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FINITE ELEMENT ANALYSIS OF A FLUID-STRUCTURE INTERACTION IN FLEXIBLE PIPE LINE

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ABSTRACT:- *This paper describes the basic theory and computing method for transient flow of liquid in flexible pipe such as rubber tubing and arterial system. A mathematical model taking into account tube wall axial and radial motion (in which the dynamic fluid pressure causes circumferential and axial motion of the tube wall) is presented. The tube wall is assumed to be elastic material and the compressibility of the liquid is neglected. Circumferential and axial strain-stress relationships for the tube are considered. The obtained mathematical system is constituted of four non-linear hyperbolic partial differential equations describing the wave propagation in both pipe wall and liquid flow. The fluid-structure interaction is found to be governed by Poisson's ratio. In this steady finite element method based on Galerkin formulation is applied. Numerical results show a good similarity with those of the literature obtained by the characteristics method.*

Key words : *Fluid-structure interaction, flexible pipe, rubber, finite element method.*

INTRODUCTION

For a long time, transient flows in elastic piping systems have received a lot of interest to predict the pressure fluctuations provoked by water hammer phenomenon. The circumstances where this pressure fluctuation appears are numerous, following voluntary disturbances or accidental disturbances (rapid valve closure or sudden pump failure in the water network or in the oil pipeline industry in northern Africa for example). The most widely of the previous investigation has considered the pipe to be quasi-rigid, such as metallic pipe, with constant diameter and thickness (Bergeron [2] et Streeter & Wylie [6],).

Transients in pipes generated by rapid changes in flow conditions, have been also investigated in the case of flexible thin walled tubing such as rubber hoses and arteries. In such systems the fluid is considered incompressible relative to elastic properties of the tube wall material. We can quote the study of Streeter & Wylie [6] where the pipe wall axial motion is neglected in the stress-strain model and the dynamic fluid pressure and the radial deformation of pipe wall were separately calculated. They presented an uncoupled mathematical model resolved numerically by the characteristics method which is based on the propagation celerity of the pressure waves and permit to obtain ordinary differential equations (Abbott [1]).

Recently the interaction between the dynamic fluid pressure and resulting dynamic circumferential and axial strain in the tube wall have been successfully investigated by Stuckenbruck & Wiggert [7], Tijsseling & Lavooij [8] and Gorman et al [4]. The authors have examined the influence of Poisson ratio on the coupling between the pipe wall and the fluid. In their works, it is showed that the fluid-structure interaction (FSI) is well governed by the Poisson coefficient of the material and the mathematical model is also resolved numerically by the characteristics method.

More Recently, an exhaustive source review (123 references) by Wiggert & Tijsseling [9], which summarize the essential mechanisms that causes (FSI): Poisson coupling which is described above, the friction coupling related to the transient fluid shear stresses acting on the pipe, and junction coupling which result from junction conditions such as closed end, miter bend, T section etc. They present the various numerical and analytical methods that have been developed to predict FSI, and relate recent contributions in the field, with primary emphasis on those published from 1990 to 2000.

In this paper we present a mathematical model to examine the FSI of the coupled system in the case of a single horizontal flexible thin walled tubing. This is an other example in which we show the importance of fluid-structure interaction phenomena (see Karra & Ben Tahar [5]). A finite element method based on Galerkin formulation is developed (Dhatt & Touzot [3], Zienkiewicz & Taylor [10]) and a non-linear matrix system is obtained. To solve this later, an iterative algorithm based on the Gaus substitution method is used.

Assumptions

Consider the case of a single horizontal flexible thin walled tubing. The wall is free to move in the radial and axial directions. In addition to the habitual assumptions of one-dimensional plane flow, the following assumptions particularly to the pipe elastic properties should be stated.

1. The pipe wall material is homogeneous, isotropic, linearly elastic and subjected to small deformations provided by the Hook's law.

2. Only small deformations of tube wall occur : $0.9 < r/r_0 < 1.1$, where r is the tube radius and r_0 is the initial tube radius.

