

Full Length Research Paper

Empirical modeling of solar radiation for selected cities in Nigeria using multivariate regression technique

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In this paper, the data for solar radiation, minimum and maximum temperatures, wind speed and evaporation for the years 1970-1995 and for fourteen stations taken over Nigeria were obtained from the archives of the Nigerian Meteorological Agency (NIMET), Oshodi, Lagos. The distributions of solar radiation with each of the four meteorological variables were observed. It was found that solar radiation was adequately monitored by each of the variables indicating that the linear combination of the variables could be used to developed model from where solar radiation can be evaluated. Consequently, a multivariate linear regression model was developed for each of the stations. The statistical indicators such as R^2 , MBE, RMSE and MPE were calculated to estimate the efficiency of the developed models. For instance, the values of the R^2 , MBE, RMSE and MPE calculated for Sokoto, a Sahelian station are 0.5349, 0.0273, 0.4154 and 0.0811, respectively, which show that the model for Sokoto is significant. Those for other stations have also been calculated with all showing high level of significance at 0.05 alpha level.

Key words: Solar radiation, meteorological variables, correlation coefficient, multivariate regression, evaporation, significance.

INTRODUCTION

Solar radiation received at the earth's surface is essential for the development and utilization of solar energy devices. It is needed for designing collectors for solar heaters and other photovoltaic equipment that depend on solar energy. Incoming solar radiation has a significant role in hydrological and crop growth modelling. For instance, it is a key input for estimating potential evapotranspiration which play a major role in the design of water supply storage reservoirs and irrigation systems (Sanusi and Abisoye, 2011). The solar radiation reaching the earth's surface depends on the climatic

condition of the specific location, and this is essential for accurate prediction and design of a solar energy system (Burari and Sambo, 2001). Technology for measuring solar radiation is costly and has instrumental hazards (Alam et al., 2005); this has led to paucity of it in many stations in the developing countries (including Nigeria). Thus, alternative methods for estimating these data are required (Al-Salihi et al., 2010). One of these methods is the use of empirical models. Accurate empirical modelling depends on the quality and quantity of the measured data used in developing it, and is a good

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Figure 1. A map of Nigeria showing four climatic regions and the studied stations.

tool for generating solar radiation of any kind (whether global, direct or diffuse) at locations where measured data are not available.

When solar radiation enters the Earth atmosphere, a part of the incident energy is removed through the processes of scattering, absorption and reflection. The scattering of solar radiation is mainly by atmospheric molecules and aerosols while the absorption of solar radiation is by stratospheric ozone, water vapor, oxygen, carbon (IV) oxide, as well as clouds. The reflection of solar radiation on the other hand, is mainly by clouds and this plays an overriding part in reducing the energy density of the solar radiation reaching the surface of the Earth (Exell, 2000).

Several models have been proposed to estimate global solar radiation. Angstrom (1924) was the first scientist known to suggest a simple linear relationship to estimate global solar radiation. Badescu (Badescu, 1999) studied existing relationships between monthly mean clearness index and the number of bright sunshine hours using the data obtained from Romania. Trabea and Shaltout (2000) studied the correlation between the measurements of global solar radiation and the meteorological parameters using solar radiation, mean daily maximum temperature, mean daily relative humidity, mean daily sea level pressure, mean daily

vapour pressure and hours of bright sunshine data obtained from different parts of Egypt; while Sfetsos and Coonock (2000) used artificial intelligence techniques to forecast hourly global solar radiation. Okogbue and Adedokun (2002) estimated the global solar radiation at Ondo, Nigeria.

The main objective of the present study is to develop predictive models, using multi-variate regression analysis technique to predict solar radiation using four meteorological parameters, that is, minimum and maximum temperature, wind speed and evaporation for the study area and other locations having similar climatic characteristics in Nigeria. This will be useful in the development and the applications of solar energy technology.

METHODOLOGY

The monthly mean solar radiation, minimum and maximum temperatures, wind speed and evaporation data were obtained from the Archives of Nigerian Meteorological Agency, Oshodi, Lagos. The data obtained covered a period of twenty-six years (1970-1995). The stations were grouped according to their respective weather condition as shown in Figure 1. The distributions of the solar radiation with each of the atmospheric parameters were observed. It should be noted here that the solar radiation data were measured using Gun-Bellani distillate. Measurements were taken in millilitres (ml) which were converted to useful format

(MJ/m²) using a conversion factor proposed by Folayan (Folayan, 1988), which is 1 ml = 1.357 MJ/m²

Relevant multivariate linear regression analysis theory

Multivariate regression analysis is a technique for modeling and analyzing several variables when the focus is on the relationship between a dependent variable and one or more independent variables. For instance, let z_1, z_2, \dots, z_r be a set of r predictors believed to be related to a response variable Y . The multivariate linear regression model for the j th sample unit has the form:

$$Y = \alpha + \beta_1 z_{j1} + \beta_2 z_{j2} + \dots + \beta_r z_{jr} + \varepsilon_j \quad 1$$

where ε_j is a random error, the β_i where $i = 1, 2, \dots, r$ are unknown parameter estimates and α is the intercept. With n independent observations, one model can be written for each sample unit or can be organized into vectors and matrices so that the model now becomes

$$Y = Z\beta + \varepsilon \quad 2$$

where

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \quad Z = \begin{pmatrix} z_{j1} \\ z_{j2} \\ \vdots \\ z_{jn} \end{pmatrix} = \begin{pmatrix} z_{11} & \dots & z_{1r} \\ z_{21} & \dots & z_{2r} \\ \vdots & \dots & \vdots \\ z_{n1} & \dots & z_{nr} \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_r \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix} \quad 3$$

Least squares estimation

Ordinary least squares (OLS) estimates are commonly used to analyze both experimental and observational data. The OLS method minimizes the sum of squared residual, and leads to a closed-form expression for the estimated value of the unknown parameter β :

$$\hat{\beta} = (Z'Z)^{-1} Z'Y = \left(\frac{1}{n} \sum z_j z_j' \right)^{-1} \left(\frac{1}{n} \sum z_j y_j \right) \quad 4$$

where $\hat{\beta}$ denote the least squares estimate of β and Z' denotes the transpose of Z .

RESULTS AND DISCUSSION

Distribution of solar radiation with meteorological variables

Figures 2 to 5 show the distribution of solar radiation with meteorological variables, which are, the minimum and maximum temperatures, wind speed and evaporation for four stations, that is one from each of the four climatic regions in Nigeria.

In Sokoto, it was observed that minimum and maximum temperatures and evaporation increased with solar radiation between January and June and also between September and November. A downward trend was discer-

nible between July-September as shown in Figures 2a-b. The increasing trends between January and June may be due to the fact that during these months, dry season condition is predominant in this station. The decreasing trend in July-September may be due to the presence of disturbances like cumulous cloud and cloud cluster which are significant enough to cause variation in weather (Adeyemi, 2004). During this period, all regions in Nigeria will be experiencing intense rainfall because the intertropical discontinuity (ITD) would have reached its maximum northern position. Wind speed on other hand shows an opposite trend to solar radiation during dry season. For instance, between January and March, solar radiation increases as wind speed decreases. However, wind speed and solar radiation show the same downward trend between June and August, the rainy season in the Sahelian zone (Figure 2c). This shows that intense rainfall decreases the intensity of solar radiation and the rate of wind flow in the zone. Meanwhile, there is increasing trends between September and November, another phase of dry season. This may due to the fact that the dry season period in the zone is characterized by mostly turbulent flow of wind. Solar radiation and evaporation show the same increasing and downward trends (Figure 2d). The increasing trends are discernible between January-May and September-November, the dry season period and downward trends between June and August, the rainy season period. This is in agreement with Graham et al. (2004) argument that the physics of evaporation shows that the evaporative demand of the atmosphere is directly dependent on the net radiation which is dependent on solar radiation.

