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ON THE PRESSURE VELOCITY AND TEMPERATURE FACTORS AND THE EFFECT OF VISCOSITY ON THE ARTERIAL BLOOD FLOW IN RELATION TO THE HYPERTENSION PATIENT, PART 1 – FLOW WITHOUT OUTFLOW

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ABSTRACT: *In this paper, we examine the effects of viscosity on the blood pressure, velocity and temperature distributions in the arterial blood flow in the absence of outflows. The governing continuity, momentum and energy equations are solved analytically by method of characteristics. Using the wavefront expansions, an equation of the form of the Riccati equation is derived. By this, explicit results about the pressure, velocity and temperature distributions are obtained. It is observed that viscosity reduces the pressure, hence, the velocity and temperature distributions. Our results may find relevance in the medical treatment of high blood-pressure problem*

Key words*: Viscosity pressure, velocity, temperature, arterial blood flow, hypertension.*

INTRODUCTION

Man is plagued with many internal flow problems. Among others, he has the problem of stenosis of the arteries and the situation where the aortic value fail to close properly. With emphasis, the latter can lead to hypertension and hypothermia in the patient. Because of the importance of human life, it becomes necessary to join hands, inter-disciplinarily to attack problem of hypertension and hyperthermia in relation to blood flow An understanding of blood flow in the arteries, dynamically, must be based on a good knowledge of physical laws governing the behaviour of the physical fluids at rest and in motion.

The mathematical theory of blood flow in the arteries is based on an unsteady one-dimensional model in which the pressure and fluid velocity are average over the cross-sectional area of the arteries. The equations describing the flow are sets of nonlinear first-order partial differential equations, which are hyperbolic and resemble those of the one-dimensional gas dynamics [5]. The equations arising from the forgoing are solved by method of wavefront expansions. As a result of a result of omitting from the model some aspects of the system that become significant at rapid flow changes, shock discontinues may develop. Therefore, the possibility of shock discontinues is a direct consequence of the non-linearity of the system [12]. [2] and [3] envisioned shock waves in the form of abrupt rise in pressure, velocity and temperature, or as spike–type perturbations which may evolve from relatively gradual changes in pressure.

Much work have been done at different dimensions on the flow of blood in the arteries. For example, [2] used the numerical approach to consider the steady flow pulses all shock wave of the arteries. They compared the linearized and nonlinearized analysis and their effect on the wavefront. [10], examined shock waves in the mathematical models of the aorta for the inviscid flow through a semi-infinite uniform distensible tube. Using the method of characteristics he obtained the distance at which shock discontinuities may occur. [4] extended problems in [10] to viscous flow in non-uniform tubes. They investigate the, conditions under which a, shock wave will form, the time and distance from the entrance of the tube wherein this will happen. They found that for a tapered tube, the shocks form earlier and nearer to the entrance than for a uniform tube, and that viscosity has a delaying effect on shock formation.

In line with [4], [l] examined the conditions under which shock is formed, however, they left out. According to [3], the energy equation need not be examined (temperature being approximately constant throughout the circulatory system) if the energy stored in the elastic walls of the artery is not accompanied by thermal effect (and in which case, the viscosity will be zero). [13] gave a relationship between heat transfer and the rheological properties of blood. They showed that the viscosity of blood in the circulatory system is a function of temperature. Allowing viscosity to depend on temperature, [4] showed that viscosity delays shock formation. What was not clear is the effect of variable viscosity on shock formation. Responding to this, [3] presented an asymptotic analysis of the temperature equation and investigated the effect of variable viscosity on the temperature distribution in a sickle cell anemia patient. They showed that a local hot spot exist in the sickle–cell anemia patient since the velocity becomes infinite at (xs, ts) and for practical purpose, the viscosity is greater than zero.

The aim of this paper is to investigate the effect of viscosity on the pressure and velocity, temperature distributions in the arterial blood flow (without side branches) in the high - blood patient.

PHYSICS OF PROBLEM

As a living organ, certain malfunctioning conditions due to diseases may arise in the artery. These are times when he aortic valve fails to close properly (i.e incompetent). Because of this, a large proportion of the blood ejected by the ventricle in systole (ventricular contraction) flows back to it in diastole (auricular contraction). The heart naturally adjusts itself to maintain peripheral circulation. In this situation, the heart is greatly enlarged, thus increasing the volume of blood ejected from it. The amplitudes of the pulse is then, very large and strong, thus, giving rise to arterial "pistol" shot. The pistol shot is the manifestation of hydraulic jump or shock in the arterial system. The jump in the pressure variable may lead to a crisis called hypertension (a body pressure that is above normal), which is invariably accompanied by hyperthermia (a body temperature that is above normal) in the patient.

