

## **Modelling the Technical Efficiency of Hospitals in South-Eastern Nigeria Using Stochastic Frontier Analysis**

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### **Abstract**

The efficient performance of hospitals is critical to cost containment and the delivery of effective health services. This paper examines the efficiency of hospitals in LIC context using a sample of 200 hospitals generated from a survey of hospitals in southeast Nigeria. The paper uses the translog production function version of the stochastic frontier model (SFA) to estimate the efficiency levels of individual hospitals and the determinants of inefficiency. The results indicate large variations in the efficiency scores of sample healthcare facilities with average efficiency of 71%. Private hospitals show greater level of efficiency than public ones. The average scale elasticity was also found to reflect constant returns to scale. The results suggest that large social welfare gains could be made by improving the efficiency of hospitals in LICs. Suggestions are made on how to achieve greater efficiency in these institutions.

Keywords: Technical efficiency, Hospitals, Stochastic Frontier Analysis, Nigeria

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## Introduction

Hospitals are central to the health system and the management of healthcare. They take up very significant level of healthcare resources. In Sub-Saharan Africa the hospital sub-sector takes up 45-69% of total health expenditure.[30-31,33] There are many possible sources of inefficiency in any production arrangement such as the hospital system. These may include technical efficiency, allocative efficiency, X-efficiency and even distributional efficiency. However, the focus of this study is technical efficiency. Technical efficiency refers to the production of maximum output of goods and services from given inputs. It suggests that output production is a technical problem requiring the management of inputs in such a way as to produce maximum output (output orientation) or to produce a given output with minimum set of inputs (input orientation). Profit-maximizing behavior requires a firm to be first technically efficient.

The objective of this study is two fold:

- (i). To estimate the determinants of technical efficiency of hospitals in southeast Nigeria and
- (ii). To measure the level of efficiency of individual hospitals using the stochastic frontier analysis (SFA).

The measurement of the level of efficiency of individual health facilities is critical for management decisions and choice of inputs and outputs in the production of health services. This study focuses on hospitals in southeast Nigeria where a recent survey has provided a fresh set of data to analyze the performance of the hospitals in this region. However, we believe that the lessons from the results of this study will be useful in the hospital sector in many low-income countries.

### *Heterogeneity Problems*

A variety of output services including in- and out-patient visits, x-rays, laboratory tests [5,14,20, 40,42,43,44,48] are used in healthcare efficiency analysis. However, heterogeneity tends to characterize healthcare outputs [25] and this generates concerns about the usefulness of econometric models in the estimation of efficiency of healthcare units [32]. Nevertheless, this approach has gained attention in the literature (see for example, [45,50]). One way of reducing heterogeneity of input and outputs across healthcare units is to construct index of input and output variables. This study assumes that hospitals produce an index of outputs: weighted admissions. This index is constructed following the common

practice in the literature of converting outpatient visits, number of X-rays and laboratory tests into weighted admissions.[2] Although the weighting factor for each component may still be contentious, we believe that this approach offers an empirical way of achieving homogeneity among the varied outputs. Unlike outputs, hospital production inputs are generally more homogeneous and therefore less problematic to measure. The two major inputs into the hospital production function are capital and labor, which can be summarized in money-metric cost terms or by use of a single measure of input which effectively reduces the model to a cost function.

In addition hospital characteristics such as its location and ownership structure may also influence the level of its efficiency.[25,12] Ownership structure will reflect the kind of expectation and external pressure that is brought to bear on management. A public hospital may be non-profit hospital but may be subject to other forms of operational constraints. A for-profit hospital may face even greater pressure in terms of expectation of profit claims of ownership from the managements.<sup>i</sup>

## Methods

In this study all outputs are measured in physical quantities as weighted outputs by transforming other outputs into weighted admissions. While there are large variations between countries in the ratio of cost of outpatient visits to inpatient days [1-2], we have followed recent literature that suggest that the cost of inpatient day is equivalent to about twice the cost of outpatient visit. [24,51] Considering that that this ratio might be higher in the context of LICs, we converted the outpatient visits to inpatient equivalent at the rate of 1 inpatient day to three outpatient visit. Similar conversion rates were applied to X-rays and laboratory services. Heterogeneous labor inputs were converted into common labor units using different weights for different categories of health workers. In the auxiliary regression, the doctors and other clinical staff are disaggregated. Bed input is used in physical quantities. However, the monetary values of drugs are used since it is difficult to use physical quantities of this variable. The effects of ownership and location on efficiency are captured using dummy variables.