In Yola, there are increasing trends in variations of solar radiation with temperatures, wind speed and evaporation between January-April and September-December, the dry season period (Figures 3a-d). This may due to the topography of the zone that is characterized by short grasses and scattered drought-resistant tree which aids incessant surface heating that caused water vapour to be transported to the higher layers of the atmosphere through buoyancy (Adeyemi et al., 2004; Aro, 1975). Also, solar radiation shows downward trends with the four meteorological variables between April and August, the rainy season period in the zone. This may likened to the presence of the localized convection due to the usual long period of humid condition in the zone. In Ibadan, the dry and the wet season occurrence are greatly influenced by its latitudinal location and solar radiation has considerable seasonal variations. There are downward trends between March and September in the distribution of solar radiation with temperatures and evaporation as shown in Figures 4a, b and d, that is, during the rainy season period. This may due to the fact that these months being the core rainy months, are characterized by incessant cloud formation and thereby depleting the amount of solar radiation reaching the

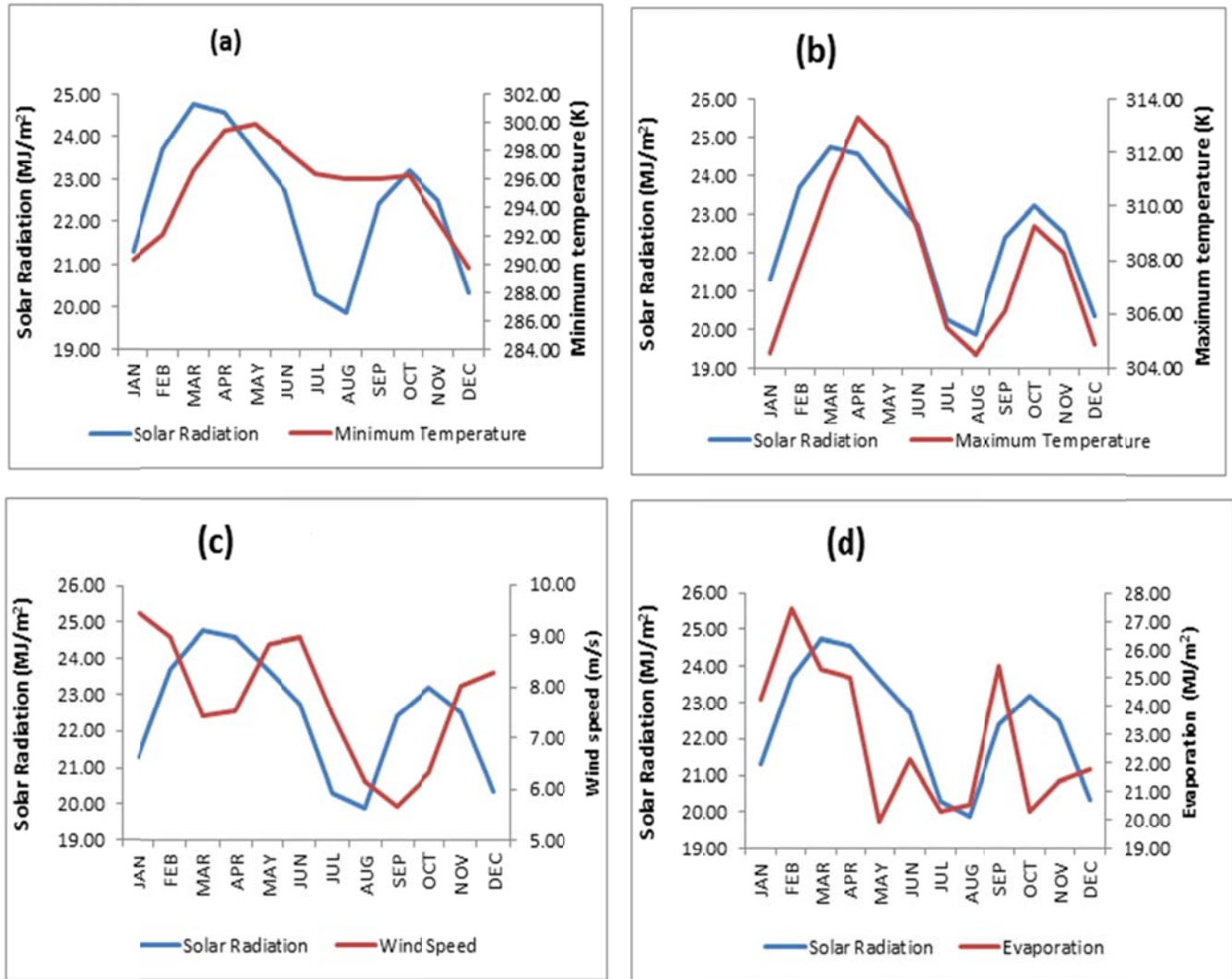


Figure 2. Distribution of (a) solar radiation and minimum temperature (b) solar radiation and maximum temperature (c) solar radiation and wind speed (d) solar radiation and evaporation at Sokoto.

Earth's surface (Ajayi and Adeyemi, 2009). The increasing trends are observed between October and March. This may be due to the prevalence of the dry CT air which characterizes the dry season. The period is characterized by low humidity and high rate of evaporation aided by the increase in solar radiation and in air temperature in the zone. On the other hand, there is a downward trend in the distribution of solar radiation with wind speed between June and August (Figure 4c). This may be due to the fact that the zone is under the influence of moist maritime south-western monsoon wind which blows inland from the Atlantic Ocean. The dry season occurs from November to February during which dry dust-laden winds blow from the Sahara desert. This period is characterized by low humidity and high rate of evaporation (Ajayi and Adeyemi, 2009). There is a general decreasing trend in solar radiation between

November and January in Sokoto, Yola and Ibadan respectively (Figure 2 to 4a-d). These are expected because November and January are harmattan months when aerosol mass loading greatly reduces the intensity of solar radiation (Babatunde and Aro, 2000).

In Port-Harcourt, solar radiation has two peak values in February and November (Figures 5a-d). This is expected because of a very high sunshine hour which is obtainable in these months due to high clearness index (Aro, 1975). Least solar radiation is observed in the months of July and August, respectively. This is expected because heavy rainfall characterizes these months in the station. Therefore, the total solar radiation recorded is quite low because of the wet atmosphere and the presence of heavy clouds. If the weather is cloudy, the solar radiation value would be largely affected.