We shall examine the effect of viscosity on the pressure, and hence, the velocity and temperature distributions of the flow in relation to the hypertension patient.

MATHEMATICAL FORMULATION

In line with [I], an unsteady one-dimensional model for arterial blood flow is presented and examined. The model is developed on the basis of the following assumptions: blood is compressible and newtonian. The arteries are straight elastic tubes (without side branches) of circular cross-sections constrained from longitudinal motions, so that only the axisymmetric bulging motions of the tube walls are considered. Since the radius of the artery is usually much smaller than the typical axial wavelength of flow, the radial acceleration and pressure force are neglected and only the longitudinal pressure gradient and axial fluid acceleration are considered. The flow and pressure pulses propagate with the local speed of sound c. The viscosity of blood vary with temperature. And, the blood constituents (solid corpuscles and plasma) flow with the same velocity.

The flow being one-dimensional, the only independent variables in the model are the axial coordinate, x and time, t. The governing continuity, momentum and energy equations for a segment x1 ≤ x ≤ x2 are:

$$\frac{d}{dt}\int_{X_1}^{X_2}\rho A dx = -\rho u A\Big|_{x_1}^{x_2} \tag{3.1}$$

$$\frac{d}{dt}\int_{x_1}^{x_2}\rho u A dx = -\left(\rho u^2 A + pA\right)\Big|_{x_1}^{x_2} + \int_{x_1}^{x_2} p\frac{\partial A}{\partial x} - \int_{x_1}^{x_2} A\frac{\partial \eta}{\partial x}dx \tag{3.2}$$

$$\frac{d}{dt}\int_{x_1}^{x_2}\left(\tfrac{1}{2}\rho u^2 + \rho C_v T\right)A dx = -\left(\tfrac{1}{2}\rho u^2 + \rho C_v T\right)uA\Big|_{x_1}^{x_2} - \int_{x_1}^{x_2} Ap\frac{\partial u}{\partial x}dx - \int_{x_1}^{x_2} Ak\frac{\partial^2 T}{\partial x} - \int_{x_1}^{x_2} A\eta\frac{\partial u}{\partial x}dx \tag{3.3}$$

Equation (3.1) – (3.3) can be written in differential form as:

$$\frac{d}{dt}(\rho A) + \frac{d}{dx}(\rho u^2 A) = 0 \tag{3.4}$$

$$\frac{d}{dt}(\rho u A) + \frac{d}{dx}(\rho u^2 A) + A\frac{dp}{dx} = \overline{f} \tag{3.5}$$

$$\rho C_v\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}\right) + k\frac{\partial^2 T}{\partial x^2} + p\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial x} \tag{3.6}$$

where ρ is the varying fluid density, u axial fluid velocity, A cross – sectional area of the tube, p the pressure difference across the wall of the tube, and F the frictional force per unit length of the tube, k thermal diffusivity, T temperature, η Stress force, Cv specific heat at constant volume. The difference between equation (3.6) and the general energy equation is the inclusion of the term $p\frac{\partial u}{\partial x}$ called the thermal effect. This gives the assumption that the energy stored in the elastic walls of the arteries is accompanied by thermal effect.

Since the arteries are distensible and tapered, the cross – sectional areas vary with pressure and instantaneously to the pressure changes in the fluid. Hence,

A = A(p, x) (3.7)

Defining the local speed of sound as

$$c_0^2 = \frac{\partial \rho}{\partial p} \quad (3.8)$$

and using it in equation (3.4), we have

$$\frac{\partial p}{\partial t} + u\frac{\partial u}{\partial x} + \rho c_0^2 \frac{\partial u}{\partial x} = -\rho c_0^2 \frac{u}{A}\frac{\partial A}{\partial x} \quad (3.9)$$