### *The Stochastic Frontier Production Function*

Stochastic Frontier analysis (SFA) is a multivariate statistical method that decomposes the error associated with each observation into traditional

whitenoise error and one-sided to estimate inefficiency. The literature on stochastic frontier is rather well developed. However, there have been debates on its appropriateness in hospital production function based on difficulties associated with specifying appropriate technical production function because of the problems of output heterogeneity and defining parsimonious inputs as dicussed above (see for example, [32,39]). In spite of these reservations, SFA has found increasing use in health economics applications. [15]

The concept of frontier is central to efficiency analysis as it characterizes the relationship between observed performance of production units and the potential or ideal performance. The deviations of outputs of individual units from the frontier or best industry practice constitute relative technical inefficiency.[17-18] Unlike Data Envelopment Analysis (DEA) that assumes no specific distribution but also assumes that every deviation from the frontier is a result of inefficiency SFA imposes a functional form on the data but also makes the reasonable assumption that deviation from the frontier are composed of two errors: random error and error due to inefficiency. The following specifies the functional form of SFA originally proposed independently by Aigner et al.[3] and Meeusen and van der Broeck [29] for cross sectional analysis:

$$y_i = x_i\beta + (v_i - u_i) \quad 1$$

Where  $y_i$  represents the output of firm  $i$ ,  $x$  are  $1 \times m$  column vector of variables and  $\beta_i$  are  $m \times 1$  row vector of associated coefficients.  $v_i$  are statistical noise that are independently and identically distributed (iid),  $v_i \sim N(0, \sigma_v^2)$

while  $u_i$  are one-sided (non-negative) random variables that measure inefficiency.<sup>ii</sup> Both  $v_i$  and  $u_i$  are assumed to be independent. Specific distributional assumptions are usually made about the distribution of the inefficiency term. The common assumptions include: truncated normal, half-normal, exponential, and gamma distributions. This basic model has undergone several modifications including those specified in [7,8,10,21,23,35,37], among others. Some of the extensions have been designed to facilitate the estimation of time-varying and time-invariant technical inefficiencies (in the case of panel data), cost and production functions.

### The Empirical Model<sup>iii</sup>

The specification of the production function implied by (1) has generally considered the Cobb-Douglas (C-D), the CES, and the translog production

functions. The translog functional form has been found to be flexible and appropriate for hospital production function. Unlike the C-D production function, it places no restrictions on substitution among inputs.<sup>iv</sup> However, the C-D production function is very popular in empirical studies. The empirical estimation in this study was based on the translog model; the C-D was used as auxilliary regression.

This study relies on the model of (1) specified in [7].

$$y_{it} = x_{it}\beta + (v_{it} - u_{it}) \quad \text{for } i = 1, 2, \dots, N \text{ and } t = 1, 2, \dots, T$$

in which the variables are as defined in (1) and  $u_{it}$  are obtained by truncation at mean  $\mu = 0$  of the normal distribution. The mean, however, is a function of a vector of explanatory variables and their associated parameters, that is  $\mu = z_u\delta$  where  $z$  is a vector of attributes of the firm that determine efficiency and  $\delta$  is the associated vector of parameters. The variance parameters are  $\sigma^2 = \sigma_v^2 + \sigma_u^2$  and  $y = \sigma_u^2 / (\sigma_v^2 + \sigma_u^2)$  For cross sectional data, the technical efficiency of the  $i$ th firm is defined by

$$u_i = TE_i = z_i\delta + \varepsilon_i \quad (3)$$

where the error term  $\varepsilon_i$  defines the truncation of the normal distribution with zero mean and constant variance ( $\sigma^2$ ) and the point of truncation is given by  $\varepsilon_i \geq -z_i\delta$ . This makes the assumptions about the distribution of the error term in the second equation, the regression equation, consistent with the assumptions made about the inefficiency distribution in the first stage. [7]

The general empirical model to be estimated is a linearized translog production function<sup>v</sup> which may be expressed as:

$$\begin{aligned} \ln admis_i = & \beta_0 + \beta_1 \ln bed_i + \beta_2 \ln staff_i + \beta_3 \ln drug_i \\ & + \beta_4 \ln beds_i^2 + \beta_5 \ln staff_i^2 \\ & + \beta_6 \ln drugs_i^2 + \beta_7 \ln bed * staff_i \\ & + \beta_8 \ln bed * drug_i + \beta_9 \ln staff \\ & * drug_i + (v_i - u_i) \end{aligned} \quad (4)$$

The technical inefficiency effects are defined by:

$$u_i = \delta_0 + \delta_1 (loc_i) + \delta_2 (ow_i) + \delta_3 (bcap_i) + \delta_4 totstaff + \varepsilon_i \quad (5)$$

The subscript  $i$  refers to the  $i$ th health facility in the sample population. The variables are defined in Table 1.