3. Poisson ratio ν is nearly to 0.5, it can be shown that the volume of the wall material by a unit of length remains constant .
   i.e.: $e\,r = e_0 r_0$, where e_0 is the initial wall thickness and e is the instantaneous wall thickness.

4. The radial inertia of the pipe wall is neglected so that the hoop stress and fluid pressure are related by:

$$\sigma_\theta = \frac{pr}{e}$$

5. The fluid is incompressible relative to the elastic properties of the wall material.

6. The ratio of wall thickness to diameter is constant in the initial static condition.

7. Convective terms are negligible: $\dot{(\ )} = \dfrac{d}{dt} = \dfrac{\partial}{\partial t}$

Mathematical model

Structure Equations

A combination of fundamental equations relating stresses and strains on the tube wall provides the following relationships:

$$\dot{\sigma}_x - \nu\,\dot{\sigma}_\theta - E\frac{\partial U}{\partial x} = 0 \quad (1)$$

$$\dot{\varepsilon}_\theta = \frac{1}{E}\left(\dot{\sigma}_\theta - \nu\,\dot{\sigma}_x\right) \quad (2)$$

where σ_x is the axial stress, σ_θ is the hoop stress, ε_x is the axial strain, ε_θ is the hoop strain, E is the Young's modulus of elasticity, ν is the Poisson ratio, U is the pipe axial velocity, t is the time and x the axial position.

The strain-displacement relations can be written as (under the assumption 3).

$$\dot{\varepsilon}_x = \frac{\partial U}{\partial x} \quad and \quad \dot{\varepsilon}_\theta = \tau\frac{\dot{r}}{r} \quad (3)$$

where $\tau = 1 - \dfrac{e_0}{r_0}$

Taking into account of the second equation of (3), the integration of equation (2) yields

$$\tau \ln\frac{r}{r_0} = \frac{1}{E}\left\{\left(\sigma_\theta - \sigma_{\theta_0}\right) - \nu\left(\sigma_x - \sigma_{x_0}\right)\right\} \quad (4)$$

where $\sigma_{x_0} = \dfrac{p_0 r_0}{2e_0}$ is the initial axial stress.

Under the assumptions 3 and 4 this equation becomes

$$\frac{p}{p_0} = \left(\frac{r_0}{r}\right)^2\left\{1 + \frac{Ee_0}{p_0 r_0}\left[\tau\ln\frac{r}{r_0} + \frac{\nu}{E}\left(\sigma_x - \sigma_{x_0}\right)\right]\right\} \quad (5)$$

For minor changes in the radius (assumption 2) this equation can be approximated by

$$\frac{r}{r_0} \approx 1 + \left[\left(p - p_0\right)\frac{r_0}{Ee_0} - \frac{\nu}{E}\left(\sigma_x - \sigma_{x_0}\right)\right] \Big/ \left(\tau - \frac{2\,p\,r_0}{Ee_0}\right) \quad (6)$$

For flexible tubes, the change of diameter as a consequence of fluid pressure change, may be significant. Under the assumptions 3 and 4, an analysis of stress-strain relationships in the tube wall will give the following expression

$$\dot{\sigma}_\theta = \sigma_\theta \left(\frac{\dot{p}}{p} + 2\frac{\dot{r}}{r} \right), \quad (7)$$

Taking into account of the equation (5), equation (7) becomes

$$\dot{\sigma}_\theta = \frac{1}{F} \frac{\dot{p} r_0}{e_0} \quad (8)$$

where

$$F = \left(\frac{r_0}{r} \right)^2 \left\{ 1 - 2\left(1 - \nu^2\right) \left[\ln \frac{r}{r_0} + \frac{\nu}{\tau\, E} \left(\sigma_x - \sigma_{x_0} \right) + \frac{p_0 r_0}{\tau E e_0} \right] \right\}, \quad (9)$$

By replacing the hoop stress in equation (1) by its expression given in equation (8) we obtain

$$\dot{\sigma}_x - \frac{\nu}{F} \frac{r_0}{e_0} \dot{p} - E \frac{\partial U}{\partial x} = 0 \quad (10)$$

The axial direction momentum equation is

$$\frac{\partial \sigma_x}{\partial x} - \rho_m \dot{U} = 0 \quad (11)$$

where ρ_m is pipe wall density.