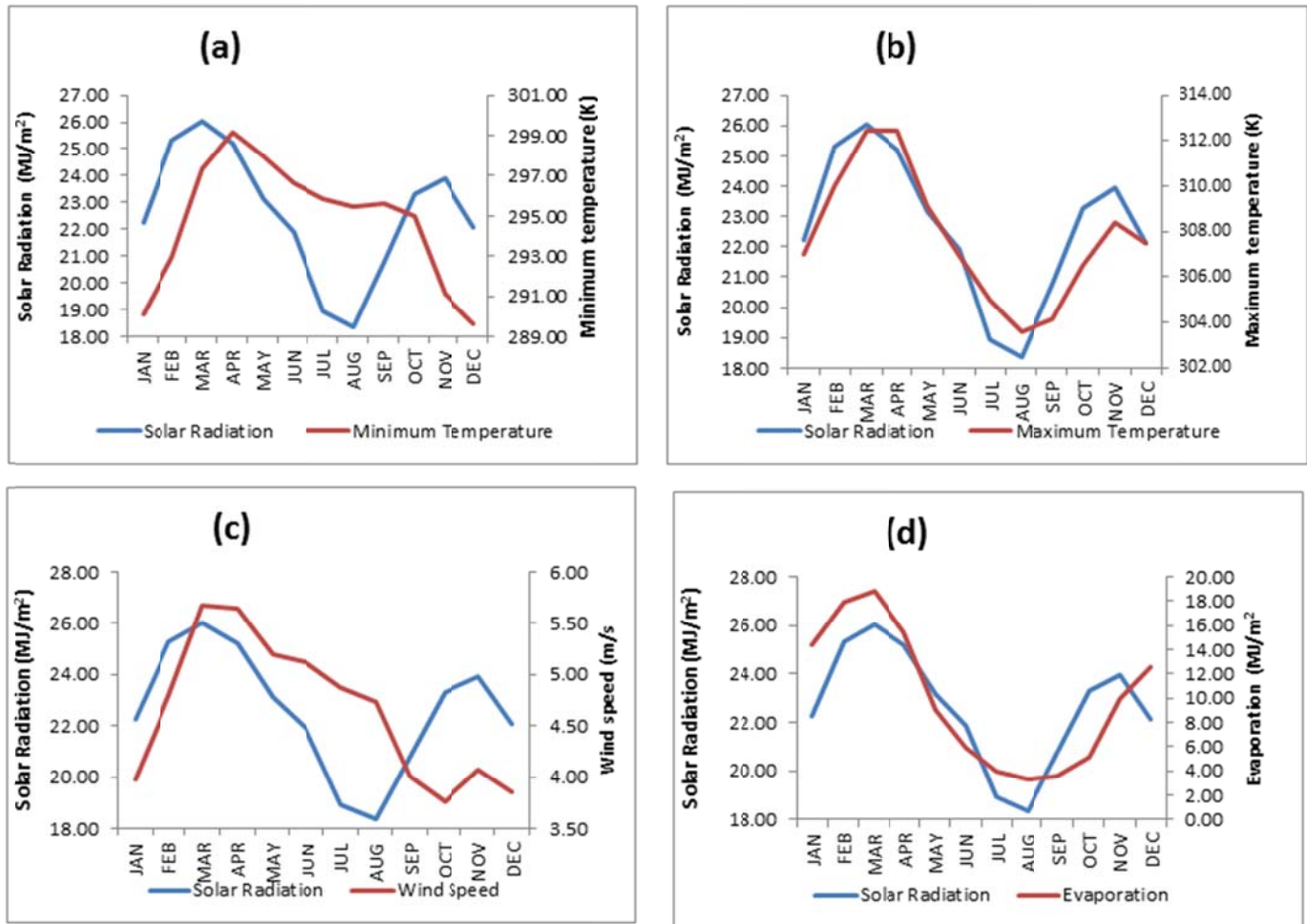


Figure 3. Distribution of (a) solar radiation and minimum temperature (b) solar radiation and maximum temperature (c) solar radiation and wind speed (d) solar radiation and evaporation at Yola.

This is because the solar radiation at normal incidence received at the surface of the earth is subjected to variations due to change in the extraterrestrial radiation and also the two significant phenomena which are the atmospheric scattering by air molecules, water and dust and the atmospheric absorption by O_2 , H_2O , CO_2 (Lam et al., 2002). Wind speed increases with solar radiation between November and February while it shows downward trends between March and August in the station as shown in Figure 5c. This may be due to excessive rainfall in these months in the zone.

Model development

The multivariate linear regression (MLR) modeling technique was then applied and a model of the form as shown in Equation 6 was developed for all stations using 1970-1990 data.

$$H = \alpha + \beta_1 T_{min} + \beta_2 T_{max} + \beta_3 W_s + \beta_4 E_p + \epsilon \quad 5$$

where α is the regression constant, ϵ is the error term, β_1 , β_2 , β_3 , and β_4 are parameter estimates of minimum temperature (T_{min}), maximum temperature (T_{max}), evaporation (E_p) and wind speed (W_s) respectively for each of the stations.

The parameter estimates for the empirical model for evaluating solar radiation for each of the fourteen stations are shown in the Table 1. For instance, Sokoto on Lat. 13.5° , Long. 5.25° model is:

$$H = -83.783 - 0.184T_{min} + 0.526T_{max} - 0.064W_s + 0.079E_p \quad (6)$$

Model testing and assessment

The models were then validated using surface data of 1991-1995 for all the parameters. The performance of the developed models was tested with coefficient of determination (R^2), root mean square error (RMSE)