**Table 1: Definition of Variables**

Variable Definitions	
$\ln admis$	Natural log of number of weighted admissions
$\ln bed$	Natural log of number of beds in the facility
$\ln staff$	Natural log of number of doctors, pharmacists, and nurses
$\ln drug$	Natural log of annual drug expenditure
$\ln beds^2$	Natural log of square of number of beds
$\ln staff^2$	Natural log of square of number of staff
$\ln drug^2$	natural log of square of drug expenditure
$\ln bed \times staff$	Natural log of the product of bed and staff
$\ln bed \times drug$	Natural log of the product of beds and drugs
$\ln staff \times drug$	Natural log of product of staff and drug expenditure
$\delta_0$	constant parameter of the inefficient equation
$loc$	location of the facility (urban or rural )
$ow$	ownership of the hospital (a dummy for public or private ownership)
$bcap$	bed capacity of the hospital as opposed to the actual number of beds
$totstaff$	number of clinical and non-clinical staff of the hospital

The technical inefficiency model nested on the stochastic frontier production model has explanatory variables that include location (i.e. whether a hospital unit is located in an urban or rural area.) It also includes potential bed capacity of the hospital (cap). The variable 'totstaff' is also included in the technical inefficiency component. It comprises both the clinical and non-clinical staff of a given hospital facility. Finally, the ownership variable is a dummy indicating whether a hospital is publicly or privately owned. Since the inefficiency equation is nested on the stochastic frontier equation, Battese and Coelli [7] suggest that the technical inefficiency component variables could also include those that are already specified in the first part. This allays any concerns about possible endogeneity problems arising from the correlation of regressors in twin equation (see also [17,26]). However, there may still be concerns about the likely endogeneity of some of the regressors in the same part of the equation. In particular, there could be correlation between the number doctors and nurses in an establishment. Doctors may, for example, influence the number of nurses that are hired in a given health facility. However, it is plausible to assume that that costs and market discipline are likely to play the critical role in determining the number of nurses that a facility hires. This implies that the correlation between doctors and nurses will not be systematic.

For  $T = 1$ , the maximum likelihood function the model is given as:

$$\ln L(\beta, \sigma^2, \gamma) = -\left(\frac{N}{2}\right) \ln\left(\frac{\pi}{2}\right) - \left(\frac{N}{2}\right) \ln(\sigma^2) + \sum \ln[1 - \Phi(z_i) - (1/2\sigma^2) \sum (\ln y_i - x_i \beta)^2] \quad (6)$$

$$\text{where } z_i = [(\ln y_i - x_i \beta) / \sigma] \sqrt{\gamma / (1 - \gamma)}$$

The distribution function  $\Phi(z_i)$  is assumed to lie within [0, 1] range and the maximization of the function gives the maximum likelihood estimate of the parameters. FRONTIER Version 4.1d obtains the maximum likelihood estimates of the parameters in  $\beta$ ,  $\sigma^2$  and  $\gamma$  3 steps:

1. It uses Ordinary Least Squares (OLS) to obtain the values of  $\beta$  and  $\sigma^2$ . The estimated parameters are unbiased though  $\beta_0$  is biased
2. It uses a two-phase iterative search employing the Davidson-Fletcher-Powell (DFP) algorithm to evaluate  $\ln L$  for values of  $\gamma \in [0,1]$
3. It uses the selected values for  $\beta$ ,  $\sigma^2$  and  $\gamma$  in the preceding steps in an iterative maximization to obtain the maximized estimates of the efficiency parameter.

#### Data

The data for this study were generated from field survey of hospital facilities in two contiguous states in southeast Nigeria, Enugu and Anambra states between January and March 2009. The combined population of the two states in 2006 was 7.5 million (National Population Council [NPC] 2006), with about 1500 public and private hospitals.

The management of public hospitals and secondary care in Nigeria is the responsibility of the state tier of government while tertiary hospitals and general stewardship is the responsibility of federal government. Public and private hospitals are regulated at state levels.

The design of the survey was guided by objective of generating a sample of hospital facilities that is representative of the population of hospitals in the states. A sample of hospitals was obtained through a random selection process using the frames obtained from the two states and sample to proportion method. A total of 99 and 101 hospitals were sampled from Enugu and Anambra states respectively, giving a total of 200 hospitals. The sample also included both the public and private hospitals in proportion to their population in the two states.

