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Modelling Rocket System Performance Parameters

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Abstract

What distinguishes the rocket system from airbreathing engines is that the rocket carries its own propellants, fuel and oxidants. The modern rocket was originally developed for ballistic missiles. The operation of the German V2 ballistic missile was not satisfactory because of excessive nose-cone heating. The Soviet Union launched Sputnik 1 to orbit the earth for 21 days. The solid and liquid propellants are widely used in rocket propulsion. Thrust production is necessary in the propulsion of the rocket. Therefore, the purpose of this study is the development of expression through thrust for the study of the performance characteristics of the rocket. Mathematical model has been developed and illustrative solutions obtained, plotted and analyzed for the relationship between parameters that affect rocket performance.

Keywords: nose-cone, thruster, propellant, booster, mass ratio, specific impulse, ballistic missile.

1. INTRODUCTION

Rockets unlike airbreathing engines carry onboard their own fuels and oxidants. This alone enables the vehicle to operate successfully in outer space where there is little or no oxidant. Rockets can reach levels in space unequalled by any other prime movers it has no competition in this area of operation. The modern rockets were originally developed as ballistic missiles to carry war heads to enemy territory. Although of limited success because of the overheating of the *nose cone* the German V2 rocket is a typical earlier version of these vehicles. Lack of knowledge of the characteristics of the *bow shockwave* created and the optimum nose cone design [1] were responsible for the V2 rocket limitations.

The space era started with the cold war. This was a competition between the United States of American and the Soviet Union. It was a supremacy war fought on the battleground of technology advancement. The Soviets were able to launch Sputnik 1 to space in October 1957 to orbit the earth for 21 days and finally burn itself out on re-entry to earth

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atmosphere. A year later, the United States quickly established the National Aeronautics and Space Administration (NASA). The space age rivalry has begun. Today, even the Chinese and Indians have all joined the race [2, 3]. Satellite and space telescope launch, space shuttle and inter-planetary missions are now taken for granted. One hopes that the National Space Agency of Nigeria would soon be able to launch its satellite with home-made rockets.

2.0 Rocket Types

The rocket may be classified as nuclear covering both fusion and fission types, electrodynamic with plasma jet, ion and photon rockets as subtypes and finally the chemical rocket made up of the free radical, the solid and the liquid propellants [4 – 9]. The chemical rockets are, however the most widely used.

The overall size and shape of the chemical rocket is affected by the type of propellant used. For example, whereas the liquid propellant rocket design requires the provision of pumps and injectors, these items are not necessary in the case of the solid propellant type. The importance of the propellants cannot be overemphasized because thrust can only be produced when these propellants are available and burnt satisfactorily. This can explain why the liquid propellant rocket combustion chamber is referred to as the thrust chamber.

2.1 The Solid Propellant Rocket

The solid propellant rocket consists of the combustion chamber, igniter and expansion nozzle. The whole of the fuel and oxidant are contained in the combustion chamber. The igniter is either a detonator or a fuse containing highly reactive primary explosive such as lead oxide. The process is self-sustaining because heat is conducted back to the propellant. The burning rate is dependent on the pressure in the chamber and increases with increasing pressure. The combustion can be controlled or uncontrolled. For maximum thrust, the whole combustion chamber is filled with the propellant and the burning resemble that of the cigarette. This however results in limited operation time and difficulty in nozzle design since the only cooling available is from external sources. The combustion rate is therefore controlled by pressure and the propellant grain (shape). The thrust and duration of operation depend on the shape, size and material of the propellant. The solid propellant comes in the following shapes: tubular, star, multi-fin, rod and tube, double anchor and double composition. Solid propellant rockets are used as booster rockets and as projectiles. In use, they are suitable for small rockets and missiles.

2.2 Liquid Propellant

The liquid propellant rocket propulsion system consists of a rocket engine and a set of tanks for the storage and supply of propellants. The rocket engine comes with one or more thrust chambers, feed mechanism for suppling the propellants from the tanks to the thrust chamber, source of power to deliver the propellants, piping and plumbing for the transfer of the propellants. There are also the control devices for propellant flow control as well as the structure to transmit the thrust. The supply of propellants is accomplished either by the use of high-pressure gases or pumps. There may also be thrust vector control system, engine condition or health monitoring subsystem. The design of any propulsion system is tailored to suit each specific mission requirement. The requirements have to do with application. The requirement for a cruise missile will be different from that of a second stage of a space launched vehicle. In addition to the mission requirements and definitions, such factors as the thrust chamber pressure, mixture ratio, nozzle exit area etc. can be analytically determined and optimized. Liquid propellants rocket engines are of two main classifications and include those for boosting payload and velocity increase and those for auxiliary propulsion such as trajectory adjustment and attitude control. There are other ways of classifying the liquid propellant rocket engines such as the reusable as in the space shuttle and single flight systems, restartable as in a reaction control engine or single firing as in space launch boosters. In the combustor, thruster or thrust chamber the liquid propellants are metered, injected, atomised and mixed for combustion. The high-pressure hot combustion gases are expanded accelerated and ejected at high velocity to impact a thrust force. The thrust chamber requires cooling and one way to achieve this is by passing the fuel around the hot chamber walls to absorb the excess heat.