Fluid Equations

For the fluid, the continuity and the momentum equations are respectively (Streeter & Wylie [6])

$$2\frac{\dot{r}}{r} + \frac{\partial V}{\partial x} = 0, \quad (12)$$

$$\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial V}{\partial t} + \frac{\lambda V |V|}{2D} = 0, \quad (13)$$

where V is the fluid velocity, ρ is the fluid density, D is the pipe diameter and λ is the fluid friction factor.

By using equations (2) and (3) and taking into account of equation (8) the continuity equation (12) becomes

$$\frac{1}{F} \frac{2\, r_0}{E^* e_0} \dot{p} - 2\nu \frac{\partial U}{\partial x} + \tau \frac{\partial V}{\partial x} = 0 \quad (14)$$

where $E^* = \dfrac{E}{1-\nu^2}$

The four equations system (10), (11), (13) and (14) where the unknowns p, V, U and σ_x are function of the distance x and the time t, take into account of the FSI. This interaction is governed by the Poisson ratio in the equations (10) and (14).

Finite element formulation

The system of equations (10), (11), (13) and (14) can be written as:

$$R(\boldsymbol{Y}) = \boldsymbol{A}\, \dot{\boldsymbol{Y}} + \boldsymbol{B} \frac{\partial \boldsymbol{Y}}{\partial x} + \boldsymbol{f} = 0,\ x \in [0, L],\ t \geq 0 \quad (15)$$

where the boundary conditions can be defined by a function f_s which depend of the example to study:

$$\varphi(\boldsymbol{Y}) = \boldsymbol{f}_s,\ x \in \{0, L\},\ t \geq 0 \quad (16)$$

and the initial conditions

$$\boldsymbol{Y}(x, 0) = \boldsymbol{Y}_0(x) \quad (17)$$

where $\boldsymbol{Y} = \begin{Bmatrix} p \\ V \\ \sigma_x \\ U \end{Bmatrix}$

$$\boldsymbol{A} = \begin{bmatrix} -\dfrac{\nu r_0}{F e_0} & 0 & 1 & 0 \\ 0 & 0 & 0 & -\rho_m \\ 0 & 1 & 0 & 0 \\ \dfrac{2 r_0}{F\, E^* e_0} & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 & -E \\ 0 & 0 & 1 & 0 \\ 1/\rho & 0 & 0 & 0 \\ 0 & \tau & 0 & -2\nu \end{bmatrix} \text{ and } \boldsymbol{f} = \begin{Bmatrix} 0 \\ 0 \\ \dfrac{\lambda V |V|}{2D} \\ 0 \end{Bmatrix}$$

To solve the system of equations (15), we use a Galerkine variational formulation (Dhatt & Touzot [3]; Zienkiewicz & Taylor [10]). Let $<\psi> = <\delta p, \delta V, \delta\sigma_x, \delta U>$ a vector of four sufficiently regular test functions. After multiplying equation (15) by $<\psi>$ and integrating over the domain [0, L], one obtains

$$\int_0^L <\psi>\{R(\boldsymbol{Y})\}\,dx = 0 \quad \forall\ <\psi> \tag{18}$$

For the discretization of variational formulation (18) we use a linear isoparametric element with two nodes and four degrees of freedom per node (P, v, σ_x, U). Linear shape functions are used to express fluid or structure variables as a function of nodal variables. The discretization permits to obtain a system of n+1 time partial derivative equations where n is the number of finite elements.