The survey was conducted using questionnaire instrument. Respondents<sup>vi</sup>, were required to provide information on several key variables on inputs and outputs of their hospitals. The key inputs included the number of admissions, the number of outpatients, the number of X-rays conducted at the X-ray department if this existed and the medical laboratory of the hospitals. The variables also included the recurrent costs of services as well as capital costs such as building, expenditures on electric generators, vehicles, etc. It also included information on ownership type, employment records and drugs. Measurements of floor areas were also taken. The level of disaggregation of the information was also important if the performance of the hospitals were to be meaningfully compared. The survey recorded a 100% retrieval rate, though some of the hospital management authorities did not provide some information because they were not available. For example, many hospitals were unable to provide information on the floor size of their establishments but provided information on the bed capacity of the hospitals. The following analyses are based on 187 hospitals that had complete information required for this analysis and had no more than 60 beds. This restriction was necessary

to obviate the problem of heteroscedasticity.

## Results

The sample statistics show that the mean number of beds was 16 with standard deviation of 8. The median was number of beds was 15. The mean expenditure on drug was 206,683.90 (= \$1782 as at the time of the interview). The mean number of staff was about 13 with standard deviation of 10. The average number of unweighted outpatients and inpatients were 118 and 28 per month respectively. These figures suggest that majority of the hospitals operate on very small scale which implies that scale economies are almost non-existent for most of the hospitals. However, there were also differences in average size between public and private hospitals. The total number of public hospitals in the sample was 46 while the total number of private hospitals was 142 which reflect the relative distribution of ownership of hospitals in the two states. However, public hospitals tended to be marginally bigger in size with average number of beds 19 as against 16.6 for their private counterparts.

A two-group mean-comparison test was conducted to know the level of difference between sizes of the two groups assuming unequal variances with Welch's degree of freedom. The result showed that the difference was statistically not different from zero with Walch degrees of freedom 63. Similar results were also obtained for a test of difference between the size of rural and urban hospitals. The fact that there are no differences in the sizes of hospitals based on ownership and location does not rule out possible differences in efficiency of hospitals based on these and other criteria. Thus these variables were also included as possible sources of inefficiency in the inefficiency equation that was estimated.

The econometric estimation involved the estimation of the stochastic frontier model and the nested inefficiency model as specified in translog form in equations (4) and (5) and auxiliary model using the C-D functional specification. The results of the translog model are shown in Table 2.

**Table 2: Estimated Stochastic and Inefficiency Models (Translog)**

<b>Stochastic Model</b>			
<b>Vaible</b>	<b>Symbpl</b>	<b>Coef</b>	<b>t-value</b>
Const	$\beta_0$	24.846	25.155***
Inbeds	$\beta_1$	3.059	3.142***
Inwstaff	$\beta_2$	0.433	0.443
Indrug	$\beta_3$	-4.376	-12.323***
Inbeds <sup>2</sup>	$\beta_4$	0.245	1.074
Inwstaff <sup>2</sup>	$\beta_5$	0.080	1.906**
Indrug <sup>2</sup>	$\beta_6$	0.224	7.220***
Inbedxstaff	$\beta_7$	-0.407	-1.610*
Inbedxdrug	$\beta_8$	-0.246	-1.981**
Instafxdrug	$\beta_9$	0.025	0.255
<b>Inefficiency Model</b>			
Const	$\delta_0$	0.761	0.729
loc	$\delta_1$	-0.004	-0.005
ownership	$\delta_2$	-0.733	-2.100**
Inbedcap	$\delta_3$	0.305	0.617
totalstaff	$\delta_4$	-0.035	-1.072
Sigma sqd	$\sigma^2$	0.527	3.889***
gamma	$\gamma$	0.400	1.557
<b>Log Likelihoof function = -176.67</b>			
<b>LR test of one-sided error = 11.48</b>			
<b>Number of restrictions = 6</b>			
<b>*** Statistically significant at 1%; ** at 5%, and * at 10% levels</b>			

The OLS estimate of  $\beta_0$  obtained in the first stage estimation is biased [13] and so the entire parameter estimates obtained in this first stage are not reported here. However, these estimates are used as starting values for the second stage estimation based on maximum likelihood estimation (MLE) and using the Davidson, Flecher, and Powel (DFP) algorithm. The result of this stage is reported as the parameter estimates of the stochastic model. The efficiency of individual hospital units is reported in the third stage estimation and the distribution of these is shown in Figure 1.