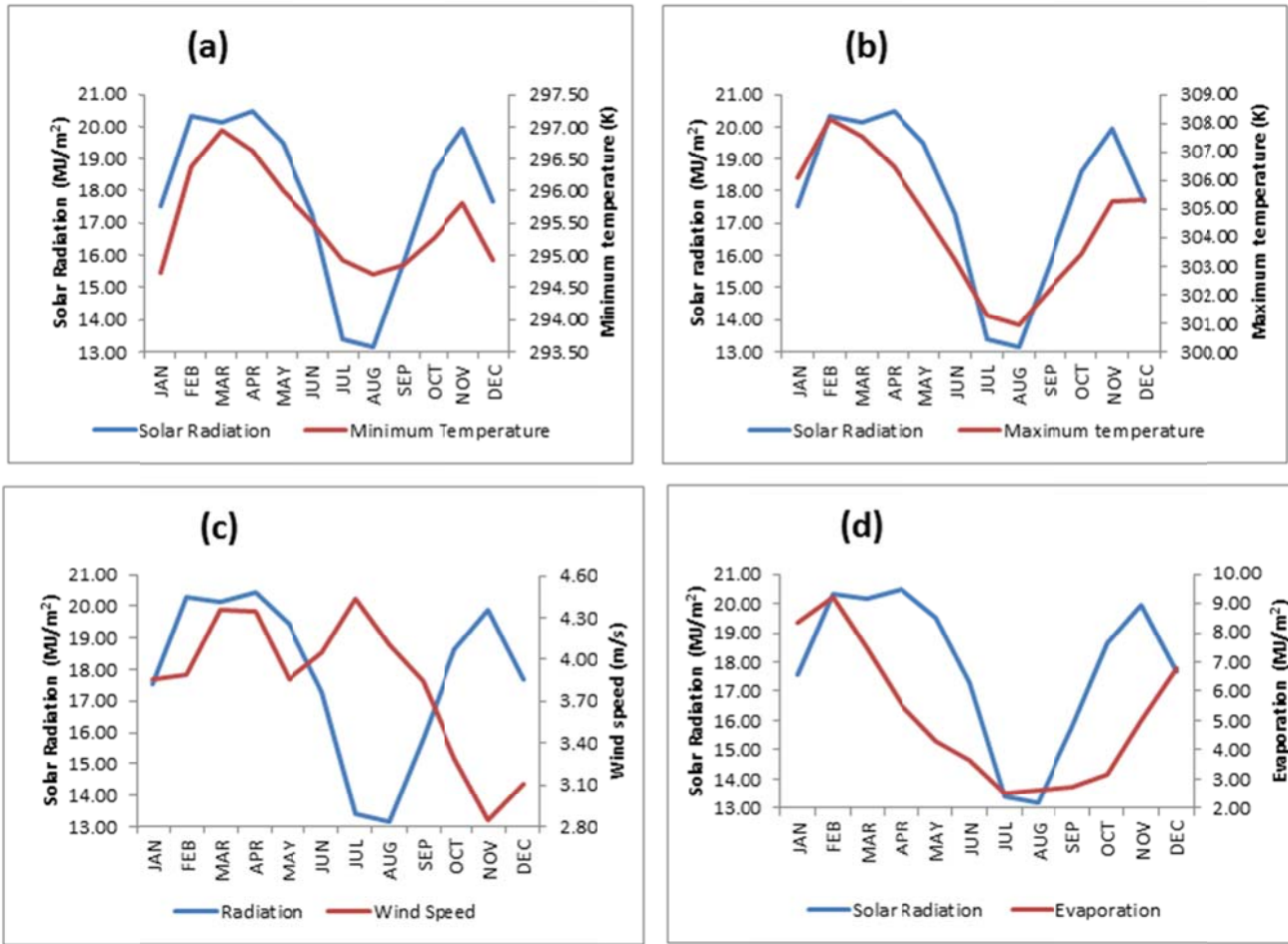


Figure 4. Distribution of (a) solar radiation and minimum temperature (b) solar radiation and maximum temperature (c) solar radiation and wind speed (d) solar radiation and evaporation at Ibadan.

and mean bias error (MBE), mean percentage error (MPE) and their values are shown in Table 2.

The expressions for evaluating the value of MBE, RMSE and MPE as stated by Okundamiya and Nzeako (2010) are shown in Equations (8-10):

$$MBE = \frac{1}{N} \sum_{n=1}^n (H_p - H_a) \tag{7}$$

$$RMSE = \sqrt{\frac{1}{N} \left[\sum_{n=1}^n (H_p - H_a)^2 \right]} \tag{8}$$

$$MPE = \frac{1}{N} \sum_{n=1}^n \left[\frac{H_a - H_p}{H_a} \times 100 \right] \tag{9}$$

$$t-test = \sqrt{\frac{(P-1)(MBE)^2}{(RMSE)^2 - (MBE)^2}} \tag{10}$$

where H_p and H_m is the predicted and measured values of solar radiation and N is the total number of observations. These indicators are mainly employed for

adjustment of solar radiation data (Halouani and Ngguyen,1993; Falayi and Rabi, 2005; Okogbue and Adedokun, 2002; Al-Saihi et al., 2010). The models developed were applied to evaluate solar radiation at the stations. The result of these was compared with the actual values obtained for all stations which are the average as shown in Table 2. The agreements are remarkable.

From Table 2, the values of the coefficient of determination (R^2) which show the proportion of variance accounted for by the correlation measured values with predicted values and the significance of the developed models to predict the solar radiation vary from 0.3017-0.7655 for the selected stations. The graphical demonstration of the correlation of measured and predicted values are shown in Figures 6 to 8. There are good agreement between the predicted values obtained from the models and the actual measured values of the surface data of solar radiation. The values of the mean bias error (MBE) indicate the average deviation of the predicted values from the actual measured values.

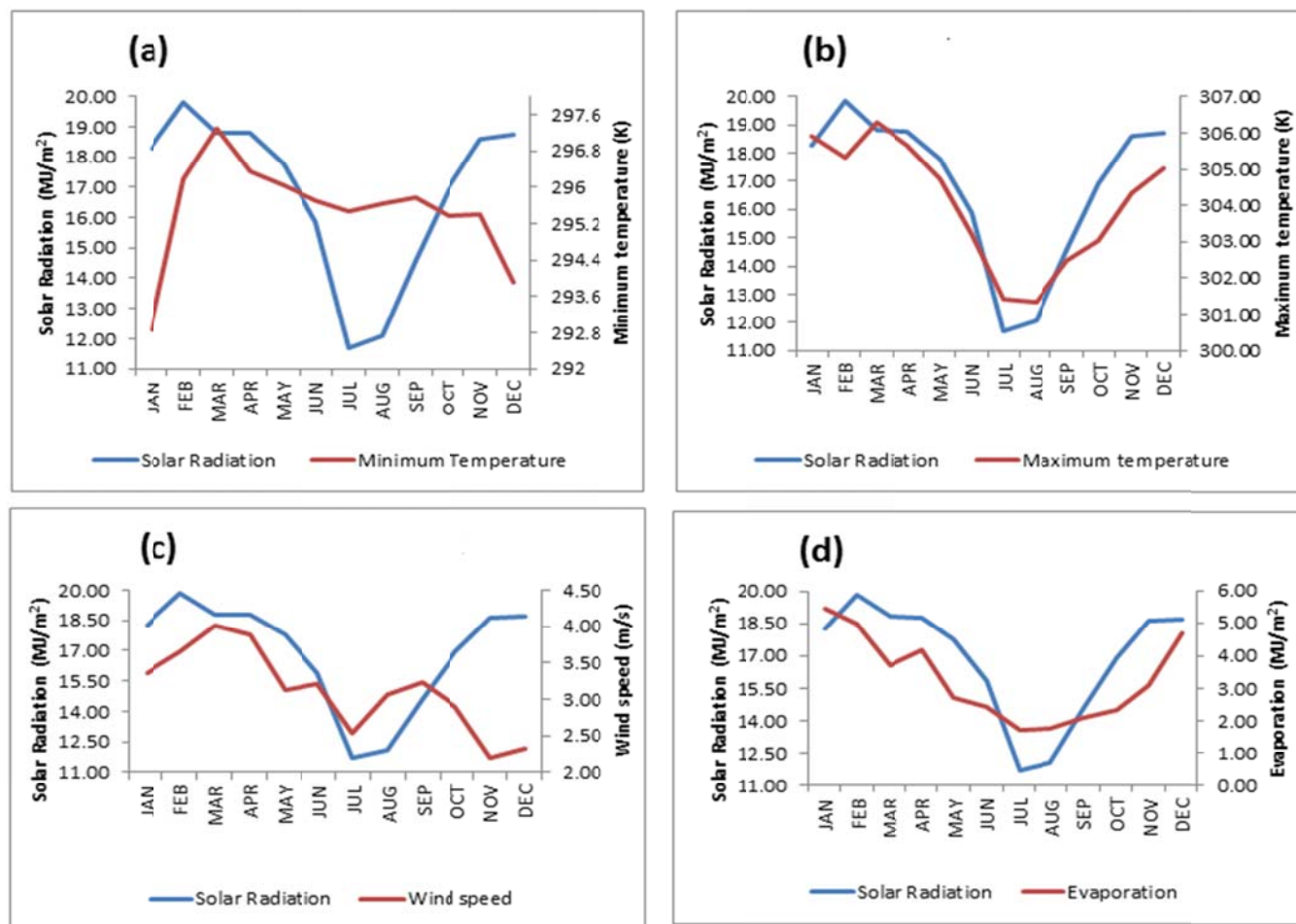


Figure 5. Distribution of (a) solar radiation and minimum temperature (b) solar radiation and maximum temperature (c) solar radiation and wind speed (d) solar radiation and evaporation at Port-Harcourt.