Semi implicit Euler method (Dhatt & Touzot [3]) is used to approximate the time derivative of fluid and structure variables i.e. $\dot{p}_i = (\overline{p}_i - p_i^{j-1}) / (\theta\ \delta t)$, where $\overline{p}_i$ is the pressure at the time $(j-1+\theta)\delta t$, (θ is a parameter which affect the numerical stability of solution, $0 < \theta \le 1$, j = 1, 2,...) and p_i^{j-1} is the pressure at the node i and the time $(j-1)\delta t$. Finally we obtain after assembly the non-linear matrix system :

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & \cdots & & & & & & 0 \\ A_{21} & A_{22} & A_{23} & 0 & \cdots & & & & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & \cdots & 0 & A_{ii-1} & A_{ii} & A_{ii+1} & 0 & \\ & & & & \cdots & \cdots & 0 & A_{i+1i} & A_{i+1i+1} & A_{i+1i+2} & 0 \\ & & & & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \; 0 \\ & & & & & & 0 & A_{nn-1} & A_{nn} & A_{nn+1} & \\ 0 & & & & \cdots & & & 0 & A_{n+1n} & A_{n+1n+1} & \end{bmatrix} \begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_i \\ Y_{i+1} \\ \vdots \\ Y_n \\ Y_{n+1} \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ f_{i+1} \\ \vdots \\ f_n \\ f_{n+1} \end{Bmatrix} \tag{19}$$

where

- $\boldsymbol{A}_{ik}$ is (4x4) matrix which is null if $k < i$-1 or $k > i$+1 else it depends of the unknowns of the problem $\boldsymbol{Y}_{i-1}$, $\boldsymbol{Y}_i$ and $\boldsymbol{Y}_{i+1}$ at the times $(j-1+\theta)\delta t$ and $(J-1)\delta t$.

- $\boldsymbol{f}_i$ is a second member vector which depends of the velocity in the nodes i-1, i and i+1 at the time $(j-1)\delta t$.

- $\boldsymbol{A}_{11}$, $\boldsymbol{A}_{12}$, $\boldsymbol{A}_{n+1\,n}$ and $\boldsymbol{A}_{n+1\,n+1}$ are (4x4) matrices which depend of the boundary conditions in the nodes 1 and n+1.

The non-linear matrix system (19) can be written as a compact form:

$$\left[A\left(\overline{Y}\right)\right]\left\{\overline{Y}\right\} = \{f\}, \tag{20}$$

where $\left\{\overline{Y}\right\}$ is the solution at the time $(j-1+\theta)\delta t$.

The solution of the system (20) is obtained using iterative algorithm based on the Gaus substitution method (Dhatt & Touzot [3]). In this method a succession of solutions $\left\{\overline{Y}\right\}_0, \left\{\overline{Y}\right\}_1, \ldots \left\{\overline{Y}\right\}_k$ are constructed, $\left\{\overline{Y}\right\}_k$ is calculated from $\left\{\overline{Y}\right\}_{k-1}$ by solving the following linear system: $\left[\mathbf{A}\left(\overline{\boldsymbol{Y}}_{k-1}\right)\right]\left\{\overline{\boldsymbol{Y}}\right\}_k = \{\boldsymbol{f}\}$, k=1, 2,... with $\left\{\overline{\boldsymbol{Y}}\right\}_0 = \{\boldsymbol{Y}\}_{j-1}$ is the solution at the time $(j-1)\delta t)$. The process is repeated until the convergence of solution. This is obtained when $\left\|\left\{\overline{\boldsymbol{Y}}\right\}_k\right\| - \left\|\left\{\overline{\boldsymbol{Y}}\right\}_{k-1}\right\| \le \varepsilon \left\|\left\{\overline{\boldsymbol{Y}}\right\}_{k-1}\right\|$, $(\varepsilon << 1)$.

The stability and the speed of convergence depend of three parameters which are the time increment δt, the number of finite element n and the value of θ. When the solution is obtained at the time $(j-1+\theta)\delta t$, a linear interpolation permit us to obtain the solution at the time $j\,\delta t$.

