The van Hiele levels of understanding of students entering senior high school in Ghana

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Abstract

This study was an attempt to measure the Van Hiele levels of geometric thought attained by SHS 1 students on entering Senior High School in Ghana. In all, 188 SHS Form 1 students from two schools were involved in this study. These students were given the Van Hiele Geometry Test adapted from the 'Cognitive Development and Achievement in Secondary School Geometry Test' items and an aptitude test, both in the fourth week of their entry to the SHS. The results showed that 59% of the students attained Van Hiele level 1. Out of 59%, 11% reached level 2 and only 1% reached level 3 by the theory. This indicates that the Van Hiele level of understanding of (i.e. over 90%) Ghanaian students before entering SHS is lower than that of their colleagues other countries.

Keywords   van Hiele levels, geometric thinking, secondary school geometry

Introduction

There has been a great deal of concern about the level of students' understanding of geometry in Ghanaian schools. In 2003, Ghana participated in Trends in International Mathematics and Science Study (TIMSS) in order to find out how the performance of her eighth graders (JSS2) in science and mathematics compared with those of other countries. The analysis of the Ghanaian students' performance in mathematics indicated that, Measurement, Geometry and Algebra were the candidates' weak content areas (Anamuah-Mensah et al, 2004).

In Ghana, mathematics is regarded as a cardinal factor in the nations' scientific and technological advancement because of its useful links to many other fields of human endeavour (Ministry of Environment, Science and Technology (MEST), 2009). Students' mathematical competencies have been closely linked to their levels of geometric understanding (Van Hiele, 1986; French, 2004). My study focuses on the geometric thinking levels of Ghanaian students in the context of the Ghanaian Curriculum. In Addition, the West African Examination Council (WAEC) Chief Examiners annual reports for the SSSCE & WASSCE from 2003 to 2006 observed that candidates were weak in Geometry of circles and 3-dimensional problems. According to their reports, most candidates avoided questions on 3-dimensional problems, where they attempted geometry questions; only few of the candidates showed a clear understanding of the problem in their working.

The teaching of high school geometry in many countries including Ghana was for a long period of time based on the formal axiomatic geometry that Euclid created over 2000 years ago (Van Hiele ,1999; French, 2004). In his era, Euclid's logical construction of geometry with its axioms, postulates, definitions, theorems, and proofs was, indeed, an admirable

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mathematical achievement (Van Hiele, 1999). However, Van Hiele (1999) expresses the view that school geometry that is presented in the traditional Euclidean fashion assumes that school children also think on a formal deductive level. Empirical evidence, however, indicates that this is not the case, as many students experience difficulty with geometry when it is presented in the Euclidean way (De Villiers, 1997; Hiele, 1999).

In response to many years of students' experiencing problems with Euclidean formal axiomatic geometry, many countries (e.g the U.S.A, the Netherlands and Russia) began to advocate reform in approaches to school geometry in their mathematics curriculum (Atebe, 2008). The changes that were implemented reflected, for most part, changes in teaching in the light of the research conducted in the late 1950s by two Dutch mathematics educators, Pierre Van Hiele and his wife, Dina Van Hiele-Geldof. The van Hiele's were experienced teachers in a Montessori secondary school in the Netherlands who noticed with disappointment the difficulties that their learners had with geometry, particularly in formal proofs. They therefore conducted research on thought and concept development among their school children. Their work was first reported in 1957 in companion doctoral dissertations at the University of Utrecht. The Van Hiele model identifies five sequential levels of thinking that learners pass through in geometry. According to the model, the learner, assisted by appropriate instructional experiences, passes through these levels in a hierarchical order, beginning with recognition of shapes as a whole (level 1), progressing to discovery of properties of shapes and informal reasoning about these shapes and their properties (levels 2 and 3), and culminating in a formal deductive and rigorous study of axiomatic geometry (levels 4 and 5) (Van Hiele, 1986; Fuys et al., 1988).