Table 1. Multivariate parameter estimates of the models for the studies stations.

Station	Location		Models' parameters estimates					p-value
	Lat (°N)	Lon (°E)	α	β_1	β_2	β_3	β_4	
Sokoto	13	5.25	-83.7827	-0.1837	0.5256	0.0642	-0.0797	0.0000
Kano	12.07	8.43	-120.731	0.0149	0.4593	0.0863	-0.0196	0.0000
Nguru	12.91	5.30	-99.1411	-0.1119	0.5013	0.4478	0.0358	0.0000
Maiduguri	11.91	13.17	-51.7748	0.4025	-0.1448	-0.3399	0.3799	0.0000
Minna	9.62	6.53	-204.529	-0.4197	1.1483	0.2715	-0.21688	0.0000
Jos	9.87	8.52	-192.513	-0.2533	0.9523	0.0370	0.08928	0.0000
Bida	9.08	6.02	-165.524	-0.3573	0.9558	0.3479	-0.06878	0.0000
Yola	9.23	12.47	-148.911	0.0379	0.5248	-0.4690	0.099828	0.0000
Osogbo	7.73	4.48	-225.887	-0.3327	1.1239	0.4135	-0.0383	0.0000
Ibadan	7.43	3.9	-208.052	-0.5119	1.2417	0.1476	-0.0944	0.0000
Ikeja	6.58	3.33	-227.319	-0.4350	1.2284	0.3887	-0.0968	0.0000
PH	4.85	7.02	-212.561	-0.3019	1.0498	0.3169	0.0216	0.0000
Benin	6.32	5.6	-205.085	-0.5191	1.2330	0.4416	-0.0476	0.0000
Enugu	6.37	7.55	-216.977	-0.4366	1.2001	0.0569	-0.0753	0.0000

Table 2. Application of the multivariate proposed model for each of the fourteen stations using 1991-1995 data.

Station	Solar radiation		Performance estimates			
	Measured	Predicted	R ²	MBE	RMSE	MPE
Sokoto	23.20	22.93	0.4086	-0.2717	1.3186	0.8668
Kano	24.74	24.96	0.5600	0.2248	1.2282	-1.1148
Nguru	25.98	24.37	0.3017	-1.2092	2.2361	4.3238
Maiduguri	24.20	25.78	0.5350	1.5808	2.2751	-6.4854
Minna	22.59	23.30	0.6209	0.7120	1.9751	-3.8081
Jos	22.63	21.72	0.7655	-1.9094	2.0708	3.0847
Yola	24.35	23.30	0.6321	-0.4261	1.9346	0.9346
Bida	21.51	22.59	0.6533	1.0763	2.1929	-6.1701
Ibadan	18.12	19.26	0.7132	1.1458	1.9122	-7.2537
Osogbo	17.77	19.15	0.4148	1.3834	2.5899	-9.5505
Enugu	17.03	19.30	0.6604	2.6714	3.1111	-17.150
Benin	15.92	18.45	0.5409	2.6234	3.0702	-17.265
Ikeja	15.19	18.45	0.3225	-2.6345	3.4697	-18.682
Port-Harcourt	16.75	18.98	0.5314	2.2307	3.0028	-15.489

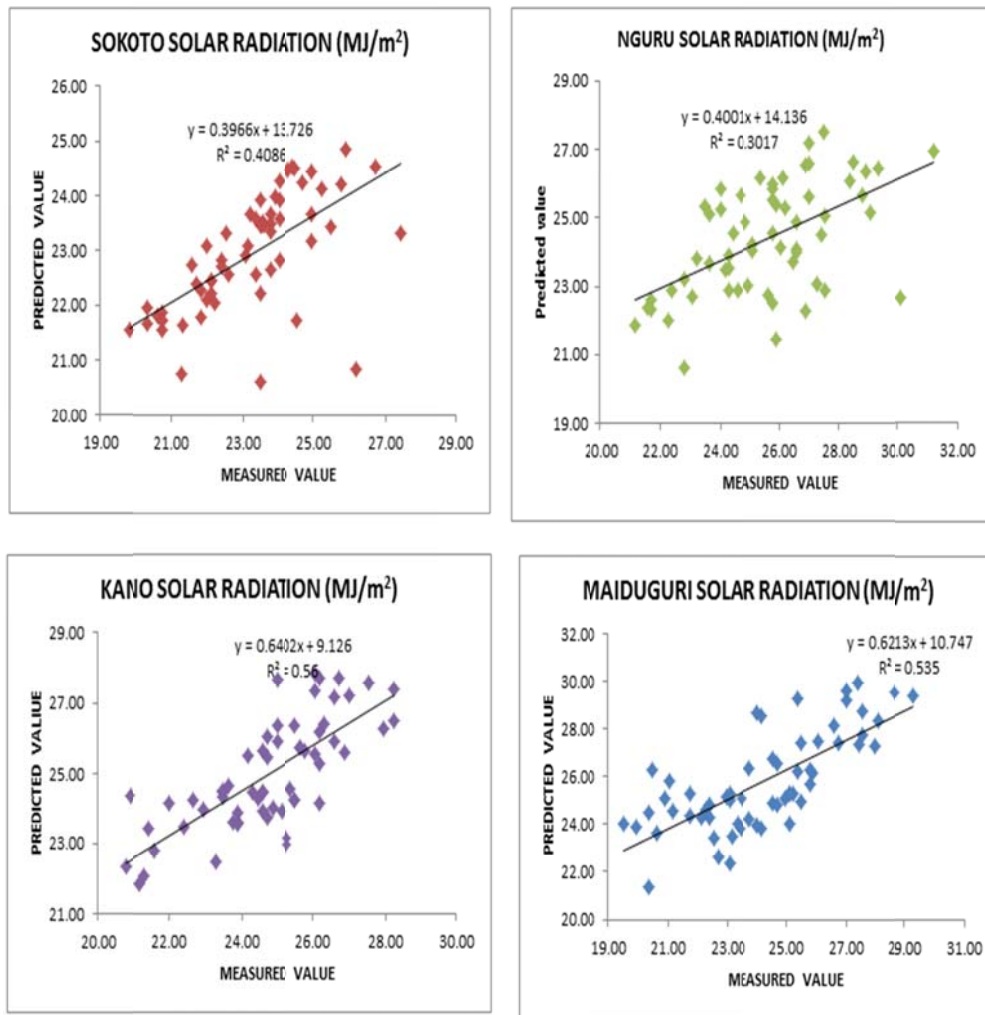


Figure 6. The correlation between the predicted and measured values of the monthly solar radiation Sokoto, Kano, Maiduguri and Nguru Stations in Nigeria.