Numerical results

In this section we present an example for validating numerical developments. It concerns an elastic rubber tube analysed by Stuckenbruck & Wiggert [7], where the characteristics are: $r_0 = 9$ mm, $e_0 = 1$ mm, $L = 1$ m, $E = 2.22$ MPa (10^6 Pa), $\nu = 0.5$, $\rho_m = 1185$ Kg/m^3, $\rho = 1000$ Kg/m3, $\lambda = 0.02$, $p_0 = 10.7$ KPa (10^3 Pa), $V_0 = 0$ m/s. The tube is tethered at the end points ($U = 0$), and the wall is free to move axially throughout the interior region. The fluid boundary conditions are the following:

At the upstream end ($x = 0$) the input excitation is given by:

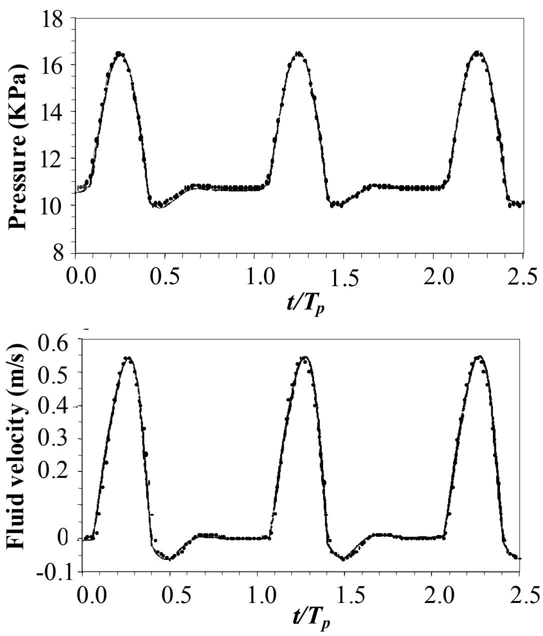
$$\begin{cases} V(t) = 0.5\ \sin(\pi\, t^*/T_s), & t^* \le T_s \\ \quad\ = 0 & T_s \le t^* \le T_p \end{cases}$$

where

$$\begin{cases} T_p \text{ is the excitation period } (T_p = 0.6\text{s}),\ T_s = 0.2\text{s} \\ \quad t^* = t - [E_{\text{int}}(t/T_p)]\ T_p,\ E_{\text{int}} : \text{integer part} \end{cases}$$

At the downstream end ($x = L$) an impedance condition is imposed $p - p_0 = (V - V_0)\sqrt{E\rho\ e_0/2\ r_0}$

Figure 1 shows predicted parameters at the mid point of the tube. The dashed lines correspond to the results obtained by the finite element method (The time step is equal to 0.001s) and the solid lines are obtained by Stuckenbruck & Wiggert [7] using characteristics method. The results prove the validity of the finite element numerical implementation and illustrate the hard coupling between the fluid wave propagation and the pipe wave propagation.