In the years since 1957, the Van Hiele model has motivated considerable research which has resulted in changes in geometry curricula in many developed countries. In Russia, for example, results from the Van Hiele's research have been applied to the school mathematics curriculum, producing appreciable improvement in students' understanding of school geometry (Hoffer, 1983; Fuys et al., 1988). In the U.S., three similar federally-funded investigations (the Oregon Project, the Brooklyn Project, and the Chicago Project) were conducted in 1979-1982 (Hoffer, 1983). The purpose of the Oregon Project was to investigate the extent to which the Van Hiele levels can serve as a model to access learners' understanding of geometry. The Brooklyn Project aimed at determining whether the Van Hiele model adequately describes how students learn geometry, and implemented four instructional modules that were detailed in accordance with the Van Hiele levels and phases (Fuys et al., 1988). In all these projects, the Van Hiele model proved to be a useful framework for accessing and unraveling students' difficulties with school geometry (Atebe & Schafer, 2008).

Despite the widespread application of the Van Hiele theory to improve mathematics curricula in many Western countries, only a few have utilized this model in an African context. My literature research indicates that there has been little investigation involving the Van Hiele model in Ghana. And as far as I have been able to ascertain, very few studies have applied the Van Hiele theory to determine the level of geometric conceptualization of Ghanaian high school students. Yet evidence abounds that many students in Ghana encounter severe difficulties with school geometry. In acknowledging the difficulties by Ghanaian students with geometry, and affirming the relevance of the Van Hiele model in ameliorating these difficulties, De Villiers (1997) for example, asserted that "unless we embark on a major revision of the primary school geometry curriculum along Van Hiele lines, it seems clear that no amount of effort at the secondary school will be successful".

Purpose of the study and research question

Personal experience had shown that the conditions available for Ghanaian students at the junior high school level does not allow them to explore geometric concepts and shapes informally prior to their high school course in geometry. Therefore, in order to inform any major revision of the junior high school curriculum, it would seem necessary first to determine the van Hiele geometric thinking levels of students entering senior high school. In this regard, the study sought to find out the stages of the Van Hiele levels of understanding Ghanaian students reach in the study of geometry before entering senior high school (SHS). In pursuance of this purpose, the following question was formulated to guide the study: - Which stages of Van Hiele levels of understanding do Ghanaian students reach in the study of geometry before entering senior high school (SHS)?

Methodology

The researcher used mainly the survey approach using test. The survey in this study was used for descriptive purposes. The researcher aimed at getting an accurate description of the Van Hiele level's geometric understanding reached by Senior High School Students. The population consists of students beginning Senior High School education in Ghana. The Winneba Senior High School and Zion Girls were sampled for the study because students in this municipality are posted from all over the ten regions of Ghana to these schools. This was a "convenience" sample. McMillan (2000) defined a convenience sample as one where a group of participants are selected because of availability. The students from the sample schools have studied mathematics in Ghana at the basic level and have all passed the Basic Education Certificate Examination (BECE) examination, which tests among other things their ability in Geometry.

 The sample comprised a total of 188 Senior High School learners with a mean age of 16 years. The ages of these learners ranged from 15 to 19, with majority of them between the ages of 15 and 16. Of the 188 learners, 62 were drawn from a private Senior High School in Winneba whilst the rest came from a government Senior High School in the Winneba Municipality. In the government Senior High School, 3 classes were chosen out of 9 form 1 classes. In order of Science, Business and Arts representing all the courses run by the school. This was done to ensure that all the major courses run by the school were represented. In the private School only one class was selected out of four classes due to the fewer number of streams.

Research instrument: Considering the nature of research questions being examined, the instruments used in the data collection was the Van Hiele Geometric Test (VHGT). The Van Hiele Geometric test items used in assessing Senior High School form one students' Van Hiele level was adapted from the Usikin (1982). The VHGT is designed to measure one's Van Hiele level in geometry. This is a well-known geometry test and it has been used in several Masters and PhD Dissertation (Hoffer, 1981; Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys et al., 1988) since it was developed. The test involves 15 item multiple-choice tests. The first five questions deal with identification, naming and comparing of geometric shapes such as triangles, squares and rectangles. The next set of five questions deal with recognizing and naming properties of geometric figures, whilst the last set deals with questions that require students to logically order the properties of figures previously identified, and begins to perceive the relationships between these properties.(Pegg, 1995).

Rubric for scoring the Van Hiele Geometric Test

First grading method: Each correct response to the 15-item multiple-choice test was assigned 1 point. Hence, each student's score ranged from 0 – 15 marks. The percentage score was calculated for each student and an item analysis of students' responses was done using SPSS.

