

Volatility Comparison of the GSE All Share Index Returns using Student t- and Normal-GARCH models

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Abstract

In a frontier equity market like Ghana, trades can be quiet for long periods increasing the liquidity risk for investors. This notwithstanding, studies into the risk dynamics of the stock market are largely lacking for the Ghana Stock Exchange (GSE). In this paper, we have undertaken to use a dynamic volatility model, Generalized Autoregressive Conditional Heteroscedasticity (GARCH) to assess the risk of equity returns in the market. A comparison is made between the student-t and normal GARCH(1,1) models to ascertain a better fit for the market data. Using a daily log of adjusted return series of the Ghana Stock Exchange All Share Index (GSE-ASI) from January 04, 2011 to December 31, 2013, there is enough evidence that the student-t GARCH(1,1) better describes the volatility dynamics of the market for the period 2011 to 2013.

Keywords: Volatility, Normal-GARCH, Student t-GARCH, Maximum Likelihood Estimation, Volatility Clustering, Market Crashes, EWMA

Introduction

Volatility modelling is considered very important in the discipline in empirical financial econometrics. It is used as a measure of risk and investors, both individual and institutional, keep an eye on its level in the market. Portfolios are rebalanced to meet investment objectives or goals and a key metric that goes into the decision is the volatilities of the returns of the asset composition of the portfolios.

Unlike other financial variables, volatility is latent. Investors have to estimate it from past returns or back it out from models using market prices of assets. That presents a challenge in terms of its relevance in investment decisions *ex ante*.

Volatility of returns fluctuates over time contrary to the assumptions of independent and identically distributed with constant variances as found in most financial models. Characteristics like non-normalities, clustering and structural breaks in financial data are difficult to capture with conventional distributions (R. F. Engle & Patton, 2001; Wang, Fawson, Barrett, & McDonald, 1998). Heavy- or fat-tails of the distributions with higher than normal probabilities particularly of losses in the left tail can surprise investors as happened on Black Monday, October 19, 1997 when the market crashed (Amihud, Mendelson, & Wood, 1990; Schwert, 1989).

In frontier markets like Ghana, markets can be quiet for long periods interspersed with periods of frantic activity with block trades which increase risks for investors (Bley & Saad, 2012; Wilma De Groot, 2012). Adu, Alagidede, & Karimu (2015) noted that returns on frontier markets exhibit characteristics that are markedly

different from markets in developed countries. Near accurate measurement of volatility in such regimes is essential as the investment space is sensitive to events in the economy. Now and then bouts of inflation, foreign current squeezes, balance of payments problems, fall in commodity prices, interest rate hikes, etc. which characterize the economies of frontier markets can alter the basis of trading and profitability of investment strategies, thus sending skittish investors fleeing. Thus, based on a country's long-term risk outlook, investors can then evaluate the long-term investment potential and risk of investing in the securities of companies listed on the stock exchange and premiums demanded in the operating country.

Markets can be jolted into violent episodes of volatile trading due to market corrections in fundamentals after prolonged periods of rising asset prices usually for prices that are divorced from the economic activities in which they are supposed to be anchored. Speculators also add to the noise through information asymmetries (L. Davidson, 1999; P. Davidson, 1998). Of course market expectations depending on market outlook can have a subdued effect or throw markets into a tailspin.

In view of these, this paper investigates the stylised facts concerning the volatility of the adjusted returns of the equities of the Ghana Stock Exchange to see whether they exhibit properties found in the broader frontier markets, as well as come up with the volatility model specification that captures the risk characteristics of the returns.

The balance of the article is as follows. We provide a background to volatility models in financial economics and GARCH mod-

els followed by a theoretical description of the normal- and student t-GARCH models. Analysis using the adjusted returns which correct for thin trading using data from the Ghana Stock Exchange All Share Index spanning January, 2011 to December, 2013 is the subject of the next section. The last section draws conclusions on the appropriate model that fits the returns of the index.

