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DETERMINATION OF THE PARAMETERS FOR DESIGN OF FLEXIBLE PLASTIC TANK

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ABSTRACT: *The need to provide emergency water supplies in remote locations and to store water in small, irregular spaces was the main motivation for the research study. The flexible plastic tank provides a good solution to the challenges of efficient water supply to remote areas but has the disadvantage of creeping incessantly during use and does not recover its original dimensions after loading. This project sought to establish the limits within which the flexible HDPE tank may be used in hot tropical climates for water storage. The creep and recovery characteristics of the material were determined for the range of loads and temperatures at which the material is likely to be applied. A set of equations was developed from the creep and recovery curves to estimate the values of creep and recovery within the temperature range 30°C - 50°C and for stresses ranging from 0.781 to 1.563 MPa just by knowing only the applied stress and the operating temperature. Estimates were also provided for the expected permanent expansion of the material under load, when loaded once and when loaded and unloaded intermittently.*

INTRODUCTION

The Need for Flexible and Potable Water Storage Vessels

Slums, refugee camps present a special challenge to engineers and planners in that they are almost always not properly designed. Some of the big challenges in slum setups are water storage and rainwater harvesting because of the low height of buildings and inadequate space between buildings for construction of rainwater harvesting and storage structures, yet there is immense potential for rainwater harvesting in these places due to the vast roof area concentrated in one place as well as the fact that there is a high population density that is usually not connected to the regular municipal water system. Apart from being affordable to most slum dwellers, the flexible plastic tanks may fit in any shape of space as may be available in slums (Onyango, 2003)

In refugee, emergency and disaster camps, the most popular mode of water supply is by trucking, but the biggest problem is usually the water storage after it has arrived at the point of use. The flexible plastic tanks may easily be folded and supplied by helicopter and dropped off from the sky and may conveniently provide the required storage. For nomadic communities, water availability during their migration has been a major problem. A design of the flexible plastic storage in the model of the age old water skin to be carried by animals may alleviate this need (Kenya Rainwater Association, 2004)

This project was therefore the first step aimed at development of a tank that may contribute in part to the solution of the problem of water supply in Slums, arid and semi arid lands (ASALs) and refugee camps. Specifically, it seeks to test the feasibility of using HDPE, which is readily available and is currently used for lining water reservoirs, to fabricate flexible tanks that can fit in small and irregular spaces and may easily transported.

Despite the advantages of this tank, the material used is comparatively weak in strength, deforms with increasing intensity as temperature and loading rise and is subject to degradation on exposure to water and solar radiation Hence it is necessary to precisely define pertinent parameters under which this tank would operate. Of utmost importance is to determine the maximum amount of water that the tank may carry without deforming beyond allowable limits at the expected temperatures of operation. Moreover, it was necessary to determine the permanent deformation of the flexible tank after storage of water for a given period of time so as to set the limits of how much water the tank may store. Inevitably then, this design project was concerned with basic principles of viscoelasticity, specifically creep and recovery. Creep under load simulates stretching when loaded with water while recovery represents the contraction as the water is unloaded from the flexible tank.

Modeling Recovery

It was important to investigate creep recovery in this study since the material used was expected to be loaded intermittently, that is, the water tank would sometimes full of water in which case the HDPE material would exposed to high loads and creep and would sometimes empty, in which case recovery would proceed. This presents a case of creep in when the reservoir is loaded and recovery is assumed after it is emptied. The irreversible damage on the material is represented by un-recovered residual strain after loading. For a linear viscoelastic material in which strain recovery may be regarded as the reversal of creep, then the material behaviour may be represented as in Figure 1. Thus the time-dependent residual strain, $\varepsilon_r(t)$ may be expressed as (Crawford, 1998):

$$\varepsilon_r(t) = \varepsilon_c(t_c) - \varepsilon_c(t, t_c) \quad (1)$$

Where:

- ε_c is the creep strain during the specified period, denoted by (tc) or (t-tc).
- t_c is the specific time when the material is unloaded at the end of the creep period
- t is the specific time of taking a recovery reading

