	-1.63
ARFIMA (1,d,2)-GARCH(1,1)	0.09	194.69	-1.63
ARFIMA (2,d,2)-GARCH(1,1)	0.33	194.34	-1.61
ARFIMA (3,d,2)-GARCH(1,1)	0.50	196.14	-1.63
ARFIMA (1,d,3)-GARCH(1,1)	0.01	194.90	-1.62
ARFIMA (1,d,1)-GJR-GARCH(1,1)	0.31	202.86	-1.70
ARFIMA (1,d,2)- GJR-GARCH(1,1)	0.21	202.63	-1.69
ARFIMA (2,d,2)- GJR-GARCH(1,1)	0.41	203.31	-1.68
ARFIMA (3,d,2)- GJR-GARCH(1,1)	0.50	204.56	-1.69
ARFIMA (1,d,3)-GJR-GARCH H(1,1)	0.28	202.34	-1.67
Student-*t* Distribution (STD)			
ARFIMA (1,d,1)-GARCH(1,1)	0.31	203.98	-1.71
ARFIMA (1,d,2)-GARCH(1,1)	0.16	204.08	-1.70
ARFIMA (2,d,2)-GARCH(1,1)	0.34	206.61	-1.71
ARFIMA (3,d,2)-GARCH(1,1)	0.05	208.59	-1.72
ARFIMA (1,d,3)-GARCH(1,1)	0.08	204.19	-1.69
ARFIMA (1,d,1)-GJR-GARCH(1,1)	0.30	210.71	-1.75
ARFIMA (1,d,2)- GJR-GARCH(1,1)	0.16	210.74	-1.75
ARFIMA (2,d,2)- GJR-GARCH(1,1)	0.37	211.56	-1.75
ARFIMA (3,d,2)- GJR-GARCH(1,1)	0.50	213.25	-1.76
ARFIMA (1,d,3)-GJR-GARCH H(1,1)	0.17	210.36	-1.74
Generalised Error Distribution (GED)			
ARFIMA (1,d,1)-GARCH(1,1)	0.31	200.71	-1.68
ARFIMA (1,d,2)-GARCH(1,1)	0.06	202.40	-1.69
ARFIMA (2,d,2)-GARCH(1,1)	0.35	202.27	-1.68
ARFIMA (3,d,2)-GARCH(1,1)	0.50	206.79	-1.71
ARFIMA (1,d,3)-GARCH(1,1)	0.00	203.16	-1.68
ARFIMA (1,d,1)-GJR-GARCH(1,1)	0.30	207.29	-1.73
ARFIMA (1,d,2)- GJR-GARCH(1,1)	0.09	208.27	-1.73
ARFIMA (2,d,2)- GJR-GARCH(1,1)	0.37	208.25	-1.72
ARFIMA (3,d,2)- GJR-GARCH(1,1)	0.50	210.46	-1.73
ARFIMA (1,d,3)-GJR-GARCH H(1,1)	0.11	208.12	-1.72

Notes: Competing models fitted to the full sample of SACPI inflation rates under three distributional assumptions, where LL and AIC denote the log-likelihood and Akaike Information criterion.

Appendix 2: Bai and Perron Multiple Breakpoint Test of L+1 VS L determined sequentially for GHCPI Inflation Series

Break points	F-statistics	Critical values
0 vs 1*	115.03	8.58
1 vs 2*	34.12	10.13
2 vs 3*	120.67	11.14
3 vs 4	8.40	11.83
Number of Break points	**Sequential**	**Repartition**
1	2004M02	1977M07
2	1984M06	1984M06
3	1977M07	2004M02

Notes: Breaking variable: GHCPI inflation series for the period between 1971M01-2014M10: Breaking point option for trimming parameter 0.15 with the maximum breaks equal to 5. * Indicate significance at 5% level (Bai and Perron, 2003).