Background

Initial attempts at capturing volatility dynamically was the result of work done by J.P. Morgan in Risk Metrics called exponentially weighted moving average (EWMA) (Morgan, 1996; Pafka & Kondor, 2001). EWMA is an improvement in the constant weighted moving average (MA) which failed to capture the volatility clusters in the return series of stock market returns. The EWMA modifies the MA weights such that the weights λ in (1) declines exponentially into the past.

Thus,

$$\bar{\sigma}_t^2 = \frac{1 - \lambda}{\lambda(1 - \lambda^{W_E})} \sum_{i=1}^{W_E} \lambda^i y_{t-i}^2$$

with $0 < \lambda < 1$.

The exponential weights λ quickly declines to zero meaning the volatility of a few days in the most recent past feeds into the current volatility.

Over the past few years markets have become particularly volatile with alternate periods of high and low volatility. As it is, the periods of the volatility clusters have

come short necessitating short estimation windows W_{-E} in (1) and exponential weights λ which attenuate rapidly. Some markets in emerging economies have shown rather prolonged periods of volatilities making the use of the exponentially weighted moving average too simplistic in capturing the volatility regime exhibited by the markets return series.

This led to the famous autoregressive conditional heteroscedasticity (ARCH) model proposed by Engle (1982) which was later generalized by Bollerslev (1986) into the generalized autoregressive conditional heteroscedasticity (GARCH) model. The GARCH model offers a framework of numerous conditional volatility models. However, only a few find empirical practical use in financial economics.

GARCH models have found acceptance with practitioners as a result of its parsimonious representation of the conditional variance in the way it captures the characteristics of the time series of returns of financial time series. Generally, GARCH processes are zero mean which are serially uncorrelated with nonconstant variances. The returns r_t in (2) at a time t is expressed in terms of the information set $I_{-(t-1)}$ available at time $t-1$. The innovation or shock term of this be can specified as:

$$\varepsilon_t = r_t - E[r_t | I_{t-1}]$$

The returns r_t are driven by these random shocks ε_t which are tied to the variances σ_t^2 ex post. In the GARCH framework this is expressed as:

$$r_t = \varepsilon_t \sigma_t$$

where,

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_{t-i} r_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2.$$

This is the result of the GARCH(p,q) model. It allows for the specification for the distributions of the returns through specifying the distribution of the ε_t in (2).

However, the returns on markets such as Ghana Stock Exchange, according to work by researchers such as Dimson (1979), Fowler & Rorke (1983), Lo & Craig MacKinlay (1990), and Scholes & Williams (1977) are biased owing to thin trading. Indeed work by Appiah-Kusi & Menyah (2003) and Mlambo & Biekpe (2005) point at the persistent thin trading on the Ghana and Nigeria equity markets. Such thin trading can bias the variables substantially rendering the GARCH coefficients inconsistent.

Miller, Muthuswamy, & Whaley (1994) recommends incorporating an autoregressive model of order one (AR(1)) into the analysis to adjust the returns, thus taking care of the bias introduced by the thin trading. Thus,

$$r_t = \beta_0 + \beta_1 r_{t-1} + \varepsilon_t$$

(5)

The residuals from this equation are used to adjust the returns as:

$$r_t^{adj} = \frac{\varepsilon_t}{(1 - \beta_1)}$$

where r_t^{adj} is the adjustment at a time t to take care of the thin trading.

Our subsequent analysis was, therefore, based on these adjusted returns.

The normal-GARCH

The normal-GARCH is the simplest of the GARCH models. It assumed that for a given return in (3), the conditional variance is given by (4), and

$$\varepsilon_t | I_t \sim N(0, \sigma_t^2).$$

(7)

The leverage effect in market trading requires that asset prices and returns respond differently to price shocks in the market. The normal-GARCH assumes the response of the conditional variance to negative shocks, is the same as its response to positive shock hence the distributions of ε_t is specified as normal with the tails symmetrical.