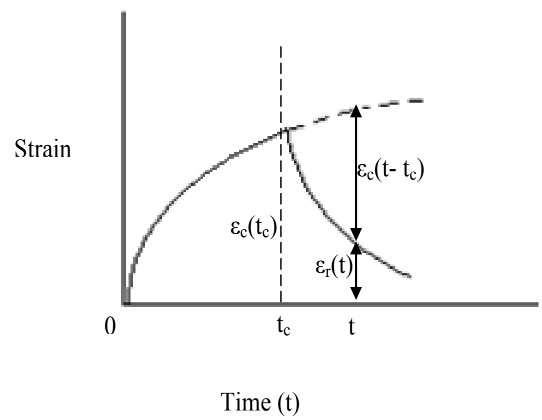

Figure 1: Typical Creep and Recovery Behaviour (Crawford, 2005)

According to Crawford (1998), there can be an infinite number of combinations of creep and recovery periods. It is therefore convenient to express this behaviour in terms of either fractional recovery or reduced time. Fractional recovery, Fr, is defined as:

$$F_r = \frac{\text{Strain Recovered}}{\text{CreepTime}} = \frac{\varepsilon_c(t_c) - \varepsilon_r(t)}{\varepsilon_c(t_c)} \quad (2)$$

Where:

$\varepsilon_c(t)$ is the creep strain at the end of creep period and $\varepsilon_r(t)$ is the residual strain at any selected time during the recovery period.

Reduced time is a dimensionless variable defined as:

$$t_R = \frac{\text{Recovery Time}}{\text{Creep Time}} = \frac{t - t_c}{t_c} \quad (3)$$

It has been shown by Crawford (1998) that if the final creep strain is not large, then a graph of Fractional Recovery Vs Reduced Time is a master curve, which describes recovery behaviour with acceptable accuracy. The relationship between Fr and tR may be derived as follows. When creep curves are plotted on logarithmic strain and time scales they are approximately straight lines so that the creep strain $\varepsilon_c(t)$ may be expressed as:

$$\varepsilon_c(t_c) = At^n \tag{4}$$

Using the relationship in Equation (1),

$$\varepsilon_r(t) = At^n - A(t - t_c)^n \tag{5}$$

Therefore, the Equation (2) for Fractional Recovery may be written as;

$$F_r = \frac{At_c^n - \left[At_c^n - A(t-t_c)^n\right]}{At_c^n} = 1 - \left(\frac{t}{t_c}\right)^n + \left(\frac{t}{t_c} - 1\right)^n \tag{6}$$

Hence,

$$F_r = 1 + t_R^n - (t_R + 1)^n \tag{7}$$

The relationship in Equation (7) is only good approximations since plastics are not linearly viscoelastic and do not completely obey the power law in Equation (4). However, the equation is favoured on account of its simplicity and is sufficiently accurate for most purposes and allows the analysis of intermittent loading.

From Equation (7) and the definition of Fractional Recovery, Fr in Equation (1), the residual strain is given by:

$$\varepsilon_r(t) = \varepsilon_c(t_c) - F_r.\varepsilon_c(t_c) = \varepsilon_c(t_c)\left[\left(\frac{t}{t_c}\right)^n - \left(\frac{t}{t_c} - 1\right)^n\right] \tag{8}$$

If there are N cycles of creep and recovery, the accumulated residual strain would be:

$$\varepsilon_r(t) = \varepsilon_c(t_c)\sum_{x=1}^{x=N}\left[\left(\frac{t_pN}{t_c}\right)^n - \left(\frac{t_pN}{t_c} - 1\right)^n\right] \tag{9}$$

Where tp is the period of each cycle and thus the time for which total accumulated strain is being calculated is t = tpN. It is also noted that that the total accumulated strain after the load application for the (N+1)th time will be creep strain for the load-on period, given by $\varepsilon_c(t_c)$ plus the residual strain $\varepsilon_r(t)$, that is:

$$(\varepsilon_{N+1})_{\max} = \varepsilon_c(t_c)\left\{1 + \sum_{x=1}^{x=n}\left[\left(\frac{t_pN}{t_c}\right)^n - \left(\frac{t_pN}{t_c} - 1\right)^n\right]\right\} \tag{10}$$

Crawford (1998) has been clearly shown that when the total strain is plotted against the logarithm of the total creep time (that is, Nt or total experimental time minus the recovery time) the resultant is a linear relationship. This straight line includes the strain at the end of the first creep period and thus one calculation, for say the 10th cycle allows the line to be drawn. The total creep strain under intermittent loading can then be estimated for any combinations of loading/ unloading times.