The results of the stochastic model based on the translog production function where three key basic inputs are used – beds, staff and drugs, and with weighted admissions as the dependent variable show that the likelihood function is -176 while the likelihood ratio test (LR test) of one sided error [ $\Pr(u > 0)$ ] is 11.6 indicating that the inefficiency effect model is appropriate for the estimation. In the

estimated stochastic model the coefficients of *Inbed*, *Indrug* and *Indrug<sup>2</sup>* are statistically significant at 1% critical level. The coefficients of weighted *staff<sup>2</sup>*, and *bedxdrug* are significant at 5% level. The coefficient of the cross-product *bedxstaff* is approximately significant at 10% level while the coefficients of *staff*, *bed<sup>2</sup>* and *staffxdrug* are not statistically significant. It turns out, however, that the coefficients of drug, *bedxstaff* and *bedxdrug* have negative signs which are contrary to expectations but the coefficients measure elasticities and have to be interpreted alongside the quadratic and cross-product terms.

However, deeper insights into the behavior of the variables could be obtained if we compute the elasticities of the output variable with respect to the input variables. The output elasticity of an input variable in the translog functional form may be computed based on not only the estimated coefficient of the input but also on the quadratic and

cross-products of that input variable. Thus, in the case of the bed variable, the elasticity is based on  $bed$ ,  $bed^2$ ,  $bedxstaff$  and  $bedxdrug$ . The calculated average output elasticity of beds is 1.184<sup>vii</sup>. In otherwords, a 1% increase in the number of beds will increase weighted admission by 1.18%. A

similar calculation for  $staff$  and  $drug$  gave average elasticities of 0.054 and 8.8 respectively. The implication is that weighted admission is elastic to number of  $beds$ , very elastic to the hospitals level of expenditure on  $drugs$  and very inelastic to the number of  $staff$ .

**Table 3: Estimated Stochastic and Inefficiency Models (Cobb-Douglas)**

Stochastic Model			
Vaiable	Symbol	Coef	t-value
Const	$\beta_0$	2.17	7.80***
Inbeds	$\beta_1$	0.14	3.25***
Indoctor	$\beta_2$	0.56	5.10***
InClinical_Staff	$\beta_3$	-4.376	-1.18
Indrug	$\beta_4$	0.33	5.77***
Constant	$\delta_0$	-21.23	-029
Sigma Sqd	$\sigma^2$	7.00	0.31
gamma	$\gamma$	0.96	6.55***

However, it is not unlikely that correlations among the variables in the form of multicollinearity may have affected the estimated elasticities of the variables. In particular, it is also reasonable to suspect that a further disaggregation of staff into medical doctors and other clinical staff could yield further insights into the elasticity behavior of the model. The simpler C-D production function was estimated. The results are shown in Table 3. It turns out that  $beds$  and  $drugs$  have reduced elasticities while the coefficient of other clinical staff increased in value, though statistically non-significant. The value of the output elasticity of doctors is now isolated as 0.56 and is statistically significant even at 1% critical value. The elasticities of  $beds$  and  $drugs$  remain highly significant indicating the importance of these inputs into the health service production process.

These results are consistent with intuition. In many hospitals in the surveyed location a key attraction for patients visiting a given hospital is the availability of drugs. Thus, hospitals that always have drugs would most likely have patients. On the contrary, output elasticity of  $staff$  is very low because increase in number of staff does not necessarily increase the number of patient visits to the hospital. This explains why most hospitals operate with minimal number of staff. In most cases, a hospital is made up of only a doctor and two or three nurses. Even when the number of patients is increasing the hospital does not usually employ more doctors or nurses. This is consistent with the observed fact that the average number of doctors in most of the

sample hospitals is less than 1.5. Hospital proprietors are generally very slow in employing extra hands. They would rather prefer to overwork the available staff than recruit new ones.

It is important however, to note that this result is a reflection of the behavior of the average hospital in the sample and may not reflect exactly the behavior of hospitals that deviate from average. In otherwords, hospitals in the sample that deviate from average may exhibit levels of elasticities different from those calculated above. Larger hospitals may for instance exhibit lower output elasticity of beds. This is because the flexibility of the translog function allows output elasticity of a given input variable to depend upon the level of the input. Thus bigger hospitals with higher levels of input may exhibit elasticities different from the average, and vice versa for smaller hospitals. It may also be the case that output elasticity may differ with respect to categories of staff which is clearly demonstrated here using the auxiliary regression.