Again by equation (3.4), equations (3.5) can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{p}\frac{\partial p}{\partial x} = f(u, A) \quad (3.10)$$

where $f = \frac{\overline{f}}{\rho A}$

Making the following replacements:

$$k\frac{\partial^2 T}{\partial x^2} = 2\xi\left(\frac{\pi}{A}\right)^{1/2}(T - T_0),\ \eta\frac{\partial u}{\partial x} = -2\overline{\mu}\left(\frac{\partial u}{\partial x}\right)^2,$$

$$\overline{\mu} = \mu(T) = \mu[1 - \alpha(T - T_0)]$$

(where ξ is the heat transfer coefficient, α is a position constant) in equation (3.6), we have

$$\rho C_V\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}\right) + 2\xi\left(\frac{\pi}{A}\right)^{1/2}(T - T_0) + p\frac{\partial u}{\partial x} - 2\left(\frac{\partial u}{\partial x}\right)^2 \mu[1 - \alpha(T - T_0)] = 0 \quad (3.11)$$

Equations (3.9) and (3.10) are wave equations. Combining them we have their characteristics forms as:

$$\left\{\frac{\partial}{\partial t} + (u + c)\frac{\partial}{\partial x}\right\}(u + F) = f - \frac{c\ u}{A}\frac{\partial A}{\partial x} \quad (3.12)$$

$$\left\{\frac{\partial}{\partial t} + (u - c)\frac{\partial}{\partial x}\right\}(u - F) = f + \frac{cu}{A}\frac{\partial A}{\partial x} \quad (3.13)$$

where F = $\int_{p_0}^{p} \frac{dp}{\rho c}$

These characteristics equations show the possibility that discontinues in the form of shock waves develop in the flow

Inviscid Flow through a uniform elastic tube

For an Inviscid flow through a uniform elastic tube, f = 0, $\frac{\partial A}{dx} = 0$. Equations (3.12) and (3.13) reduce to:

$$\left\{\frac{\partial}{\partial t} + (u + c_0)\frac{\partial}{\partial x}\right\}(u + F) = 0 \quad (3.14)$$

$$\left\{\frac{\partial}{\partial t} + (u - c_0)\frac{\partial}{\partial x}\right\}(u - F) = 0 \quad (3.15)$$

showing that (u + F) and (u – F) are invariant on C+ and C- characteristics given by

C+: $\frac{\partial x}{dt} = u + c$

C-: $\frac{\partial x}{dt} = u - c$

Equations (3.14) and (3.15) have simple wave solutions. For a semi-infinite tube (x ≥ 0), which is initially undisturbed, we have

p0 = p(x, 0) = constant, u(x, 0) = 0, $\rho_0 = \rho_1(x,0) =$ constant, T0 = T(x,0) = constant. (3.16)

At x = 0, t ≥ 0, we shall assume
p(0,t) = p0 + rt + O(t2), r > 0 (3.17)

Since u = 0 in the undisturbed region (x, t ≥ 0)
U – F = - F0 = constant (3.18)

Now combining equations (3.9) and (3.10) we have

$$\left\{\frac{\partial}{\partial t} + (u + c)\frac{\partial}{\partial x}\right\}p = 0 \quad (3.19)$$

This implies that p is constant on c^+ characteristics given by

$$\frac{\partial x}{dt} = u + c = F(p) - F(p_0) + c(p) \quad (3.20)$$

C^+ are straight lines and in particular C_o^+ is the straight line x = c_ot

Discontinuities in the first derivatives occur along the wavefront characteristics c_o^+ emanating into the region x, t > 0. If the strength of the discontinuity becomes infinite at some point (x_s, t_s) on c_o^+, a shock is fitted into the solution, for x ≥ x_s, t ≥ t_s, to avoid a multivalued solution. The point (x_s, t_s) is the point nearest to origin on the envelop formed by the characteristics starting from (0, t) on the positive t - axis.

In this paper, we are not interested in the time and location where the shocks are developed.

METHOD OF SOLUTION

We shall consider viscous flow in a non-uniform elastic tube. We have seen that our governing equations are wave equations. However, an elementary solution like those of the simple wave equations will not be possible here. Hence, we shall use the wavefront expansions method, wherein we consider the flow as a perturbation of the undisturbed state.

The jump in the first derivative of the solution propagates along a wavefront characteristic, C_0^+ through the origin, and which may become infinite at some point (xs, ts) and hence, we have the formation of shock waves

The $C^{\pm}$ characteristics are given by $\frac{dx}{dt} = u \pm c$. The wavefront characteristic, C_0^+ separates the undisturbed region in the x, t – plane from the disturbed region. Thus, along

C_0^+ : u = 0, p = p0, $\rho = \rho_0$, T = T0, A = A0.

The variables with subscripts zero are constants.