The broad objective of the study was to determine the core parameters that would contribute to the design of an un-reinforced HDPE flexible tank for water storage. The specific objectives were:

1. To obtain the maximum amounts of water that an un-reinforced flexible HDPE tanks may carry when modeled as a thin cylindrical shell under pressure at different temperatures.

2. To determine the maximum lifespan of the HDPE tank when subjected to intermittent loading and unloading based on the residual unrecovered creep after loading.

MATERIALS AND METHODS

The specimens used in this project were obtained from HDPE plastic manufactured by A-Plus Ltd. in Nairobi at different times. The material used in the tests was 0.8mm thick. The rest of the dimensions are as shown in Figure 2 as per ASTM D 638 (2003) for Type II specimens. Specimens in both the axis parallel to the direction of extrusion and in the axis perpendicular to the direction of extrusion were used in the tests in equal proportions so as to cancel the effect of material orientation.

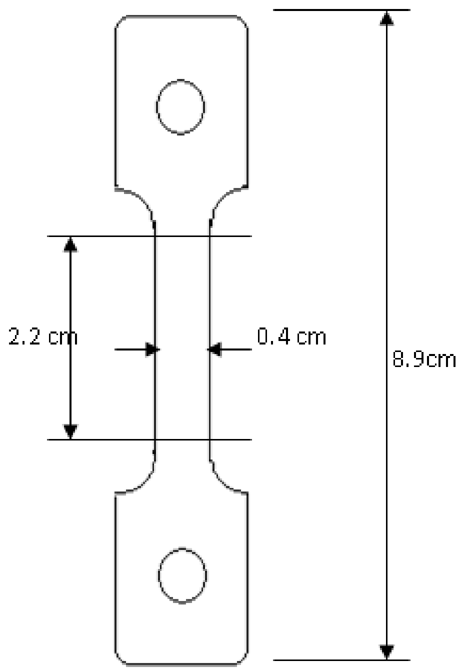

Figure 2: Specimen Geometry – Die Type II Dimensions used in the research study

The testing program comprised of exposing HDPE samples to the oven heated to prescribed temperatures. Three temperatures of 30^o, 40^o, 50^o C were used. The temperatures were preset in the oven for a long time so as to stabilize. The specimens were then loaded in creep at stress levels of 1.56, 0.94 and 0.78 MPa for each of the temperature levels. These creep levels were taken to be about 10% of the material strength so as to mitigate excess creeping. The data was collected by measurements of extension of the specimen due to stress applied a period of 168 hours. Thereafter, the specimens were unloaded and allowed to undergo recovery. The readings of recovery were taken for a period of two weeks. The results were then analyzed so as to obtain the maximum water storage capacity at the given temperatures and the approximate length of time that the flexible tank may be used before its dimensions increase (due to permanent deformation) beyond acceptable levels.

To determine stress acting on the walls at any time, the equation of thin cylindrical shells was used as (Timoshenko and Goodier, 1983):

$$\sigma_c = \frac{\mathrm{Pr}}{t} \quad (11)$$

Where:

- P is the internal pressure exerted by the stored water
- t is the wall thickness
- r is the inside radius of the cylinder.
- σ_c is the hoop stress acting on the material.

Substituting for water pressure given as $\rho g h$ where ρ is density, g is gravitational pull and h is height substituted for radius r since the shape is cylindrical lying on the side, the equation becomes

$$\sigma_c = \frac{\rho g h^2}{t} \quad (12)$$

Hence for any applied stress σc, it was possible to calculate by Equation 13, the maximum radius and length of the cylindrical tank that would contain an amount of water that would cause the stress σc on the tank wall.