The conditional distributions at every point in time are specified as:

$$r_t | I_{t-1} \sim N(0, \sigma_t^2)$$

(8)

and the GARCH conditional volatility is defined by the annualized square root of the conditional variance. For the conditional variance in (4) to remain positive, we restrict parameters for the normal-GARCH as follows:

$$\omega > 0, \alpha, \beta \geq 0, \alpha + \beta < 1.$$

(9)

Parameter Estimation for the normal-GARCH(1,1)

Volatility models are estimated by maximum likelihood (ML) where parameter estimates are obtained by numerically maximizing the likelihood function. The Normal-GARCH(1,1) model is written as:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2.$$

(10)

Following the formulation of the error process as normal with expectation zero and variance σ_t^2 , the density is specified as:

$$f(r_t) = \frac{1}{\sqrt{2\pi(\omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2)}} \exp\left(-\frac{1}{2} \frac{r_t^2}{\omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2}\right).$$

(11)

We could write likelihood function thus

$$f(\alpha, \beta, \omega | r_t) = \prod_{t=2}^T \frac{1}{\sqrt{2\pi(\omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2)}} \exp\left(-\frac{1}{2} \frac{r_t^2}{\omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2}\right) \quad (12)$$

where the parameters α, β and ω are to be maximized.

The log-likelihood function is therefore:

$$\log L = -\frac{T-1}{2} \log(2\pi) - \frac{1}{2} \sum_{t=2}^T \left(\log(\omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2) + \frac{r_t^2}{\omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2} \right). \quad (13)$$

Parameter estimation of the student t-GARCH(1,1)

The student t-GARCH(1,1) model seeks to fit returns in which market shocks have non-normal conditional distribution. Bollerslev (1987) assumes market shocks to have t-distributions with typical skewed and fat tails. An additional parameter to be estimated in the student t-GARCH is the degrees of freedom. The specification of the GARCH model does not change but the likelihood does change (Andersen, Bollerslev, Diebold, & Ebens, 2001).

Thus:

$$f_v(t) = ((v-2)\pi)^{-\frac{1}{2}} \Gamma\left(\frac{v}{2}\right)^{-1} \Gamma\left(\frac{v+1}{2}\right) \left(1 + \frac{t^2}{v-2}\right)^{-(v+1)/2}. \quad (14)$$

The log-likelihood is given by:

$$\ln L(\theta) = - \sum_{t=1}^T \left(\ln(\sigma_t^2) + \left(\frac{v+1}{2}\right) \ln\left(1 + (v-2)^{-1} \left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right) \right) \quad (15)$$

where θ denotes the parameters of the conditional variance equation.

When the returns are sampled at higher frequencies, the t-GARCH enables extreme non-normality to be captured in the tails of the distribution of financial assets.

Data and Analysis

The frequency of data for GARCH models is important. All GARCH models seek to capture volatility clusters in the financial time series. In empirical work daily or intraday data and sometimes weekly data is used. Monthly data is never used because the volatility clustering effects in the data series disappear when the data is collected over long intervals of time.

The price series of the Ghana Stock Exchange (GSE) All Share Index is collected daily from January 02, 2003 to December 31, 2013 giving a total of two thousand four hundred and eighty-seven data points. There was rebasing of the GSE All Share Index in January 04, 2011. Our analysis is based on data from this moment (January 04, 2011) to December 31, 2013 giving seven hundred and forty-two data points. The sample size is reasonably large. Financial market do go through switching from one regime to another.

Extremely large sample sizes in volatility analysis are prone to regime overlaps which might confound findings (Ang & Timmermann, 2012).

The log of the adjusted returns of the prices series was calculated. The return according to Dowd (2002) provide desirable statistical properties. Table 1 is a summary of the statistics of the returns.