We shall introduce new variables T and τ , where t = T (x) on C_0^+ ,

$$T(x) = \int_{p_0}^{p} \frac{d\zeta}{c(p_0,\zeta)} \tag{4.1}$$

and $\tau = t - T = t - \frac{x}{C_0}$, $c_0 = c(p_0)$

On the C_0^+ , τ = 0, gives x = c0t. In the undisturbed region, τ < 0, while in the disturbed region, τ > 0.
In the immediate neighbourhood of co+ we shall assume the following wavefront expansions:

For $\tau > 0$.
For $\tau < 0 : u = 0, p = p_0$, the disturbed region

$$p = p_0 + \tau p_1(t) + \frac{\tau^2}{2} p_2(t) + \ldots\ldots$$

$$\rho = \rho_0 + \tau\rho_1(t) + \frac{\tau^2}{2}\rho_2(t) + \tag{4.2}$$

$$u = u_0 + \tau u_1(t) + \frac{\tau^2}{2} u_2(t) + \ldots\ldots.$$

T = T0 + τ T1 (t) + $\frac{\tau^2}{2}$ T2 (t) +…
where ρ_1 , p1, u1 and T1 are the measure of jumps in the normal derivatives of ρ , p, u, and T, respectively.

Furthermore, we consider the following notations, expansions and transformations:

c (po, x(t)) = co

$$\frac{\partial c}{\partial p}(p_{0,}x(t)) = c_P(p_{o,}x(t) = c_p^o$$

$$\frac{\partial c}{\partial x}(p_{0,}x(t)) = c_x(p_{o,}x(t) = c_x^o$$

A(po, x(t)) = Ao

$$\frac{\partial A}{\partial x}(p_{0,}x(t)) = A_x(p_{o,}x(t) = A_x^o$$

$$A(p, x(t)) = A_o + \tau(p_1(t)A_p^o - c_o A_x^o) + \ldots.$$

$$A_x(p, x(t)) = A_x^o + \tau(p_1(t)A_{px}^o - c_o A_{xx}^o) + \ldots.$$

$$c(p, x(t)) = c_o + \tau(p_1(t)c_p^o - c_o c_x^o) + \ldots.$$

$$f(u, A) = f^o + uf_u^o + (\tau u_1(t) + \frac{\tau^2}{2}u_2(t) + \ldots)f_u^o + (p_1(t)A_p^o - c_0 A_x^o)f_A^o$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \frac{\partial}{\partial \tau}, \quad \frac{\partial}{\partial x} = -\frac{1}{c_o}\frac{\partial}{\partial \tau} \tag{4.3}$$

Using equations (4.2) and (4.3) in equations (3.9) - (3.11) respectively, and equating the coefficients of the powers of τ to zero, we have for

For τ^o :

$$p_1 - \rho_0 c_0 u_1 = 0 \tag{4.4}$$

$$p_1^1 + p_2 - \frac{p_1 u_1}{c_0} + 2\rho_0 c_0 u_1 c_x^0 - 2\rho_0 u_1 c_p^0 p_1 - \rho_1 c_0 u_1 - \rho_1 c_0 u_2 + \frac{\rho_0 c_0^2 u_1 A_x^0}{A_0} = 0 \tag{4.5}$$

$$\rho_0 C_V T_1 - \frac{p_0 u_1}{c_0} - \frac{2\mu u_1^2}{c_0^2} = 0 \tag{4.6}$$

For τ :

$$\begin{aligned} & p_1 - \rho_0 c_0 u_1 = 0 \\ & \rho_0 c_0 u_1^1 + \rho_0 c_0 u_2 + \rho_1 c_0 u_1 - \rho_0 u_1^2 - p_2 - \\ & \rho_0 c_0 (p_1 A_p^0 - c_0 A_x^0) f_A^0 - \rho_1 c_0 u_1 f_u^0 = 0 \end{aligned} \tag{4.7}$$

$$\rho_0 C_V T_1^1 + \rho_1 C_V - \rho_0 \frac{C_V u_1}{c_0} + 2\mu\frac{u_1^2\alpha}{c_0^2} - 2\xi(\frac{\pi}{A})^{1/2} T_1 - \rho_0 C_v T_2 - \left(\frac{p_0 u_2}{c_0} + \frac{p_1 u_1}{c_0} + \frac{4\mu u_1 u_2}{c_0^2}\right) = 0 \tag{4.8}$$