DATA ANALYSIS AND DISCUSSION

The tensile strength of the HDPE material had previously been determined as 12 MPa. The proposed shape of flexible tank to be used was a cylinder. The essence of the experiments was to relate the amount of water and the pressure there from to the stress acting on the walls of the cylindrical tank.

To find the maximum tank radius □c was taken as 12 Mpa , the material strength and substituted in equation (13) together with a material thickness, t of 0.8mm, this yielded a maximum tank radius of 0.989m, giving a maximum volume of 4.052 m3 per meter length at the specified thickness. Hence it was determined that the maximum amount of water that would cause instant breaking of the tank is 4,052 liters. By equation (13), an increase in thickness would increase the maximum radius and volume of water stored.

Each of these loads corresponds to a given volume of water in the tank so as to apply an equivalent pressure to induce the specified stress. From Equation (13) it was possible to calculate the corresponding maximum tank radii and volumes that will contain water to apply the equivalent of experimental loads. These are presented in Table 1.

Table 1: Maximum tank radii and volumes for applied creep loads

(a)	(b) Applied stress (MPa)	(c) Strain in 168 (hours)	(d) Unrecovered strain	(d) Maximum radius, r (m)	(e) Maximum Volume (m^3)	(f) Increase in volume (%)	(g) Permanent damage (%)
30 °C	1.563	1.80	0.55	0.422	0.530	965	27.38
	0.937	1.45	0.35	0.324	0.603	588	21.93
	0.781	0.95	0.22	0.255	0.695	427	20.26
40 °C	1.563	2.55	0.60	0.363	0.199	1070	30.46
	0.937	1.80	0.42	0.280	0.092	636	26.75
	0.781	1.65	0.38	0.255	0.069	445	23.41
50 °C	1.563	3.60	1.60	0.365	0.205	2989	20.07
	0.937	3.30	0.95	0.282	0.094	1321	19.77
	0.781	2.45	0.80	0.257	0.071	1085	17.00

Table 2: Variation of thicknesses with temperature under constant loading

Temperature	**Lateral strain**		
	0.7813 MPa	**0.9375 MPa**	**1.563 MPa**
25°C	0.000	0.025	0.063
30°C	0.063	0.088	0.150
40°C	0.100	0.125	0.188
50°C	0.213	0.250	0.375
60°C	0.313	0.375	0.438