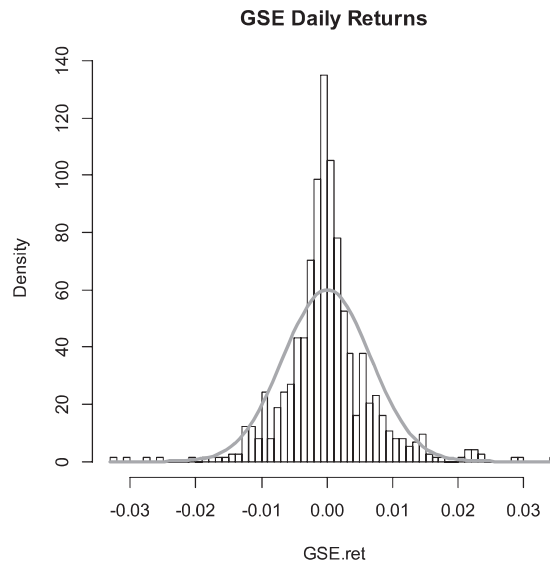
Table 1: Summary statistics of daily log-return series of GSE-ASI

Observations	741
Minimum	-0.0271
Quartile 1	-0.0013
Median	0.0006
Arithmetic Mean	0.001
Geometric Mean	0.001
Quartile 3	0.0031
Maximum	0.027
SE Mean	0.0002
LCL Mean (0.95)	0.0006
UCL Mean (0.95)	0.0015
Variance	0
Stdev	0.0057
Skewness	0.3471
Kurtosis	4.5646

A plot of the histogram and the superimposition of the normal distribution curve is shown in Figure 1.

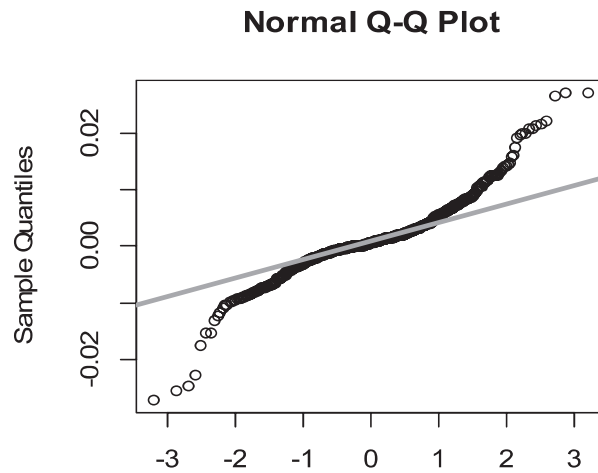
Figure 1: Histogram superimposed with distribution curve of daily log-return series of GSE-ASI

Figure 2: Quantile-quantile plot of daily log-return series of GSE-ASI



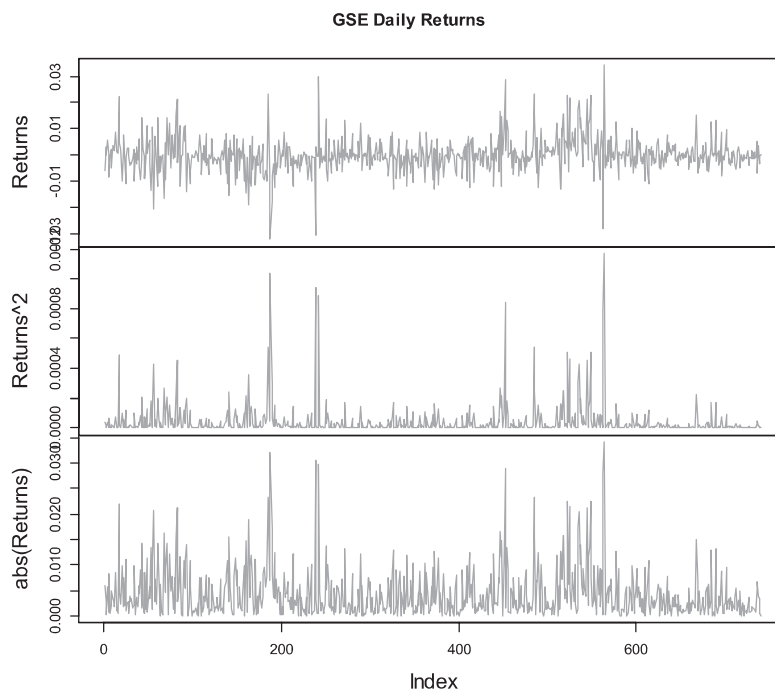
It is noticed that the histogram is peaked in the middle and right tail is heavy. The skew and kurtosis of the distribution are 0.8 and 5.74 respectively showing slight skewness to the right and fat-tails. A quantile-quantile plot in Figure 2 shows deviations from normality. A Jarque-Bera test returned a chi-square of 628.73 with a p-value of zero showing that the data series is far from normally distributed.