Rocket propulsion is achieved through the propellants by burning fuel and oxidant to produce high pressure, high temperature combustion gases that give rise to thrust. Since it is the thrust that is responsible for the propulsion of the rocket, any study for the optimum performance of the vehicle cannot ignore rocket parameters that give definition to thrust and its production. On this account of this fundamental link with propulsion, the purpose of this study is to develop parametric expressions with which to study the performance of the rocket. The foregoing expressions with modifications to suit the current study, are developed based on the fact that thrust is the net external force that can be calculated as the rate of change of momentum [10]

3.0 Expressions of Equations

$$M_m = mv \qquad (1)$$

where M_m = momentum, m = mass and v= velocity

$$\text{Thrust} = F = ma = \frac{dMm}{dt} = m\frac{dV}{dt} + V\frac{dm}{dt}$$

$$m\frac{dV}{dt} + V\frac{dm}{dt} = m\dot{V} + V\dot{m} \qquad (2)$$

Equation (2) describes a situation where the propellant mass is depleting due to combustion and the velocity is in a varying combustion rate. Force can also be expressed as

$$F = PA \qquad (3)$$

Where P = pressure and A relevant area

At the nozzle exit with nozzle exit

Pressure = P_e and ambient pressure = Po:

$$F_{nozzle} = (P_e - Po)A \qquad (4)$$

Substituting for $m\dot{V}$ in eqn(2) from eqn(4) since variation in pressure is due to variation in velocity (from Bernoulli's equation)

$$(P_e - Po)A + V\dot{m} \qquad (5)$$

This equation is very important in rocketry because the exhaust pressure and velocity are determined by a combination of the propellant, the throat and nozzle design

Impulse

Thrust as discussed earlier is the force generated by the rocket that propels it along its trajectory through the air and space. It can be said that the main purpose of the rocket engine is to accelerate the exhaust gases

Impulse, *I* is defined as the total integrated force with respect to time.

$$I = \int Fdt \qquad (6)$$

Where F is force and dt is increment of time

From Newton's second law of motion, force, F is the rate of change of momentum, therefore

$$I = \int Fdt = \int \frac{dMm}{dt} dt$$

$$= \int dMm = \Delta M_m \qquad (7)$$

Eqn(7) is the Impulse – momentum theorem and integrating this eqn(6) yields

$$I = \int Fdt = \Delta M_m \qquad (8)$$

Consider a force generated by changing mas at constant velocity to obtain the impulse as:

$$\int_{m_f}^{m_i} v \frac{dm}{dt} dt$$

$$I = \int_{m_f}^{m_i} v \frac{dm}{dt} dt = \Delta M_m$$

$$= (m_i - m_f)v \qquad (9)$$

For a rocket that has to change its course, momentum change applied exit of the nozzle would be necessary. Using the relevant parts of eqn(9) at nozzle exit

$$I = (m_i - m_f)v_e = \Delta m_{prt} \qquad (10)$$

Where the suffix *prt* refers to propellant v_e = velocity at nozzle exit

Solving

$$V_e = \frac{I}{\Delta m_{prt}} \qquad (11)$$

Defining equivalent velocity/ effective exhaust velocity, U in eqn (5)

$$(P_e - Po)A + V\dot{m} = mU \qquad (12)$$

Replacing V_e by the effective exhaust velocity U then eqn((11) becomes

$$U = \frac{I}{\Delta m_{prt}} \qquad (13)$$

Defining Specific Impluse I_{sp} as

$$I_{sp} = \frac{I}{\Delta m_{prt} g} = \frac{U}{g} \qquad (14)$$

The Specific Inpulse has the dimention of time

$$F_{nozzle} = \dot{m}U = \frac{dM}{dt} U \qquad (15)$$

Newton's second law may also be defined based on total mass (vehicle + propellant), *M, U= equivalent exhaust velocity* and acceleration, *a*

$$F = Ma = M \frac{dV}{dt} \qquad (16)$$

Where *V* is the rocket velocity. By Newton's third law, setting eqn(15) and eqn(16) equal and opposite

$$\frac{dM}{dt}U + M\frac{dV}{dt} = 0 \qquad (17)$$

Simplifying

$$dv = -U\frac{dM}{M} \qquad (18)$$

with boundary conditions: initial rocket velocity = V_o and final velocity = V_f , and similarly, initial total mass of rocket = M_o and final mass -= M_f . Integrating both sides of equation (4)