Appendix 3: Bai and Perron Multiple Breakpoint Test of L+1 VS L determined sequentially for SACPI Inflation Series

Break points	F-statistics	Critical values
0 vs 1*	59.72	8.58
1 vs 2*	24.85	10.13
2 vs 3*	54.53	11.14
3 vs 4*	35.41	11.83
4 vs 5	0.00	12.50
Number of Break points	**Sequential**	**Repartition**
1	1999M07	1999M07
2	2006M06	2003M06
3	2009M08	2006M06
4	2003M06	2009M08

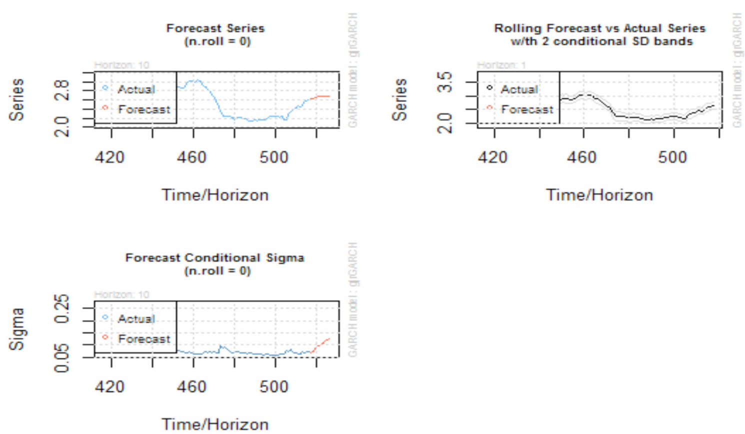
Notes: Breaking variable: SACPI inflation series for the period between 1971M01-2014M10: Breaking point option for trimming parameter 0.15 with the maximum breaks equal to 5. * Indicate significance at 5% level (Bai and Perron, 2003).

Appendix 4: Predicted values for GHCPI inflation rate using ARFIMA(3,0.26,1)-GJR-GARCH(1,1) Model under Generalised Error Distribution (GED)

Observations	**Conditional standard deviation, σ**	**Predicted conditional mean inflation**
2014M01	0.06	2.62
2014M02	0.07	2.64
2014M03	0.08	2.65
2014M04	0.09	2.66
2014M05	0.09	2.67
2014M06	0.10	2.68
2014M07	0.10	2.68
2014M08	0.11	2.69
2014M09	0.11	2.69
2014M10	0.12	2.69
Forecast Accuracy	**Roll-0**	**Roll-1**
MSE	0.00	0.00
MAE	0.05	0.04
DAC	1.00	1.00
N	10.00	9.00

Notes: MSE denotes the Mean Square Errors; MAE: Mean Absolute Error; DAC: Directional Accuracy test and N depicting the number of in-sample forecast observations and denotes the conditional standard deviation. All these forecast performance measures point to the closeness of the predicted mean to actual conditional mean of 3.13 respectively for GHCPI inflation series.

Appendix 5: Residual plots for the 10-month forecast of ARFIMA(3,0.26,1)-GJR-GARCH(1,1) Model for GHCPI Inflation Series under GED



Appendix 6: Predicted values for SACPI inflation rate using ARFIMA(3,0.50,2)-GJR-GARCH(1,1) Model under Student-*t* Distribution (STD)

Observations	Conditional standard deviation,	Predicted conditional mean inflation
2014M02	0.06	1.78
2014M03	0.07	1.79
2014M04	0.08	1.80
2014M05	0.08	1.81
2014M06	0.09	1.81
2014M07	0.09	1.82
2014M08	0.09	1.82
2014M09	0.09	1.83
2014M10	0.09	1.83
2014M11	0.09	1.84
Forecast Accuracy	**Roll-0**	**Roll-1**
MSE	0.00	0.01
MAE	0.06	0.06
DAC	1.00	1.00
N	10.00	9.00

Notes: MSE denotes the Mean Square Errors; MAE: Mean Absolute Error; DAC: Directional Accuracy test and N depicting the number of in-sample forecast observations and denotes the conditional standard deviation. All these forecast performance measures point to the closeness of the predicted mean to actual conditional mean of 1.68 respectively for SACPI inflation series.

Appendix 7: Residual plots for the 10-month forecast of ARFIMA(3,0.26,1)-GJR-GARCH(1,1) Model for SACPI Inflation Series under STD