Figure 2: Quantile-quantile plot of daily log-return series of GSE-ASI



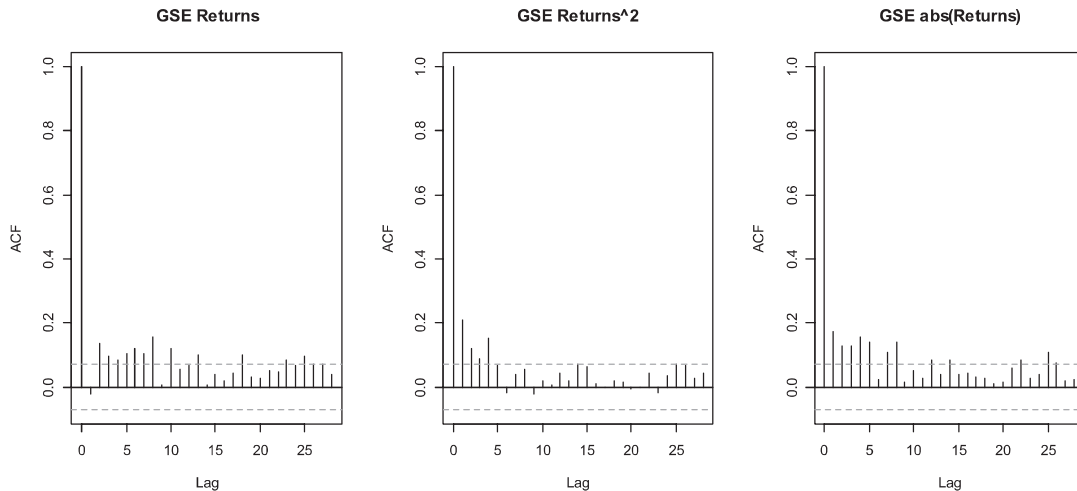
A time series plot of the return is shown in Figure 3. The returns exhibit marked volatility clusters throughout the period under review. It can also be observed from the series departures from the constant variance regime. Markets were particularly volatile in the late 2012 to early 2013.

Figure 3: Time series plot of daily log-return series of GSE-ASI



A plot of the autocorrelation function of the returns, square of the returns and absolute returns is shown in Figure 4.

Figure 4: Autocorrelation plot of the returns, square of the returns and absolute returns of the GSE-ASI



Assuming the mean returns is zero; Bekaert, Erb, Harvey, & Viskanta (1998), Merton (1980) and Thorbecke (1997) we can see in the squared returns the persistence of the volatility in as far as twenty-five lags.

For the test of the presence of conditional heteroscedasticity or ARCH effects, two approaches are available. First, a Ljung-Box statistics $Q(m)$ is conducted on the square of the residuals ε_t (McLeod & Li, 1983). The null hypothesis of the test statistic is that autocorrelation coefficient of the first q lags of the residuals is zero. The second test is the Lagrange multiplier (LM) test of Engle (1982). The test is similar to the use of F statistics for testing $\alpha_i=0$ ($i = 1, \dots, m$) given the linear regression:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \dots + \alpha_m\varepsilon_{t-m}^2 + a_t, \text{ for } t = m + 1, \dots, T \quad (16)$$

where a_t is the error term, m is a pre-specified positive integer and T is the sample size. The null hypothesis is $H_0 : \alpha_1 = \dots = \alpha_m = 0$ against the alternative hypothesis $H_A : \alpha_i \neq 0$ for some i between 1 and m .