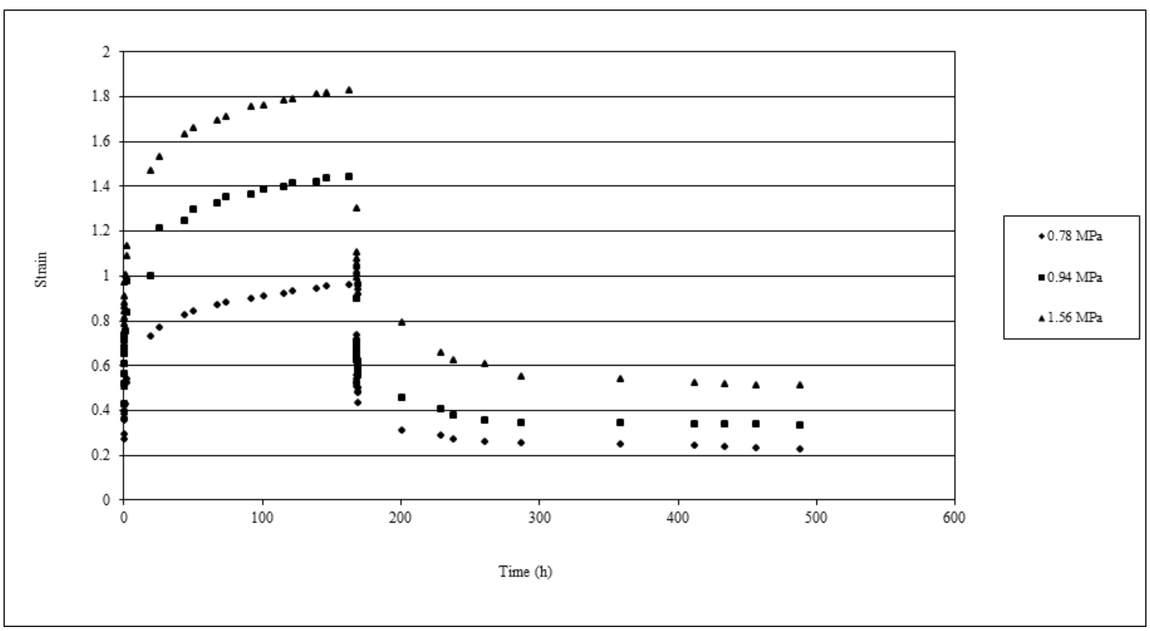

Figure 3: Creep and Relaxation curve at 30°C

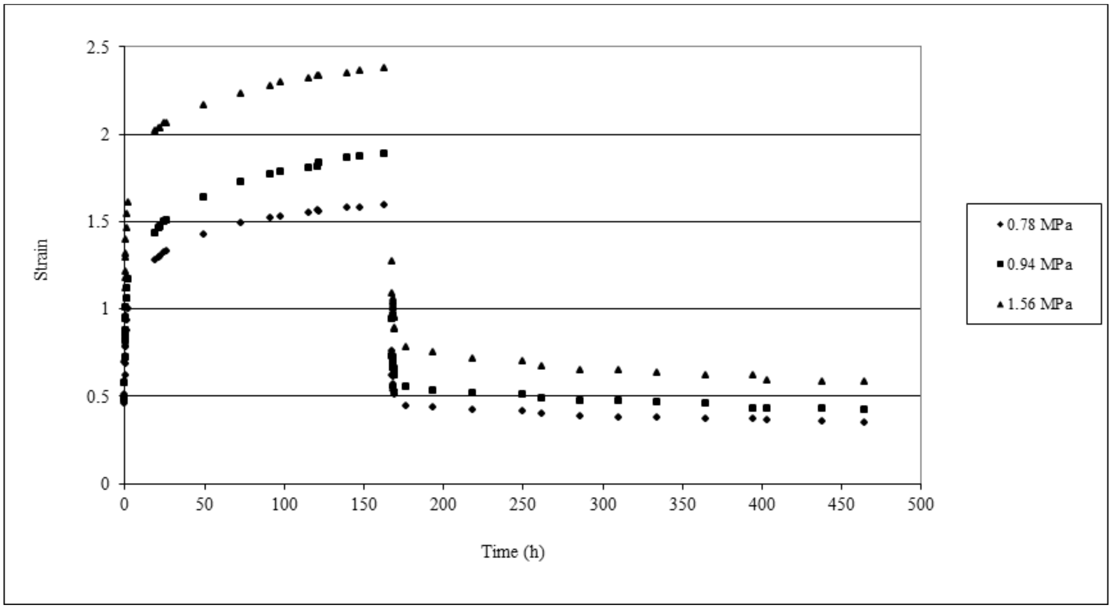

Figure 4: Creep and Relaxation curve at 40°C

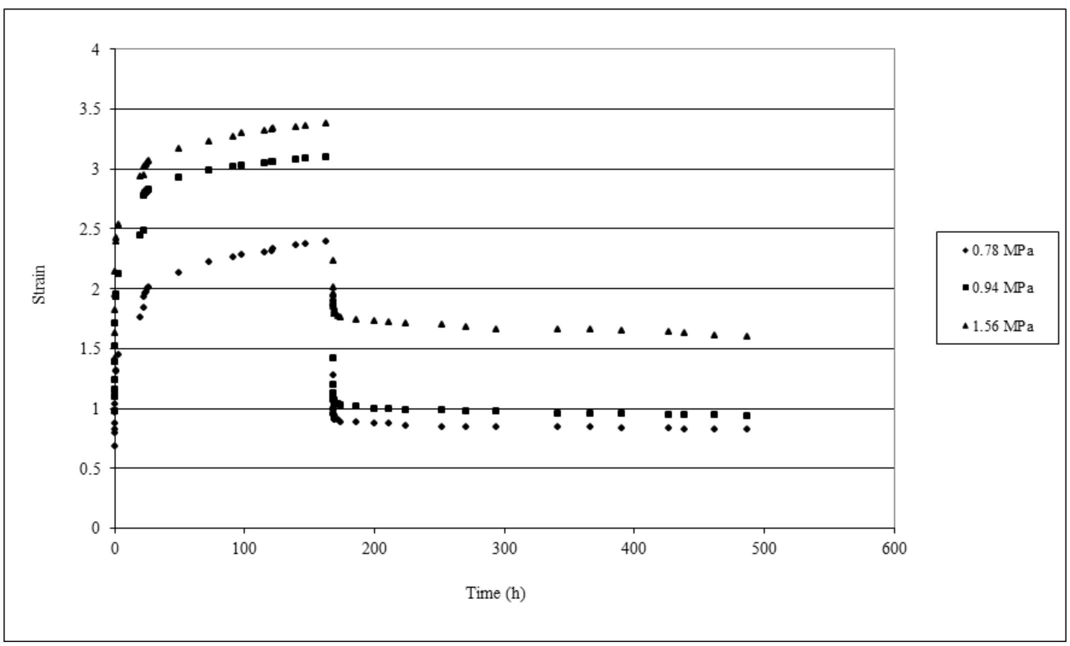

Figure 5: Creep and Relaxation curve at 50°C

Table 2 shows the results of variation in the thicknesses of specimen after elongation in different temperature conditions and varying loads. It was observed from Table 2 that the strain in the axis of loading was directly proportional to the lateral strain.

However, the research project used loads approximately 10% of the material strength so as to investigate the effect of creep namely 1.56, 0.94 and 0.78 MPa. The resulting creep and recovery curves are presented in Figures 3 – 5. The both the creep and recovery curves were fitted with power law to very high values of R2.

The power law was in the form of Equation (4), Where:

$\varepsilon_c(t_c)$ = Time dependant strain

A&n = Constants and be ± depending on whether the curve is of creep (+) or relaxation (-)

t = time elapsed (hours)

This implies that that theory on recovery presented in Crawford (1998) may be used in analysis of the data obtained in this project since in both cases, data may be described by the power law. The parameters A, n and R2 for each of the creep and relaxation curves at the specified temperatures and applied loads are presented in Table 2. It is usually important to obtain mathematical relationships for the different parameters so as to be able to interpolate or estimate values of different