For the nested inefficiency model, it is observed that only the  $ownership$  dummy variable is statistically significant at 5% level. The coefficient of other variables,  $location$ ,  $bed\ capacity$  and  $total\ staff$  are not significant. That the location dummy is statistically insignificant implies that there is no evidence of differences in the level of efficiency of hospitals located in urban and rural areas. The  $ownership$  dummy is however statistically significant with an indication that state owned hospitals tend to be more inefficient than private hospitals. Bed

capacity and total staff<sup>viii</sup> are weak and indeterminate and therefore do not confirm whether hospitals with larger number of staff and bed space are likely to be more efficient than others. However, the negative magnitude of total staff seems to suggest that hospitals with larger number of staff tend to be less efficient than those with smaller number.

The estimated variance  $\sigma^2$  and  $\gamma$  are of special importance in the nested inefficiency model. It was noted above in the specification of the empirical model that whereas  $\sigma^2 = \sigma_v^2 + \sigma_u^2$  and  $\gamma = \sigma_u^2 / (\sigma_v^2 + \sigma_u^2)$ . When the parameter  $\gamma = 0$  the variance of the nested inefficiency effect model is zero which implies that the parameters  $\delta_0, \delta_1, \dots, \delta_2$  are unidentified, implying further that the model reduces to conventional mean response function<sup>x</sup>. The closer the value of  $\gamma$  is to 1, the greater the indication of the appropriateness of the inefficiency model. In the estimated model it is observed that the value of the variance parameters  $\sigma^2$  and  $\gamma$  are 0.527 and 0.40 respectively. While  $\sigma^2$  is statistically significant at 1% level the  $\gamma$  could be barely significant at 10% level. This is not unexpected given that only the ownership variable is significant at 5% level among the variables included in the inefficient effect model. It does indicate however that the variables included in the model do not explain all the observed inefficiency. It is important to note that while there are differences in technical efficiency between hospitals based on ownership, there are no differences between urban and rural hospitals. This is not unexpected given that only the *ownership* variable is significant at 5% level among the variables included in the inefficient effect model.

#### Scale Elasticity of Hospital Production Function

Scale elasticity relates to the characteristic of production technology at a given level. In a single-input single-output case, it is the responsiveness of output to a small change in the quantity of the input variable at a given level of production or to an equally small change in all variables in a multi-input case. If the elasticity of a hospital unit is less than, equal to, or greater than unity, then its production technology exhibits decreasing, constant, or increasing, returns to scale respectively. Scale efficiencies may be calculated using the sum of output elasticities. When the unit is operating increasing returns to scale it implies that the average product of the input variable is increasing with quantity of input and the reverse is the case for decreasing returns to scale, while in the case of constant returns to scale the average product remains constant. Scale elasticity of the sample hospitals were calculated using both the estimated translog and C-D production functions and the values were 0.963 and 0.958 which are respectively not different from unity. This suggests that both models consistently predict that the hospitals in the southeast Nigeria are generally operating constant returns to scale. It could be concluded in this case that with respect to weighted admissions, the hospitals in the region are not gaining economies of scale in their production of services.

#### Efficiency of Individual Hospitals

The efficiencies of individual hospitals were also estimated. The kernel distribution of these individual efficiencies is shown in Figure 1. The mean efficiency score is 0.71, which translated to 71% efficiency level. The standard deviation is 0.12 while the minimum and maximum scores are 0.26 and 0.94 respectively. This implies a large variance between the efficiency levels of the hospitals.