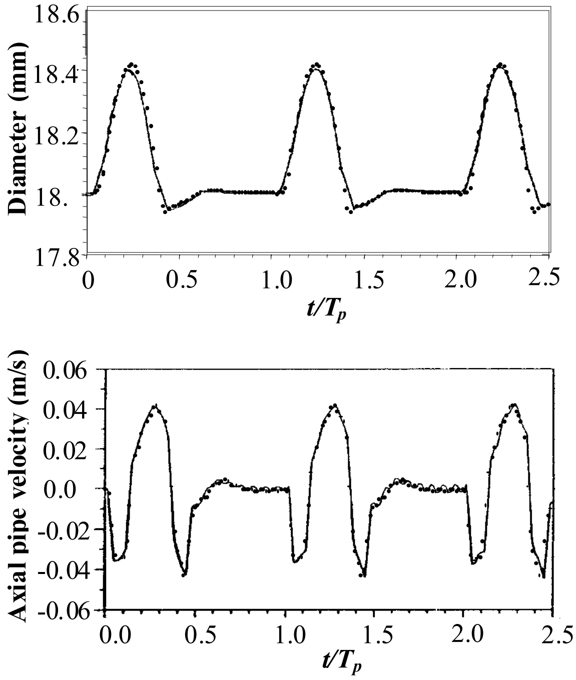


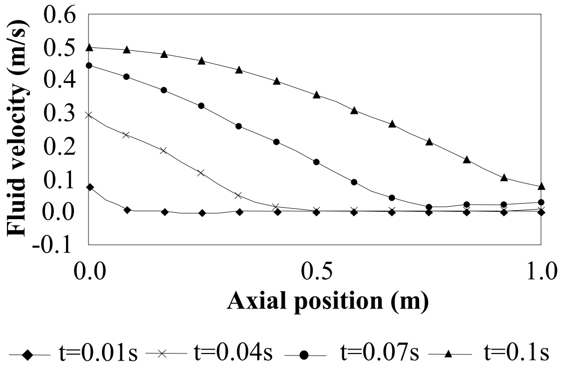
Figure 1: Predicted parameters at the mid point of the tube: —— solution of Stuckenbruck, [7]; . . . present solution based on finite elements formulation

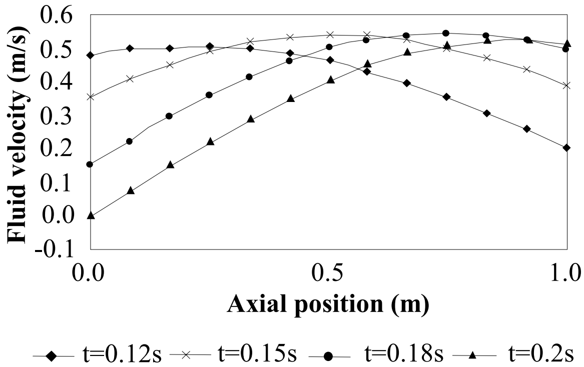
To follow these two waves (over the first time period) we have plotted in Figures 2 and 3 the fluid velocity and the axial pipe velocity as a function of the axial position. Such as the input excitation is periodic of a period equal to T_p = 0.6s, one sees clearly that the curves obtained at 0.61s time are identical to that obtained at 0.01s time. The results illustrate also that the wave speed of the pipe (≈ 50m/s) is about five times that of the fluid wave (≈ 10m/s). Indeed, the speeds of the fluid wave and the pipe wave are given respectively by Stuckenbruck & Wiggert [7]:

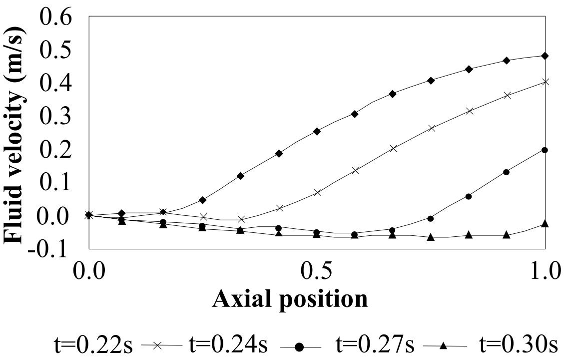
$$c_f = \sqrt{\frac{F\tau e_0 E}{2 r_0 \rho}} \quad \text{and} \quad c_s = \sqrt{\frac{E^*}{\rho_m}}$$

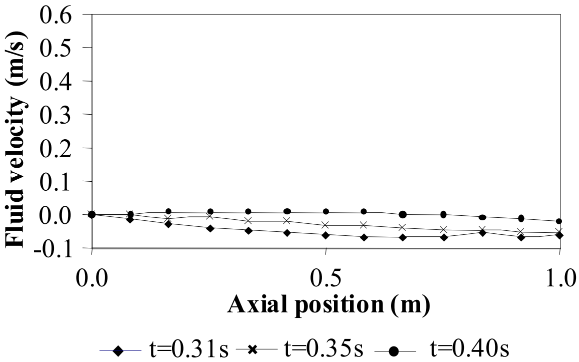
The same example is also studied with other fluid boundary conditions. At the upstream end ($x = 0$) the pressure is constant and equal to 30 KPa and at the downstream end ($x = L$) a sinusoid input excitation is given by: $V(t) = 0.5\ \sin(2\pi\, t/T_p)$, ($T_p$= 0.6s: the period). Figure 4 shows the fluid and structure variables at the mid point of the tube as a function of time (until five periods).

An other time the results illustrate the coupling effect between the fluid wave and the pipe wave which is governed by the Poisson ratio ν.