$$\int_{V0}^{Vf} dV = -U\int_{M0}^{Mf}\frac{dM}{M} \qquad (19)$$

Integrating and applying the limits

$$V_f - V_0 = -U(\ln(M_f) - \ln(M_0))$$

$$= U\ln\frac{Mo}{Mf} \qquad (20)$$

Setting $Vf - V_0 = \Delta V$ then

$$\Delta V = U\ln\frac{Mo}{Mf} \qquad (21)$$

But M_o/M_f is the mass ratio (Propellant mass ratio), MR

$$\Delta V = U\ln(MR) \qquad (22)$$

Dividing both sides of eqn (22) by *U*

$$\frac{\Delta V}{U} = \ln\frac{Mo}{Mf} \quad \text{that is}$$

$$\ln\frac{Mo}{Mf} = \frac{\Delta V}{U}$$

Therefore

$$\frac{Mo}{Mf} = e^{\frac{\Delta V}{U}} \qquad (23)$$

But $U = g.I_{sp}$ (from eqn (14)) (24)

Substituting for U in eqn (23) and simplifying

$$M_o = M_f e^{\frac{\Delta V}{gIsp}} \qquad (25)$$

Now consider a derivation in which the effect of gravity is considered

$$F = Ma = M\frac{dV}{dt} = F_{thrust} - Mg$$

Substituting for F_{thrust} from eqn(15)

$$M\frac{dV}{dt} = U\frac{dM}{dt} - Mg \qquad (26)$$

Integrating after simplification

$$\int_{V0}^{Vf} dV = -U\int_{M0}^{Mf}\frac{dM}{M} - g\int_{t0}^{tf} dt$$

$$V_f - V_0 = -U(\ln(M_f) - \ln(M_0)) - g(t_f - t)$$

Which becomes

$$\Delta V = U\ln\frac{Mo}{Mf} - gt_{br} \qquad (27)$$

Where t_{br} is Δt, propellant burn out time.

But $U = g.I_{sp}$ (from eqn (14), therefore

$$\Delta V = g.I_{sp}\ \ln\frac{Mo}{Mf} - gt_{br}$$

$$\ln\frac{Mo}{Mf} = (\Delta V + gt_{br})/\ g.I_{sp}$$

Taking exponent on each term, one gets

$$M_o = M_f e^{\frac{\Delta V + gtbr}{gIsp}} \qquad (28)$$

Equation 28 is useful as it can be used to design the parameters for a mission. Taking the natural log on both sides one obtains

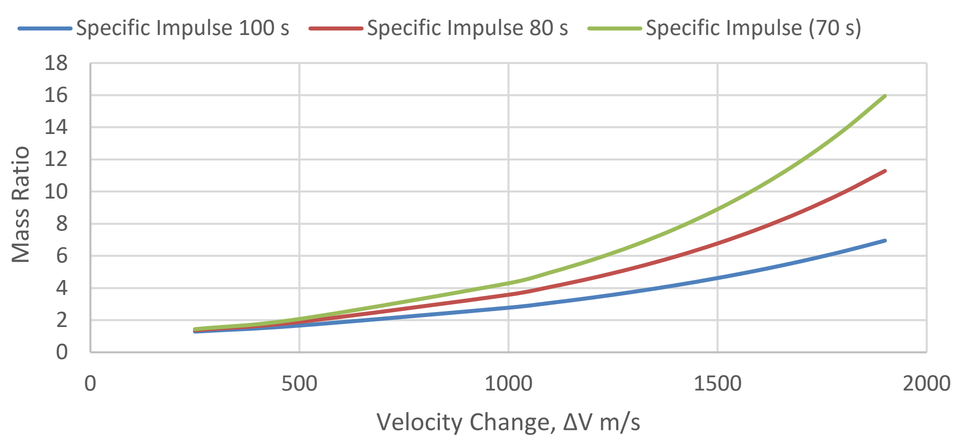
$$t_{br} = \ln(M_o/M_f)Isp = \Delta V/g \qquad (29)$$

Where M_o/M_f is the mass ratio.

4.0 Results and Discussion

4.1 Results

Figures 1, 2 and 3 have been solved and plotted to illustrate the capabilities of equations 28 and 29 as tools for the study of the performance of the rocket system and the associated design optimisation. Fig. 1 illustrates the relationship between the mass ratio and change in velocity. Fig. 2 is a plot of mass ratio against specific impulse and finally fig. 3 is a plot of burnout time against mass ratio.