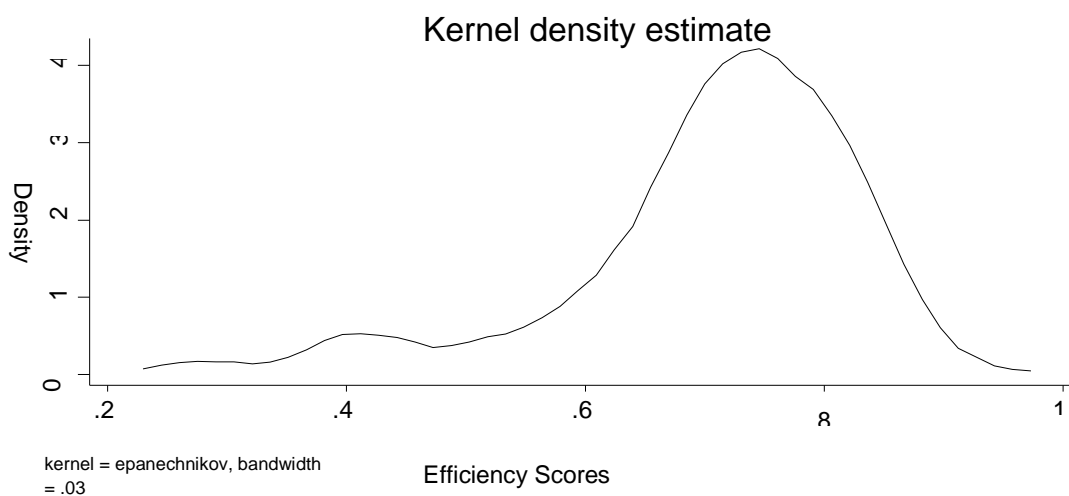


Figure1: Error! Main Document Only.: Kernel Density of Efficiency Scores



The graph shows that the distribution has a long left tail with a hump between 0.6 and 0.9. Fifteen hospitals scored 0.50 or less. The mean score among this group is 0.39. Fifty (50) hospitals had efficiency scores between 0.5 and 0.7 with a mean of 0.63 and standard deviation of 0.05. Eighty seven (87) hospitals had efficiency scores between 0.70 and 0.80 with mean efficiency of score of 0.75 and standard deviation of 0.03. Thirty six (36) hospitals had efficiency scores of 0.80 with mean efficiency of 0.84 and standard deviation of less than 0.3. Only one hospital had efficiency score above 0.90. Thus the low standard deviations indicate how tightly the scores are distributed within the range between 0.5 and 0.9.

The practical effect of the wide deviations from the

frontier is that the hospitals are using up critical social resources and producing less than desired amount of output. It suggests that infact the hospitals could produce the present level of output using only about 70% of present resources currently deployed in the sector. This is a significant resources wastage considering the health need of the population and scarce resources socially available to meet these health needs.

In order to explore further the efficiency distribution among the sample hospitals, the differences in efficiency between hospitals based ownership structure and state were investigated using Two-Group Mean-Comparison tests. The results of the tests are tabulated in Table 4.

**Table 4: Group Mean Comparison Tests**

Group Variable	Ownership		State	
	Public	Private	Enugu	Anambra
<b>Obs</b>	46	142	90	97
<b>Mean</b>	0.58	0.75	0.69	0.72
<b>Std Dev</b>	0.147	0.079	0.121	0.127
<b>Diff of means</b>	- 0.171		-.028	
<b>Diff of mean t-value</b>	t-value = -10.122***		t-value = -1.57	

There is no significant difference in the efficiencies of hospitals in the two states of the southeast covered in this study which is consistent with the fact that both states operated under similar regulatory environment until recently.

Ownership structure appears to be a major basis for differences in efficiency behavior of hospitals. The large difference, between the efficiency scores of public and private hospitals in the sample is equivalent to 17 percentage points in favor of the latter. The difference is highly significant. This is not surprising, and infact is predicted by economic theory. In the first place there is the dominance of for-profit hospitals in the sample area which is reflect in their overwhelming numerical strenght in the sample in which they constitute over 70% of total sample, excluding the eight not-for-profit private hospitals in the sample. The profit motive provides the incentive for efficiency, though this is not equivalent to patients' satisfaction. Public hospitals are, on the other hand, likely to suffer the effects of civil service procedures and hierarchical command which could greatly rub-off on efficiency. The result may seem to support the current argument of International Finance Corporation (2008) in favor of more institutional support for private hospitals in Africa. However, this study is not intended as evidence of this policy, and further

evidence of this superior efficiency need to be demonstrated. Moreover, efficiency is not the only issue in the public versus private debates.

It is difficult to compare the results of this study with similar those from studies in other LICs, largely because most such studies have generally used the DEA approach, and also because results from DEA model is influenced by the number of input and/or output variables included in the estimation. The inclusion of more input and/or outputs tends to inflate the average efficiency score in DEA model. However, the results from this study compare well with those estimated in [4,22,28]. These studies using DEA estimated average technical efficiencies of hospitals ranging from 65 – 85% in Zambia, Kenya and Ghana respectively.

### Conclusion

Increasing concerns about efficiency in the health sector need empirical evidence that will help to chart policy direction for achieving greater efficiency in the utilization of health resources given that these resources have serious opportunity costs both within the health sector itself and in other development sectors. This study using stochastic frontier method has provided further evidence of serious deviations from efficiency frontiers of

hospitals in LICs. More specifically, the results indicate that current health service achievements of the hospitals in Nigeria could well be attained with only 70% of resources available to the hospitals. This would release large amount of resources for other healthcare needs or other development needs of the society. However, while there is a large shortfall in achievement on average, some hospitals have much more shortfalls than others. Efficiency performance of the samples hospitals has very large variance with the least efficient operating on efficiency rate of 0.26 while the most efficient is 0.90. The 15 least efficient hospitals had average efficiency rate of about 0.39. In other words, their current output could as well be achieved with less than 40% of their current resources. The implication is that some hospitals are virtually wasting social resources.