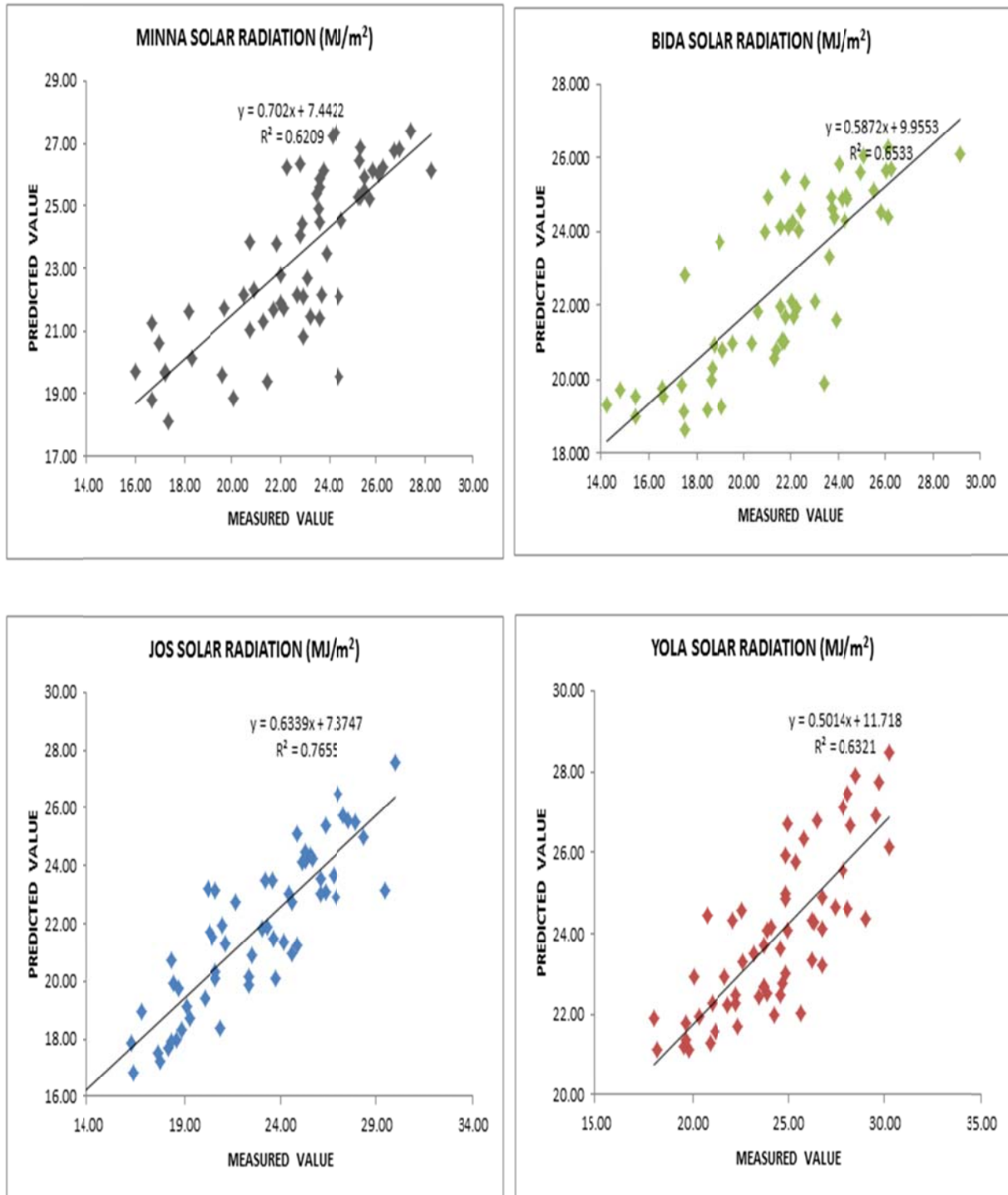


Figure 7. The correlation between the predicted and measured values of the monthly solar radiation for Bida, Minna, Jos and Yola Stations in Nigeria.

The MBE values vary from negative values to the positive values (lowest for Ikeja and highest for Benin). The low values of MBE generally show that the developed models are significant for predicting solar radiations. The MBE

values are suitable for the models for each of the fourteen stations (Okundamiya and Nzeako, 2010).

The root mean square error (RMSE) values indicate a measure of the variation of predicted values from the

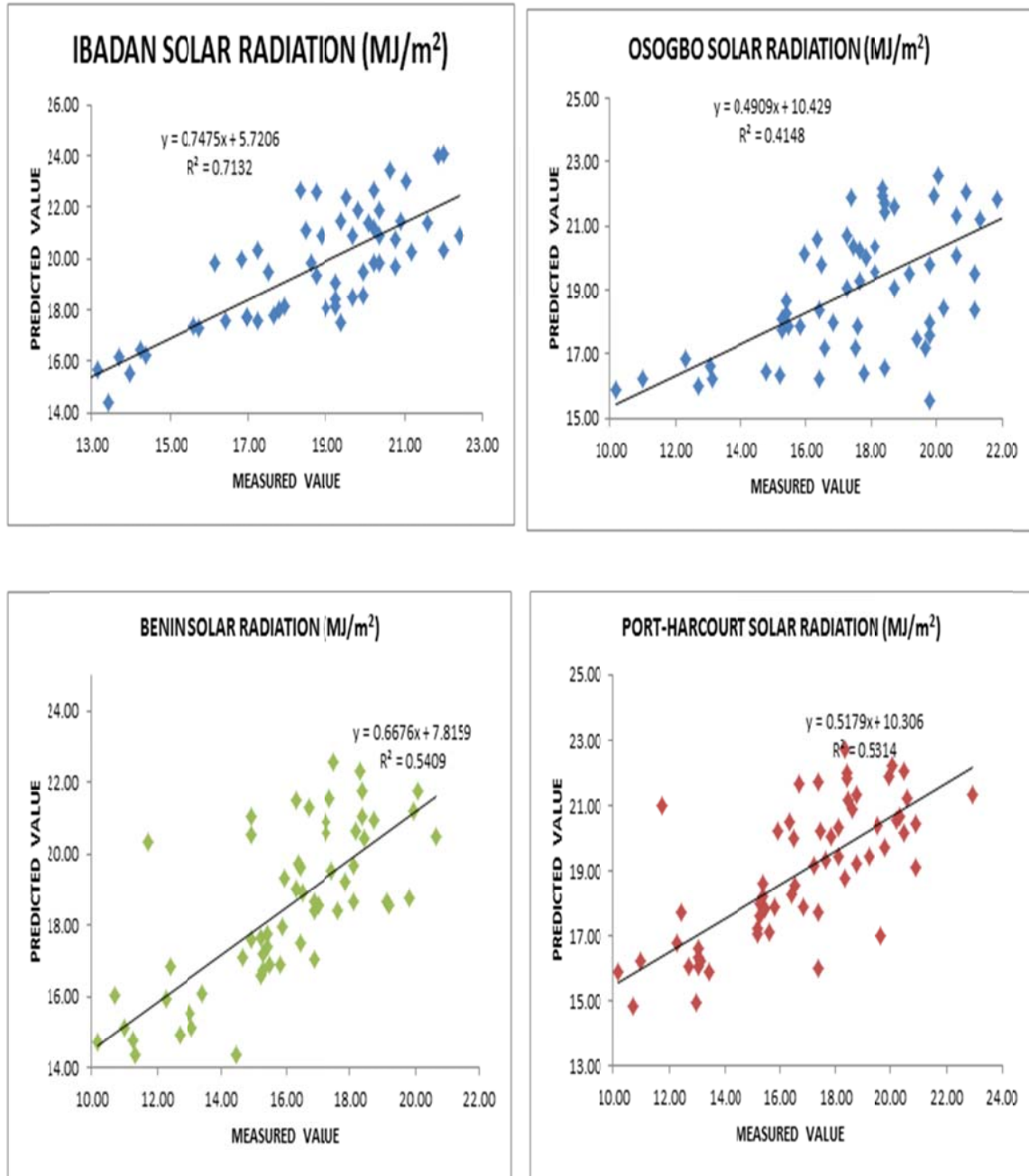


Figure 8. The correlation between the predicted and measured values of the monthly solar radiation for Osogbo, Ibadan, Port-Harcourt and Benin Stations in Nigeria