Second grading method: The second method of grading the Van Hiele geometric test (Part B) was based on the "3 of 5 correct" success criterion suggested by Usiskin (1982,p33). By this criterion, if a student answered correctly at least 3 out of the 5 items in any of the 3 subtests within the Van Hiele Geometric Test, the student was considered to have mastered either levels 1,2 or 3. Using this grading system developed by Usiskin (1982), the learners were assigned weighted sum scores in the following manner:

- 1 point for meeting Van Hiele level 1 items 1 -5
- 2 points for meeting Van Hiele level 2 items 6 – 10
- 4 points for meeting Van Hiele level 3 items 11 – 15

Thus, the maximum point obtainable by any student was $1 + 2 + 4 = 7$ points. The method of calculating the weighted sum makes it possible for a person to determine upon which Van Hiele levels the criterion has been met from the weighted sum alone. For example, a score of 3 indicates that the learner met the criterion at level 1 and 2. The second grading system served the purpose of assigning the learners into various Van Hiele levels based on their responses.

Data Analysis: This study aims to determine the Van Hiele geometric thinking levels of the participating learners. Consistent with the practice and results of many earlier Van Hiele researchers (e.g. Usikin, 1982; Mayberry, 1983, Atebe, 2008) their research generated mainly quantitative numerical data in the form of the test from the participants. Therefore the use of Statistical Package for Social Sciences (SPSS) was applied for the analysis of data. The data for the VHGT and attitude test were coded and keyed into the SPSS for the statistical analysis. In addition, given that this study is a survey, I also employed descriptive data analysis in an attempt to understand, interpret and describe the experiences of the research participants in terms of their levels of geometric conceptualization. In specific terms, various descriptive statistics such as frequency distribution, charts, measures of central tendency, and correlation coefficients were used to analyse, describe and compare separate sets of quantitative data in this study.

Findings

Overall participants' performance in the VHGT

Table 4.1 presents the overall participants' performance on each item in the VHGT. As can be seen in the table each level had five items with four multiple choice options. For each item, the number in bold font represents the total number of students who answered that item correctly. In this section the participants' overall performance on the items in the three subtest are discussed

Table 1 van Hiele geometry test: item analysis for each level per school

Level 1	Choice items	1	2	3	4	5
	A	3	8	27	34	58
	B	**159**	2	1	**42**	13
	C	0	24	**140**	28	72
	D	24	**145**	3	60	6
	E	2	9	17	24	**40**

Level 2	Choice items	6	7	8	9	10
	A	19	40	**36**	21	29
	B	**17**	9	27	18	**34**
	C	108	27	46	**108**	46
	D	38	22	33	10	37
	E	6	**90**	46	31	42
Level 3	Choice items	11	12	13	14	15
	A	31	47	**15**	**17**	55
	B	33	**62**	14	16	**34**
	C	**40**	21	10	54	35
	D	21	17	4	18	17
	E	63	40	145	83	47

NB: The figures in bold represent the total number of students who answered that item correctly.

Performance on Subtest 1: Van Hiele level 1

The students performed well only in the first three items of subtest 1. The table 1 shows that 159 (85%), 145 (77%), 140 (75%) of the students managed to answer items 1, 2 and 3 in that order, compared to item 4 and 5, 42 (22%), and 40 (21%) which was not very encouraging. In Box 1 is a sample items from Subtest 1. The correct answer for this item in the Box is choice E. Table 4.1 shows that only 40 (21%) of the students in the subsample had this correct, that is, knew that all the given quadrilaterals can be referred to as parallelograms.
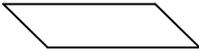

Item 5   Which of these are parallelograms?

a.   J only
b.   L only
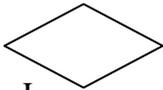c.   J and M only
d.   None of these are parallelograms
e.   All are parallelograms

Box 1 Sample item in Subtest 1

This shows lack of knowledge about 'class inclusion' in 149 (79%) of the students who participated in this research study.

Performance on Subtest 2: Van Hiele level 2

Students performed fairly well on items 7 and 9. Of the 188 students 91 (48%) and 108 (57%) respectively answered items 7 and 9 correctly, while 97 (52%), and 80 (43%) of the students were unable to answer the same items correctly. Students did not do well in items 6, 8 and 10. Of the 188 students only 17(9%), 36 (19%) and 34 (18%) of the students were able to answer questions on these items respectively.