Given the current desire of government to attain the MDGs by 2015, a policy that addresses this large waste in health sector is clearly desirable. Of the 15 hospitals that had efficiency score of less than 0.50 efficiency 13 were public hospitals. Indeed none of the public hospitals in the sample scored above 0.70. This result is robust to the nested regression and group-means test results. This implies that public policy instruments can actually be used to address the efficiency question in these hospitals directly. While this study did not investigate the causes of such high level of inefficiency in state owned hospitals, it is not unlikely that lack of incentive to be efficient, government bureaucracy and other related procedural bottlenecks may be responsible for their relative inefficiency. This gap in information needs to be further explored. However, it is clearly desirable that urgent steps be taken to improve the level of inefficiency which have great social welfare costs. It is often the case that in government hospitals one healthcare input may be over-supplied while the other is not supplied at all. It is also the case that substitutability of inputs may be very low if at all. For example, availability of drugs can hardly substitute for the presence of a doctor and vice versa. Whichever is the case, public hospital managements need to work with greater efficiency incentives and set targets if they are to improve their efficiency levels. It is possible that public hospitals make up for their relatively low inefficiency by providing higher quality care. This was not investigated in this study but it is doubtful that difference in quality of care could account for the large difference in efficiency scores between the two groups given that there are no observed major differences in the average staff between public and private facilities.

But while the private hospitals are relatively more efficient than state owned hospitals, they are still

largely inefficient. The average efficiency of private hospitals is only 0.75 (as against 0.58 of state-owned hospitals) which implies that the present level of output could be sustained with only 75% of resources currently used up by these hospitals. The results also indicate that most of the hospitals operate at constant returns to scale implying that there are no gains in scale economies of the hospitals. The smallness of a large proportion of the hospitals would seem to confirm this conclusion. Again, it is noted that the results of this study only reflect the situation with technical efficiency of the hospitals not their allocative efficiency or even their ability to undertake complex medical care requiring sophisticated equipment and complex operational procedures. The attainment of these other goals and economies of scale would clearly require policies that provide incentives for merger and greater co-operation among the hospitals both private and public. The need to consolidate healthcare in the area is also argued by the increasing sophistication of healthcare requiring the cooperation of teams of experts in the field.

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<sup>i</sup> Both the environment and structure of ownership are captured by X-efficiency

<sup>ii</sup> Note the  $u_i$  is positive in the case of production function because it indicates required level of increase in output for an inefficient firm to meet with its peers at the frontier. On the other hand the  $u_i$  is negative in the case of the cost function because it indicates by how much and inefficient firm must reduce its cost to be at the efficient cost frontier.

<sup>iii</sup> In specifying the econometric model of hospital production function, it is assumed that there are no endogeneity problem between the input regressors and output of hospitals. In other words, it is being assumed that the error term which is the difference between the observed and expected outputs are not related to the input variables. This is a common assumption in literature.

<sup>iv</sup> Cobb-Douglas production function assumes the output elasticity of inputs will be constant irrespective of level of inputs. This may not be very realistic in the context of hospital production function (van Montfort 1981). It may be expected that the output elasticity of capital with fixed labor inputs will decrease. The substitution elasticity between doctors and nurses and between labor and capital may not be also constant. The CES is a generalized version of Cobb-Douglas function.

<sup>v</sup> Note that the C-D production function is only a more restricted version of the translog in which the square and cross-products are restricted to zero.

<sup>vi</sup> The respondent was usually the chief medical officer of the hospital or someone delegated by him

<sup>vii</sup> Note that an estimated coefficient in a double log-function is in fact the elasticity which could be evaluated at a given point to obtain the elasticity at that point. In a single-input single-output model, i.e given  $y = f(x)$  the output  $y$  point elasticity w.r.t the input ( $x$ ) may be defined as  $\eta_{yx_0} = \frac{d \ln y}{d \ln x} = \frac{dy}{dx} \cdot \frac{x}{y}$ . For single output multiple output case, that is, the point elasticity for the multiple input case is given by

$$\eta_{yx_0} = \sum_{i=1}^n \frac{\partial \ln y}{\partial \ln x_i} | x = x_0$$

<sup>viii</sup> Total staff includes the clinical and nonclinical staff and may as well be a proxy of the size of the hospital

<sup>ix</sup> See footnote 6 in Battese and Coelli (1995)