actual measured values. The RMSE values vary from negative values to the positive values (lowest for Kano and highest for Ikeja). The MPE values are suitable for the models for each of the fourteen stations. Therefore, all the developed models give long term performance (Babatunde and Aro, 2000). The RMSE values are suitable for the models for each of the fourteen stations. Therefore, term by term comparison is allowed for all the developed models (Sanusi and Abisoye, 2011).

The values of mean percentage error MPE give long term performance of the developed models. They also

vary from negative values to the positive values (lowest for Ikeja and highest for Nguru). The MPE values are suitable for the models for each of the fourteen stations. Therefore, all the developed models give long term performance (Babatunde and Aro, 2000).

Added to these, the graph of variations plotted for the measured solar radiation for each of the four climatic regions along with their predicted values showed that solar radiation was well monitored at all the regions by the models (Figures 9 to 11).

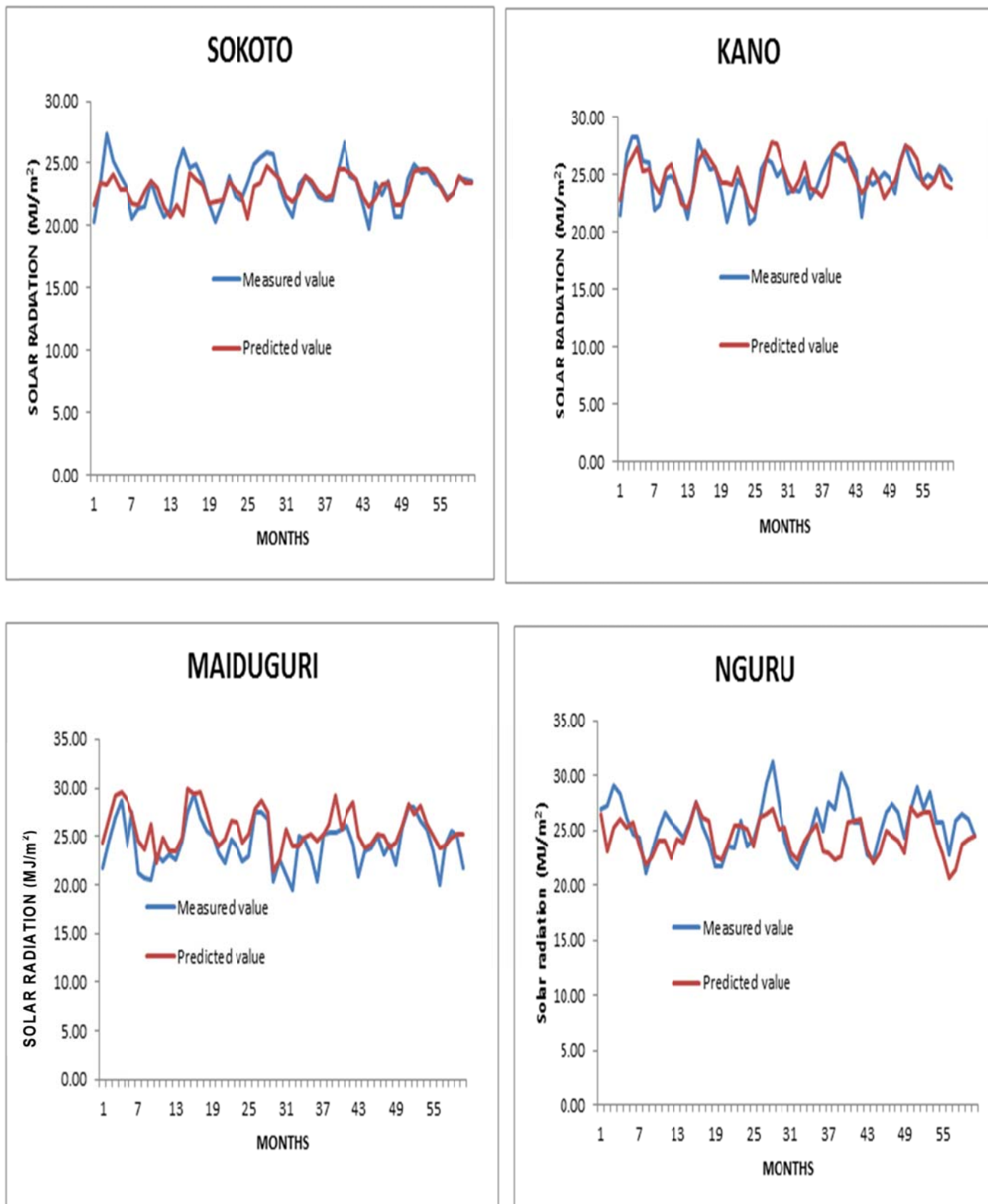


Figure 9. The variations of the predicted and measured values of the monthly solar radiation for Sokoto, Kano, Maiduguri and Nguru stations in Nigeria.