Item 8 A <u>rhombus</u> is a 4- sided figure with all sides of the same length. Here are three examples.
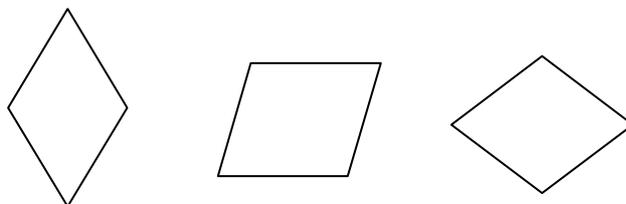
Which of (A) – (D) is not true in every rhombus?
  a. The two diagonals have the same length.
  b. Each diagonal bisects two angles of the rhombus.
  c. The two diagonals are perpendicular.
  d. The opposite angles have the same measure.
  e. All of (A) – (D) are true in every rhombus.

Box 2 Sample item in Subtest 2

In Box 2 is a sample items from Subtest 2. Of the given choices for the item in the box, choice E is the correct answer. Table 4.1 indicates that only 36 (19%) of the students who attempted this question answered the item correctly. This means that 81% of the students answered it wrongly. This reveals students' lack of knowledge about the properties of a rhombus.

Performance on Subtest 3: Van Hiele level 3

In general, the performance of the students for Subtest 3 was very poor. For items 11, 12, 13, 14 and 15, out of 188 students who took part in the test 40 (21%), 62 (33%), 15 (8%), 17 (9%) and 34 (18%) respectively answered the said item correctly. Subtest 3 is about students knowing the properties of given figures and using these to place figures with common properties in one class. Of the three items of the subtest, item 13 was extremely poorly attempted by students. This item is presented in the Box 3.
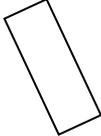
Item 13   Which of these can be called rectangles?

a.   All can.
b.   Q only
c.   R only
d.   P and Q only
e.   Q and R only

P   Q   R

Box 3 Sample item in subtest 3

In Box 3 correct choice is A. From Table 1 it can be seen that only 15 (8%) of students correctly answered the item. This situation means that 82% students did not know that rectangles have common properties with squares. This suggests that students have difficulties in understanding 'class inclusion'.

Van Hiele levels of understanding Ghanaian students reach in the study of geometry before entering Senior High School

The first question raised in this study was to find out the stages of the Van Hiele levels of understanding Ghanaian students reach in the study of geometry before entering Senior High School (SHS). In Figure 1, the students overall performance in the VHGT was presented in a bar chart.
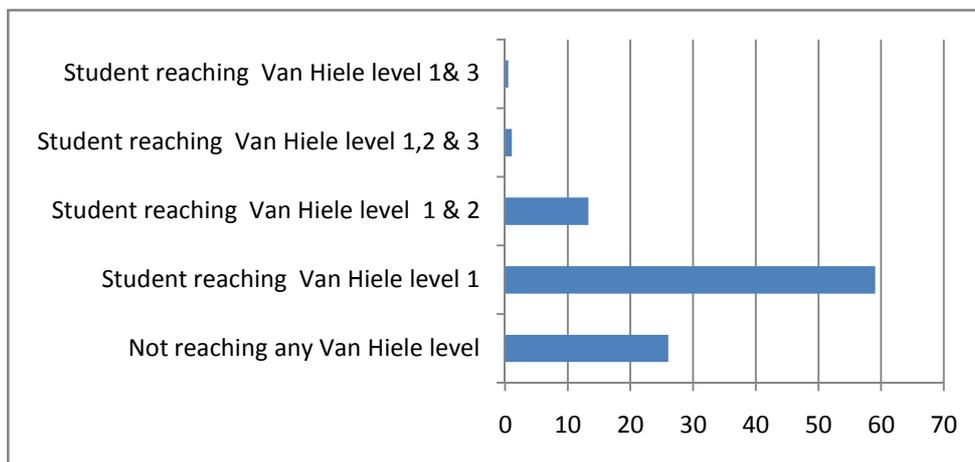
Figure 1   **A bar chart showing the overall performance in the VHGT**

As shown in Figure 1, 59% of the students attained Van Hiele level 1, 13% attained Van Hiele level 1 & 2. In addition, 1 % of the students attained Van Hiele levels 1, 2 & 3, whilst another 1 % of the students attained Van Hiele level 1 & 3. 26% of this sample did not attain any of the Van Hiele levels of intellectual development.

The Van Hiele Geometric Test (VHGT) was further analyzed by comparing the students' achievement in one level to other levels. Table 2 shows the cross-tabulation of students reaching Van Hiele level 2 by those reaching level 1.