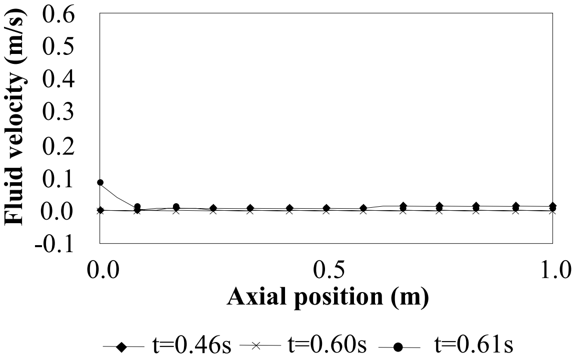
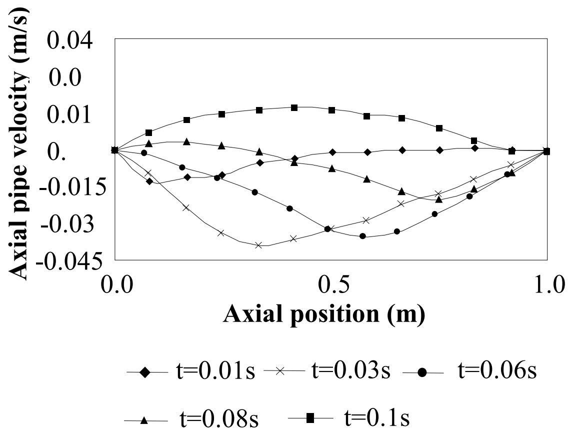
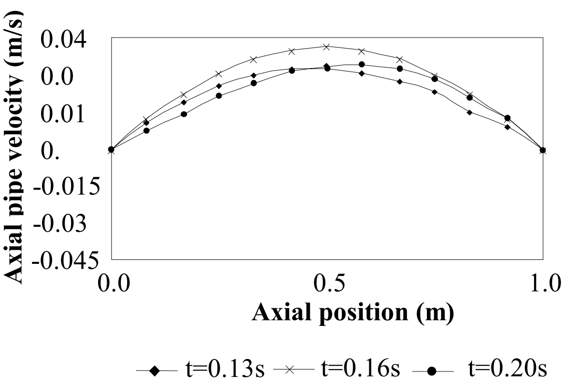
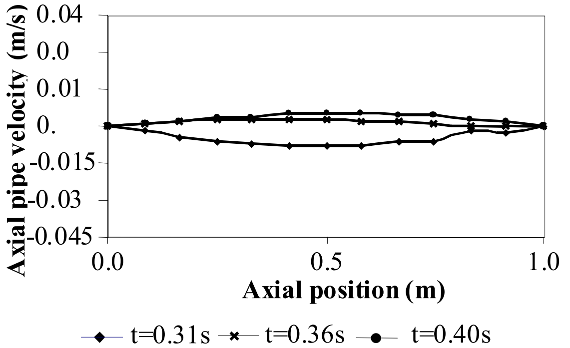