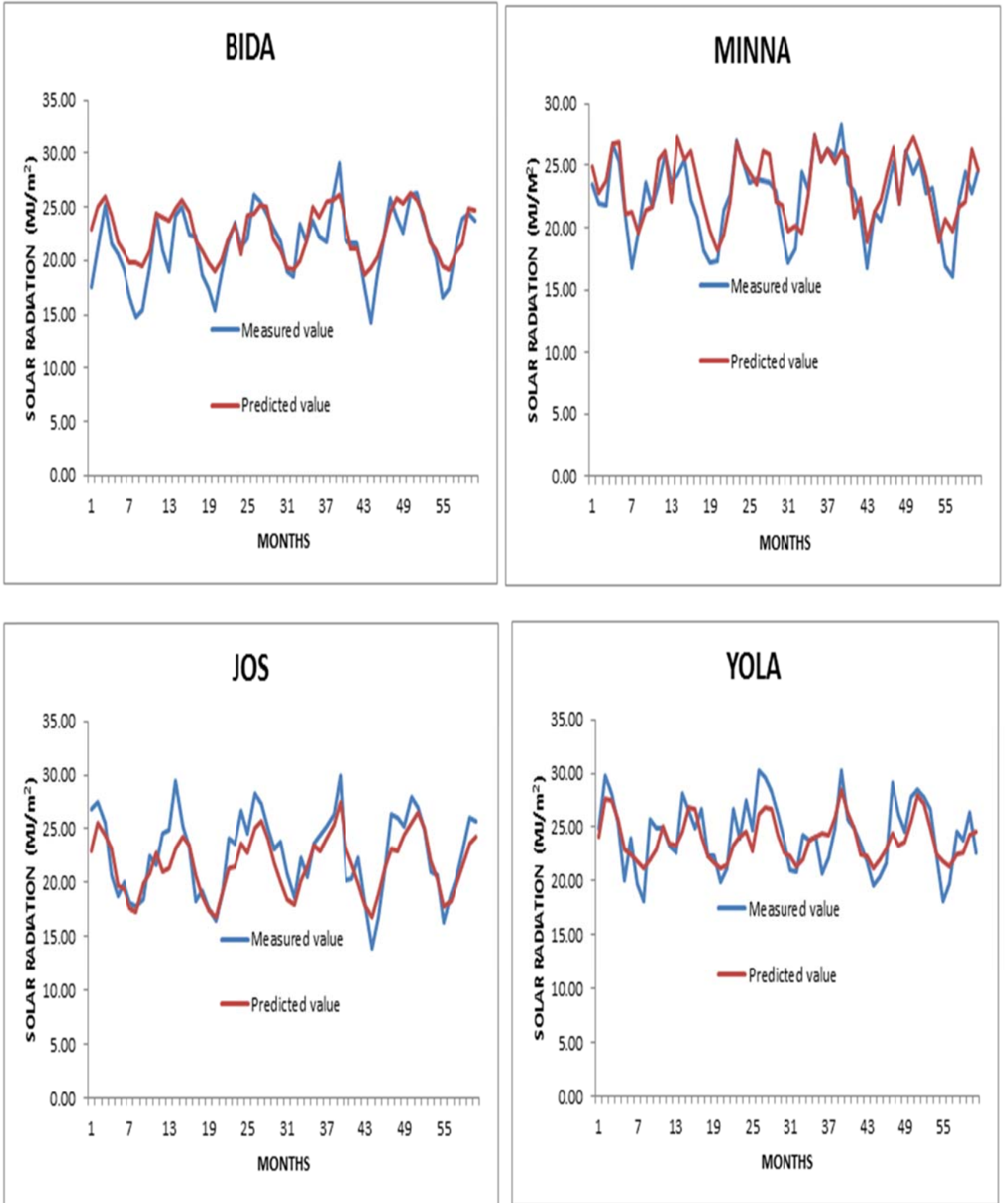


Figure 10. The variations of the predicted and measured values of the monthly solar radiation for Bida, Minna, Jos and Yola stations in Nigeria.

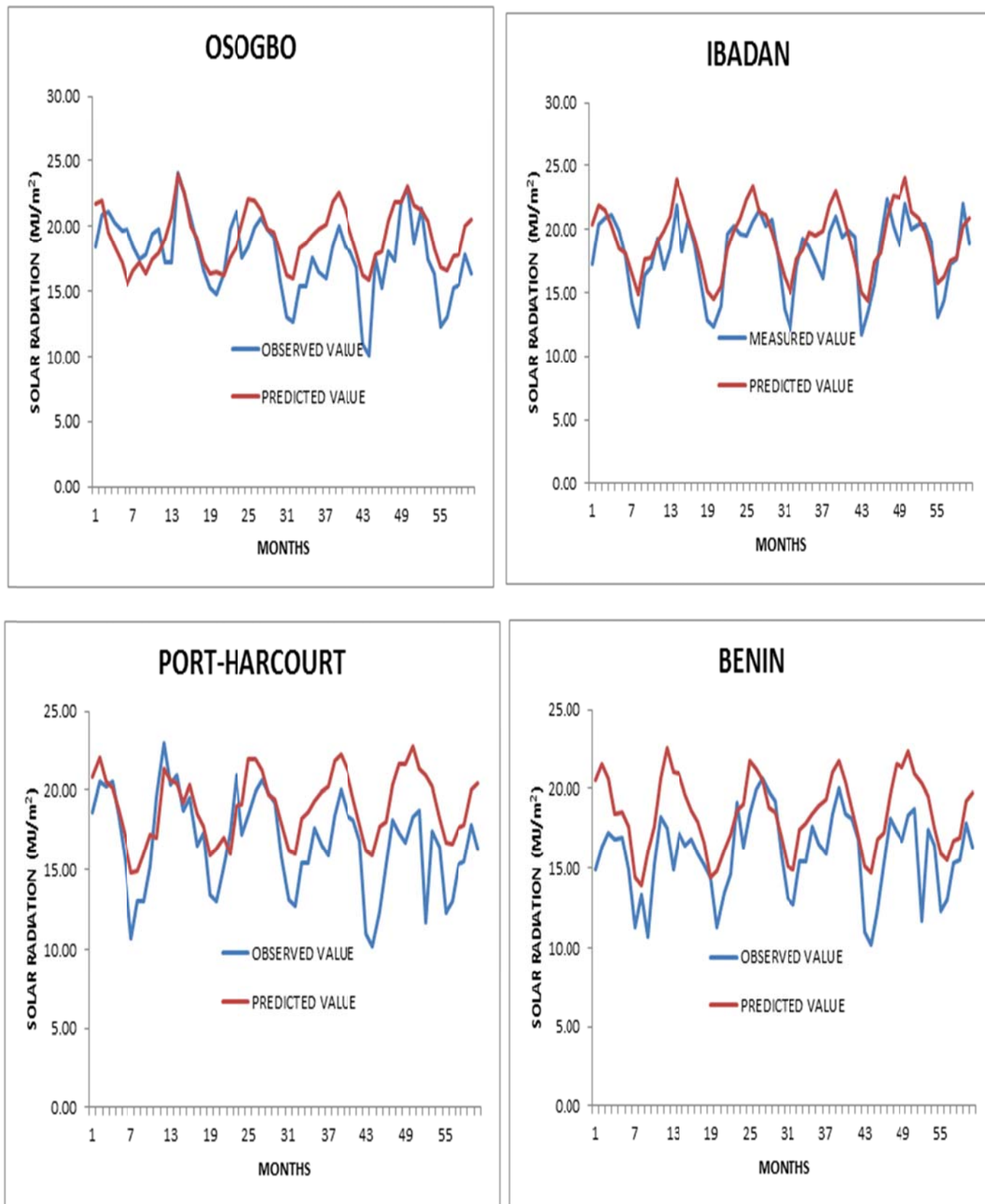


Figure 11. The variations of the predicted and measured values of the monthly solar radiation for Osogbo, Ibadan, Port-Harcourt and Benin Stations in Nigeria.

Conclusion

The distribution of solar radiation with meteorological variables for fourteen stations which represent climatic conditions in Nigeria shows that the linear combination of these variables can be used to predict solar radiation. Analysis have shown that strong correlation exists between solar radiation and all the meteorological parameters at the selected stations. The statistical indicators have also confirmed the efficiency of the developed models. For instance, the values of R^2 , MBE, RMSE and MPE for Sokoto, a Sahelian station are 0.5349,-0.0273,0.4154 and 0.0811 respectively, showing that the model for Sokoto is significant. Those for other stations have also been calculated with all showing high level of significance at 0.05 alpha levels. Therefore, the multivariate linear models developed in this research work for each of the fourteen stations is suitable for estimating solar radiation data in the stations and their proximate cities.

Conflict of Interests

The author(s) have not declared any conflict of interests.

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