Using the Ljung-Box gave $Q(12) = 74.859$ with a p-value almost zero. Applying the LM test with $m = 12$, we have chi-square = 55.611 ($F = 8.223$) with the p value of 1.406×10^{-07} . Thus, both tests confirm strong ARCH effects in the daily return series of the GSE All Share Index.

A normal-GARCH(1,1) and a student t-GARCH(1,1) models were built using the return series. The result are shown in Table 1 and 2 for the normal- and student t-GARCH respectively.

Table 2: Results of the Normal GARCH(1,1) model

	Estimate	Std. Error	t-value	p-value
mu	-0.000124	0.000244	-0.5089	0.6108
omega	0.000004	0.000001	4.1819	0.00003
alpha1	0.14778	0.017623	8.3855	0
beta1	0.77291	0.03473	22.2577	0

Table 3: Results of the t-GARCH(1,1) model

	Estimate	Std. Error	t-value	p-value
mu	-0.000124	0.000176	-0.70666	0.47978
omega	0.000004	0.000005	0.92387	0.35556
alpha1	0.197491	0.075939	2.60067	0.00930
beta1	0.801422	0.094280	8.50041	0.00000
shape	2.735543	0.404893	6.75621	0.000000

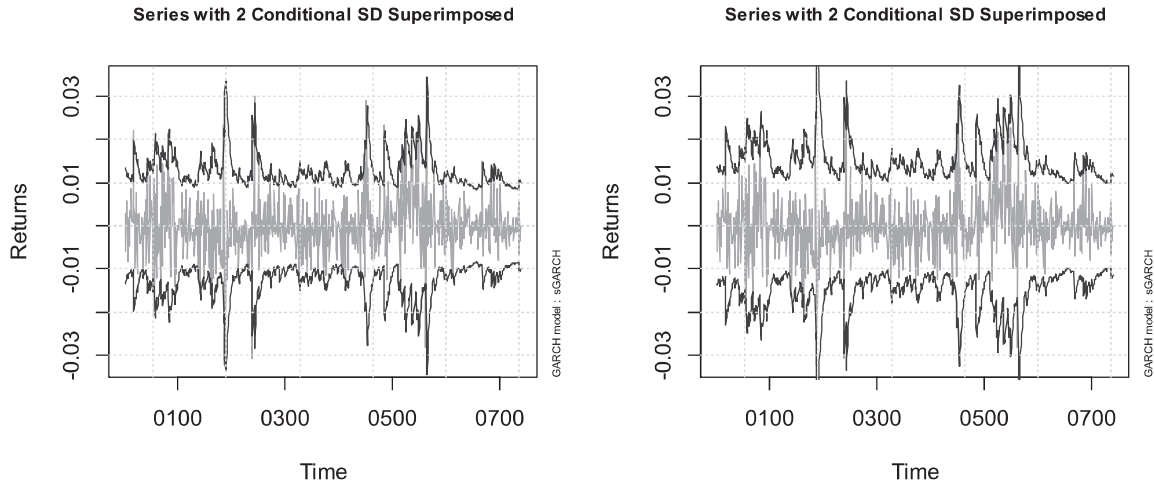
All the GARCH parameters of the Normal-GARCH are highly significant. With the student t-GARCH only the alpha, beta and shape parameters are significant. The Akaike Information Criteria (AIC) for the Normal- and Student t-GARCH are -7.3259 and -7.4844 respectively show that the Student t-GARCH is better. Again, comparing the log-likelihood, the student t-GARCH's value of 2777.97 is marginally higher than that of the normal GARCH of 2718.24, confirming the superiority of the student t-GARCH model.

The properties of the behaviour of the models are summarized in Table 4.