Fig,1:Mass Ratio Velocity Change Relationship

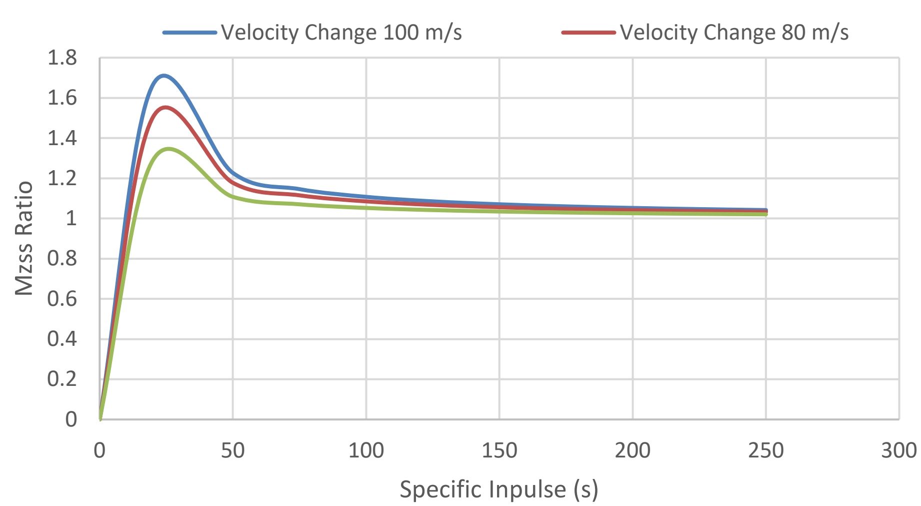


Fig. 2 Mass Ratio - Specific Impulse Relationship

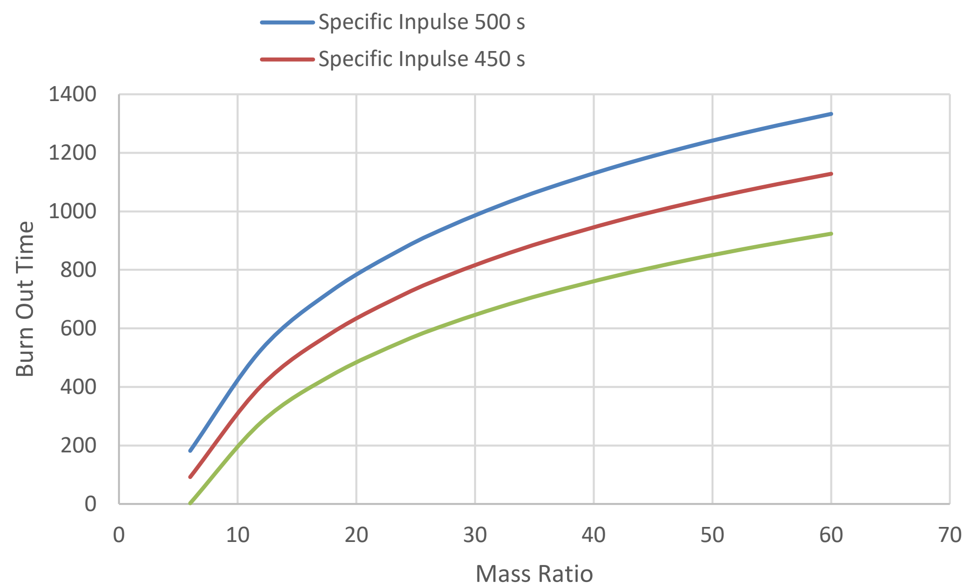


Fig. 3: Burn Out Time Relationship with Mass Ratio

4.2 Discussion

Figure 1 shows curves of Mass Ratio against Velocity Change for a given Specific Impulse. Each of the curves rises asymptotically with Velocity Change. The Specific Impulse is inversely proportional to the Mass Ratio. This is consistent with the definition of the Specific Impulse (see equation 14). The behaviour and relationship between the Specific Impulse and Velocity Change are opposite to those between the Mass Ratio and Velocity Change. For a given Mass Ratio, the Specific Impulse is proportional to the Velocity Change.

Fig. 2 shows the plot between Mass Ratio and Specific Impulse for given Velocity Change. Each of the curves rises rapidly from the left to a peak and then falls sharply midway to the right. Further descent from the midway position is gradual. For a given Specific Impulse, the Mass Ratio rises with Velocity Change. When the Mass Ratio is held constant, the Velocity Change decreases with the Specific Impulse on the left of the curve but increases on the right. The shape of the curves on the right appear to depict a region of stable operation.

Fig. 3 is a plot of Burn Out Time against Mass Ratio. Each of the curves, as would be expected, rises with the Mass Ratio. For a given Burn Out Time, the Specific Impulse falls with increasing Mass Ratio. When however, the Mass Ratio is kept constant, the Burn Out Time increase with the Specific Impulse**.**

5.0 Conclusion

Mathematical expressions have been successfully developed for the study of the performance characteristics of the rocket system and illustrative samples have been solved, plotted and analysed for parametric relationships.

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