Using equation (4.4), substituting u1 in terms of p1 into equations (4.5) and (4.7) and combining the results, p2 and u2 are eliminated, and we have

$$p_1^1 - \frac{p_1^2}{2c_0}\left(\frac{1}{\rho_0 c_0} + 2c_P^0\right) + \frac{p_1}{2}\left(2c_x^0 + \frac{c_0 A_x}{A_0} - \rho_0 c_0 A_P^0 f_A^0 - f_u^0\right) + \frac{\rho_0 c_0^2}{2} A_x^0 f_A^0 = 0 \tag{4.9}$$

This equation has the form of the Riccatti equation. This equation must satisfy the initial condition p1 (0) = r, (where $r = \frac{dp}{dt}$ is the rate of pressure rise in the ventricle at the moment when the aortic valve opens, and p is the ventricular pressure)

Also, from equation (4.6), we have

$$T_1 = \frac{p_0 u_1}{\rho_0 c_0 C_V} - \frac{2\mu u_1^2}{\rho_0 c_0^2 C_V} \qquad (4.10)$$

Specific Cases of Flow

Inviscid Flow in exponentially tapered tubes

Here, $\mu = 0$, hence, $f_A^0 = 0, c_x^0 = 0, A(p,x) = A_P e^{-\beta x}$, where $-\beta = \frac{A_x}{A_0}$, (β is a measure of the tapering of the tube. β > 0 corresponds to tubes which becomes narrower as the axial distance increases, as for blood flow in the systemic arteries, and β < 0 is for tubes which becomes wider as the axial distance increases, as for blood flow in the pulmonary arteries.)

By these, equation (4.9) reduces to:

$$\frac{dp_1}{dt} - \alpha p_1^2 - \lambda_1 p_1 = 0, \quad P1\ (0)\ = r \qquad (4.11)$$

where $\lambda_1 = -\frac{c_0 \beta}{2}$, $\alpha = \frac{1}{2c_0}\left(\frac{1}{\rho_0 c_0} + 2c_p^0\right)$

The solution is $\frac{1}{p_{1(1)}} = \frac{1}{r} e^{\lambda_1 t} - \frac{\alpha}{\lambda_1}(e^{\lambda_1 t} - 1)$ (4.12)

Viscous flow in exponentially tapered tubes

Here, using the well known Hagen – Poiseuille formula for frictional force in laminar flow to approximate the frictional force, f≠ 0, $f_u^0 = -\frac{8\pi\mu}{\rho_0 A_0}$, μ > 0, $f_A^0 = 0$, $c_x^0 = 0$,

A (p, x) = $A_{(P)} e^{-\beta x}$

By these, equation (4.9) becomes

$$\frac{dp_1}{dt} - \alpha p_1^2 + \lambda_2 p_1 = 0, \quad p1(0) = r \qquad (4.13)$$

where $\lambda_2 = \left(\frac{4\pi\mu}{\rho_0 A_0} - \frac{c_0 \beta}{2}\right)$

The solution is $\frac{1}{p_{1(2)}(t)} = \frac{1}{r} e^{\lambda_2 t} - \frac{\alpha}{\lambda_2}(e^{\lambda_2 t} - 1)$ (4.14)

(The second subscripts attached to the p1's denote the flow cases considered)

RESULTS

A comparison of the equations (4.12) with (4.14) shows that:

$$\frac{1}{p_{1(2)}} > \frac{1}{p_{1(1)}}$$

yielding

p1(2) < p1(1)

u1(2) <u1(1) u = u (p, t) (5.1)

By equation (5.1), equation (4.10) gives

$T_{1(2)} < T_{1(1)}$ (5.2)

Discussion of results

The primary aim of this study is to investigate the effect of viscosity on the flow of blood in the artery (where arterial side branches are neglected) in relation to hypertension. Therefore, in this section, we shall look at the effect of viscosity on the flow, dynamically and clinically.

Equation (5.1) shows that the pressure in the viscous flow is less than that in the inviscid flow, and hence, the velocity in the viscous flow is less than that in the inviscid flow. From equation (4.10), since the temperature is also velocity – dependent, it follows that the temperature in the viscous flow is less than that in the inviscid flow. This yields equation (5.2)

From the foregoing, there is a clinical implication. With drugs capable of raising appreciably the viscosity of blood of the hypertension patient, the problem of hypertension will be ameliorated

CONCLUSION

The analysis shows that the viscosity of blood reduces the pressure, and hence, the velocity and temperature of the arterial blood flow. No doubt, our results will be of great importance to the biomedical and pharmacological scientists.

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