Table 4: Summary of the properties of the Normal-GARCH(1,1) and t-GARCH(1,1) models

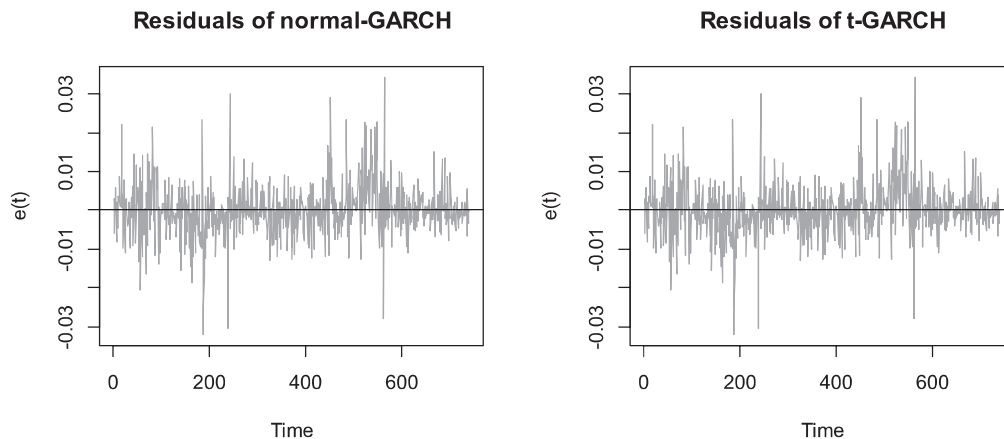
	normal GARCH(1,1)	t-GARCH(1,1)
unconditional mean	-0.0001242972	-0.0001242969
unconditional variance	0.00004.725284	0.003953814
Persistence	0.9206801	0.9989134
half life	8.387281	637.5576

Figure 5: Residual plots of Normal-GARCH(1,1) and t-GARCH(1,1) with SD superimposed



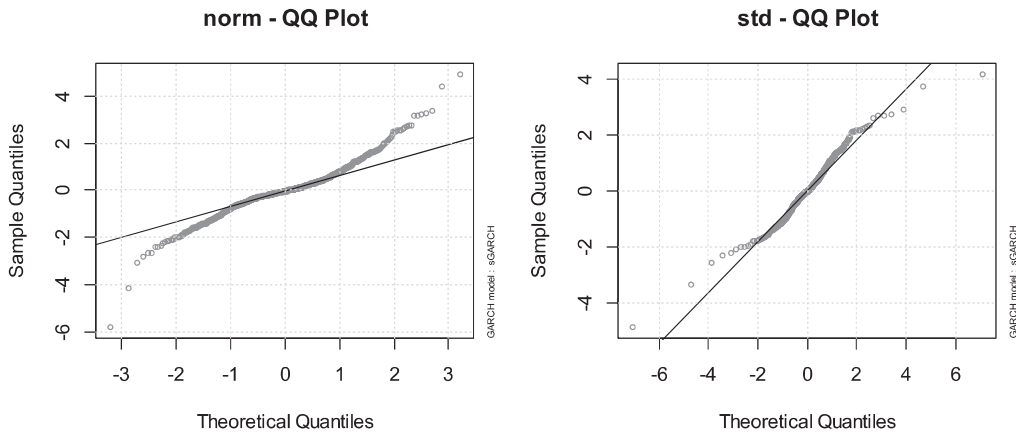
The residual plots of both models (Figure 5) show homoscedasticity of the variance spanning the interval $-0.02 < \sigma_{\varepsilon}^2 < 0.01$ and mean of zero. These residual plots do not differentiate between the models.

Figure 6: Residual plots of Normal-GARCH(1,1) and t-GARCH(1,1)



A plot of the residuals as shown in Figure 6 above confirms that the t-GARCH is the appropriate model in characterizing the volatility of the index returns of the Ghana Stock Exchange during the data period.