Figure 2. Fluid velocity versus to the axial position at different







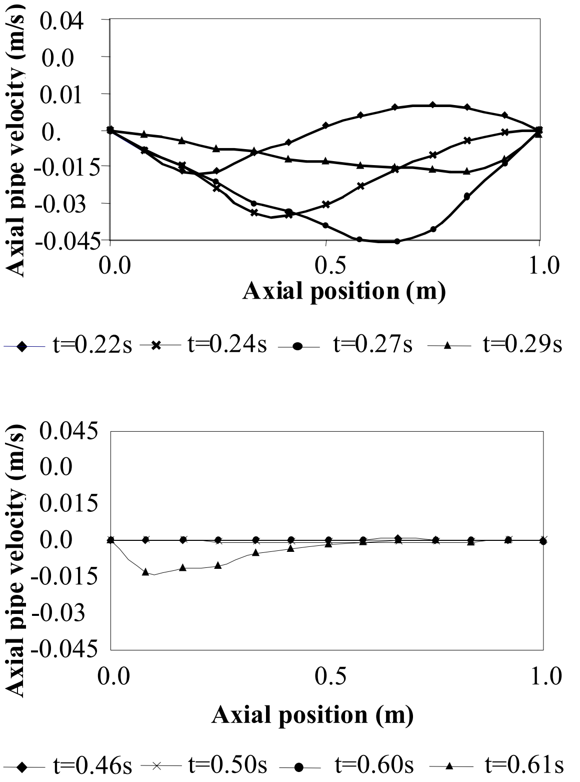


Figure 3: Axial pipe velocity versus to the axial position at different times

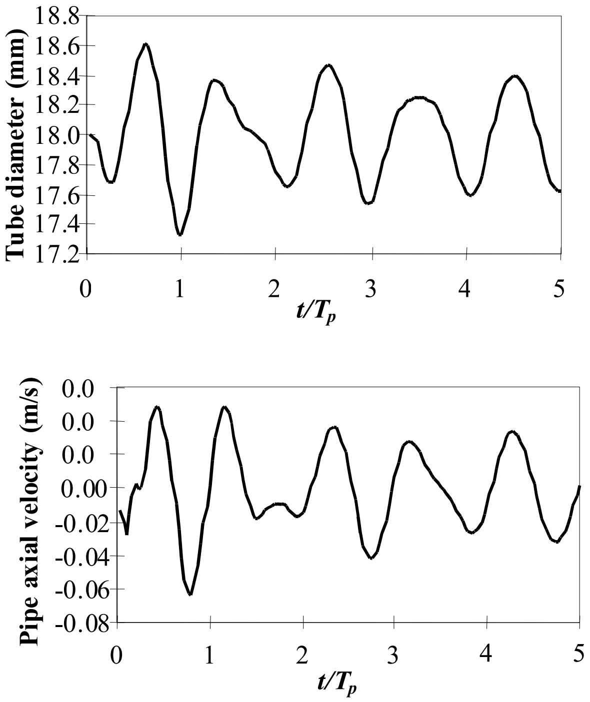
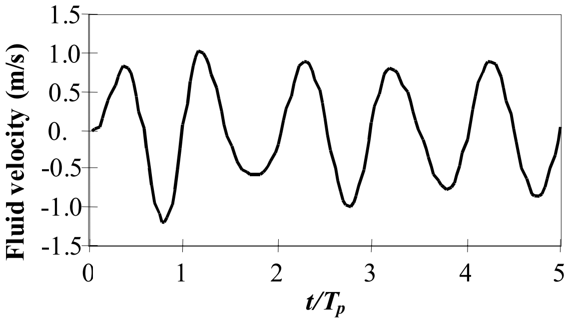
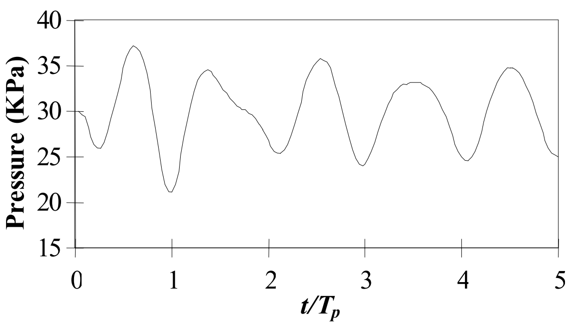


Figure 4: Predicted parameters at the mid point of the tube (sinusoid input excitation)





CONCLUSION

To expect from the calculations, the FSI between transient flow and a flexible pipe line, a finite element method based on Galerkin formulation was investigated. The computer program of this formulation takes account of some fluid boundary conditions such as an imposed pressure, an imposed velocity or an imposed impedance condition. The numerical results predicted by this formulation show a good similarity with those obtained by the characteristics method. The numerical results illustrate also the importance of interaction between the fluid wave and the pipe wave.

It is to note that the finite element analysis program developed has the advantages to predict the overcharges of pressure and can be used as a tool of help of decisions at the level of manoeuvres often experienced in the industry like the oil pipeline industry in northern Africa.

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