parameters for the temperatures and loads not actually used in the experiments. It was noted from Table 3 that the value of A generally increased with temperature for both creep and recovery. It was also noted that at a particular temperature, the values of A increased as the applied stress increased for both creep and recovery. To investigate this observation, curves of applied stress versus A values were plotted for both creep (Figure 6) and relaxation (Figure 7) data.

From Figure 6, the relationship between applied stress and values of A for creep at different temperatures was seen to be best represented by exponential equations as presented in the curves and yielded quite high value of R2. On the other hand, From Figure 7, the relationship between applied stress and values of A for recovery at different temperatures was seen to be best represented by linear equations as presented in the curves and yielded high value of R2.

Table 3: Summary of the parameters form the creep and relaxation curves

Temp °C	Applied stress (MPa)	CREEP				RECOVERY			
		A	n	MEAN n	R^2	A	n	MEAN n	R^2
30	**1.563**	0.991	0.126	0.137	0.964	0.931	-0.098	-0.105	0.971
	0.937	0.751	0.133		0.980	0.582	-0.098		0.986
	0.781	0.451	0.153		0.994	0.463	-0.121		0.994
40	**1.563**	1.341	0.124	0.127	0.939	0.941	-0.078	-0.070	0.986
	0.937	1.002	0.128		0.975	0.646	-0.067		0.984
	0.781	0.860	0.128		0.990	0.528	-0.066		0.989
50	**1.563**	2.161	0.097	0.115	0.956	1.855	-0.023	-0.023	0.960
	0.937	1.796	0.119		0.979	1.075	-0.022		0.948
	0.781	1.272	0.128		0.994	0.944	-0.024		0.948