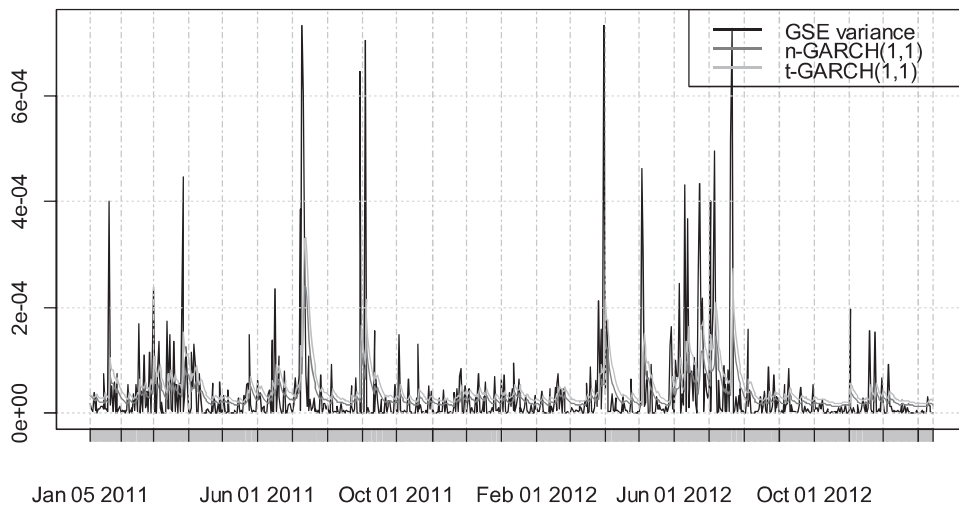
Figure 7: Quantile-quantile plots of the two models



To confirm the performance of the Student t-GARCH over the Normal-GARCH(1,1) we plotted the variance of the original returns series and superimposed the GARCH(1,1) models as shown in Figure 8.

Figure 8: Plot of the variances of Returns, n-GARCH(1,1), and t-GARCH(1,1)

Variances of Returns, n-GARCH, t-GARCH



From the Figure 8 above, the Student t-GARCH(1,1) tracks the variance of the original returns series closely than the Normal-GARCH(1,1). Filtering out the noise that is invariably present in market data, we are able to conclude that the Student t-GARCH(1,1) describes better the volatility dynamics of the market for the period 2011 to 2013.

Trading on the Ghana Stock Exchange occur in spurts led by the financial and energy stocks. This is characterised by the α value of 0.1974 which confirms that the market is not very sensitive to market events. Block trading from institutional investors is the main source of liquidity in the financial and energy stocks in the market. Alexander (2008) asserts that the GARCH error parameter α above 0.1 indicates volatility is very sensitive to market events, something that has been empirically observed. Market activity can be quiet for so long on the GSE. This thin trading clearly shows in seemingly low volatility in the asset returns. The GARCH lag parameter β of 0.8014 points to mild volatility which dies out quickly in the market. Overall, the sum of volatility parameters ($\alpha+\beta$) above 0.9, according to Alexander (2008), points to a market with a relatively flat volatility. This confirms the peculiarities observed in frontier and emerging markets with thin trading.

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Concluding Remarks

Equity markets exhibit a pattern of behaviour across markets whether developed, emerging or frontier. These stylised facts make it easier for investors to strategise when venturing out of their home countries especially from the developed markets into the other markets. In this paper, we have shown that these stylised facts exist in the Ghanaian equity market.

We have also illustrated the how GARCH models can be used to describe the evolving volatility of the stock index of the Ghana Stock Exchange. We modelled both the normal-GARCH and student t-GARCH by assuming the normal- and student t-distributions characterise the adjusted returns of the Ghana Stock Exchange All Share Index.

Our conclusion is that the volatility of the index is evolving, albeit subdued as a result of thin and nonsynchronous trading on the Ghana Stock Exchange. We found out that the volatility in the markets was captured by the student t-GARCH(1,1) model. This is in line with what researchers have found out in other markets of similar characteristics (Alberg, Shalit, & Yosef, 2008; Atoi, 2014; Gökbulut & Mehmet, 2014). By so doing, we have contributed to the stream of research on the volatility characteristics of equity markets within the larger framework of frontier markets.

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