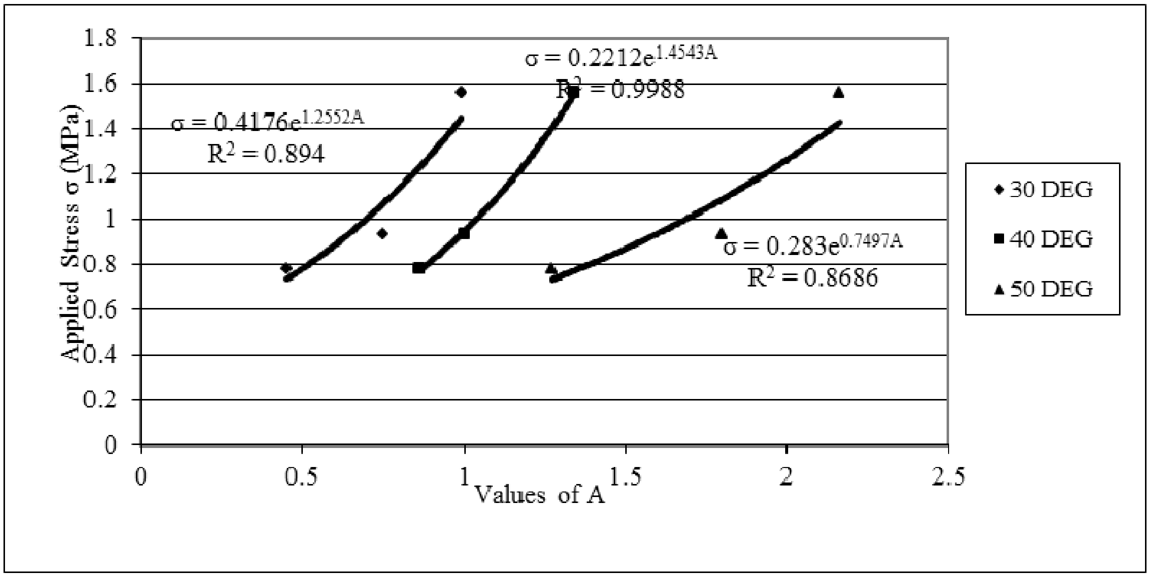

Figure 6: Relationship between applied stress and values of A for the creep curves at different temperatures

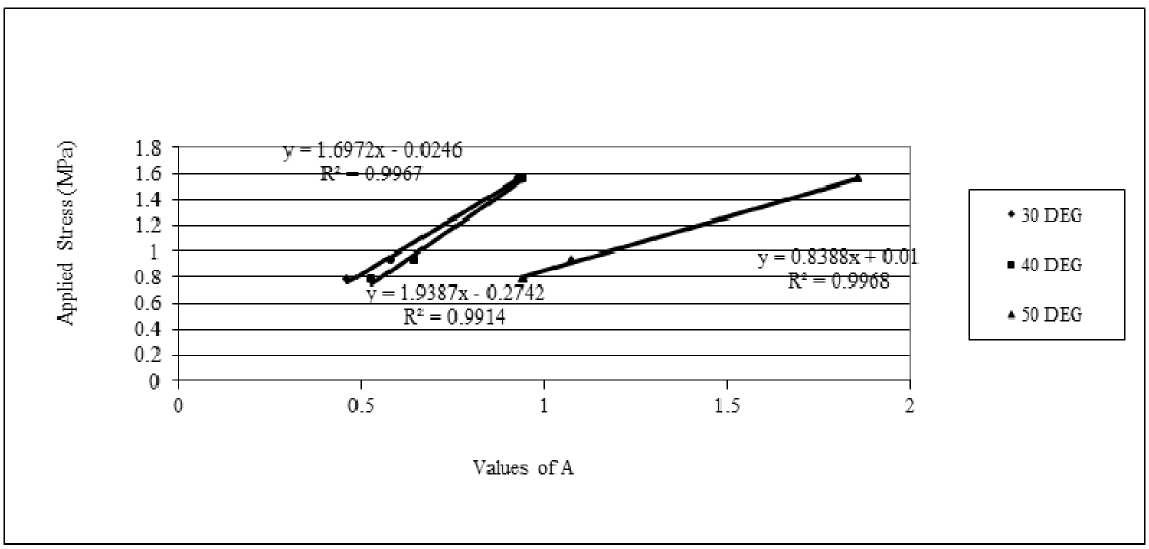

Figure 7: Relationship between applied stress and values of A for the relaxation curves at different temperatures

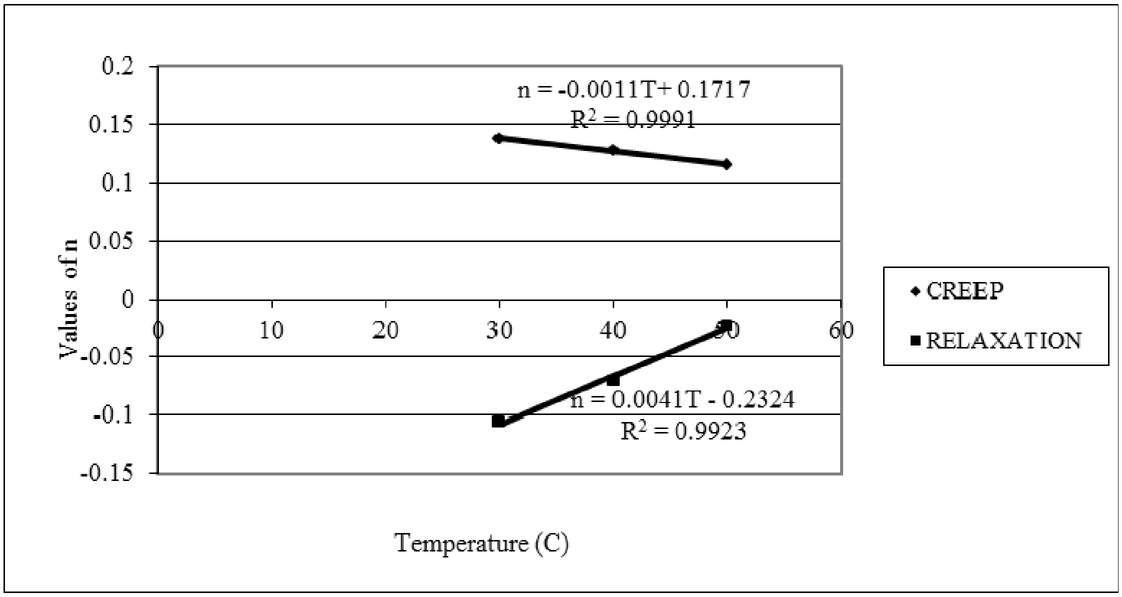

Figure 8: Relationship between values of n and temperature (T) for creep and relaxation

It was also noted in Table 3 that the values of n were almost constant for any applied stress at a particular temperature for both creep and recovery. Therefore for each temperature an average value of B was obtained. It was further observed that the average values of B decreased as temperature increased for creep and that the values of B increased as temperature increased during recovery. In Figure 8, a curve of the relationship between the values of B and temperature was plotted and yielded almost perfectly straight lines. It was therefore concluded that the values of B were independent of applied stress but were linearly related to the operating temperature.

It was concluded that, from the three curves presented in Figure 6, 7 and 8 it is possible to estimate the value of creep and recovery within the temperature range 30°C - 50°C and for stresses ranging from 0.781 to 1.563 MPa just by knowing only the applied stress and the operating temperature. This is because from either Figure 6 or 7, it is possible to obtain the values of A in creep and recovery by knowing the value of the applied stress (service loads) and from Figure 8, it is possible to obtain the values of n by knowing the operating temperature. These can then be used to estimate the expected creep and recovery strains from the power law in Equation (4) for the range of temperature and loads covered in the project.

It was further concluded that, by equations (10) and (11), by this project it is possible to obtain the expected permanent expansion of the material under load, when loaded once and when loaded and unloaded intermittently when given the expected volume of water to be stored. This is because from to be stored, the load that the water applies on the material may be calculated. Using Figures 6, 7 and 8, the values of constants A and n are obtained to be used in Equation (4) to calculate the corresponding value of $\varepsilon c(tc)$, which can be used together with tp, the period of each cycle and the number of cycles N, to obtain the accumulated residual strain in equation (10) and the total accumulated strain in equation (11)

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