Table 2 Analysis of the VHGT according to the van Hiele Levels (Levels 1 &2)

	Van Hiele level 2	
van Hiele level 1	Not reached	Reached
Not reached	95%	5%
Reached	81%	19%

From Table 2 it can be seen that fifty two (52) out fifty five (55) students representing 95% who did not reach Van Hiele level 1 did not also reach Van Hiele level 2. Whilst, three (3) students representing 5% of the students, who did not attain Van Hiele level 1 reached Van Hiele level 2. In addition, out of a sample of one hundred and thirty three (133) students. One hundred and eight (108) of this students, representing 81% of students who reached Van Hiele level 1 did not also reach level 2 of Van Hiele level of intellectual development. Whilst the remaining 25 representing 19% reached both Van Hiele level 1 and 2. This suggests that only 11% of the students reached level 2 by the theory (i.e. 19% of the 59% reaching level 1).

Table 3 shows the cross-tabulation of students reaching Van Hiele levels 3 by those reaching level 1.

Table 3 Analysis of the VHGT according to the van Hiele levels (levels 1 & 3)

	van Hiele level 3	
van Hiele level 1	Not reached	Reached
Not reached	93%	7%
Reached	97%	3%

Furthermore from Table 3, students who did not reach Van Hiele level 1 were fifty one (51) out of fifty five (55) representing 93% could not reach Van Hiele level 3. Only four (4) of this number who did not reached Van Hiele level 1 also reached Van Hiele level 3 representing 7% of this students. Also out of one hundred and thirty two (132) students who attained the Van Hiele level 1 of intellectual development, one hundred and twenty eight (128) representing 97% of the students could not attain Van Hiele level 3, but four (4) students representing 3% attained Van Hiele level 3.

Table 4 shows the cross-tabulation of students reaching Van Hiele levels 3 by those reaching level 2.

Table 4 Analysis of the VHGT according to the Van Hiele Levels (Levels 2 &3)

	van Hiele level 3	
van Hiele level 2	Not reached	Reached
Not reached	97%	3%
Reached	89%	11%

In addition, out of one hundred and fifty nine students (159) who took part in the VHGT test, one hundred and fifty four (154) of these students who did not reach Van Hiele level 2 also did not reach Van Hiele level 3 representing 97% of the student. Interestingly the remaining five (5) students representing 3% of the students who did not reach Van Hiele level 2 reached Van Hiele level 3. Finally out of twenty eight (28) students who took the test, twenty five (25) of them representing 89% of the students who reached Van Hiele level 2 did not reach Van Hiele level 3, the remaining three (3) students reached both level's 2 & 3 representing 11

% of the students who took part in the test. These results therefore show that 59% of the students attained Van Hiele level 1 and from this proportion of students, 11% reached level 2 and only 1% reached level 3 by the theory

Discussion on Findings

The results of VGHT show that 59% of the students attained Van Hiele level 1. From this proportion of students, 11% reached level 2 and only 1% reached level 3 by the theory. This indicates the stage of the Van Hiele level of understanding reached by most (i.e. over 90%) Ghanaian students before entering SHS is lower than what most students at this stage (or age) reach in other countries in the study of geometry.

It clear from the results that 59% of the students attained Van Hiele level 1. From this proportion of students, 11% reached level 2 and only 1% reached level 3 by the theory, whilst 29% did not attain any of the levels suggesting that most students were at pre-recognition level of Van Hiele level 1 before beginning senior high school geometry. These findings concur with those of the previous research studies (Usiskin, 1982; Burger & Shaughnessy, 1986; Senk, 1989; Pusey, 2003; Siyepu, 2005; Atebe & Schäfer, 2008). The findings of the studies mentioned here, indicated that the majority of their students were found to be operating at the pre-recognition level, and that a very small number of students operated at Van Hiele levels 2. This is problematic, since in Ghana, level 3 skills are required to successfully begin senior high school geometry. Teaching and learning in geometry is mainly focused on Van Hiele levels 1 & 2, with a small amount of geometry work being done at level 3. Since most students only operate at the pre-recognition level, level 1 and level 2, it is quite clear that many students will be unsuccessful in doing high school geometry.

Conclusion

This study was an attempt to measure the Van Hiele levels of geometric thought among SHS 1 students in Ghana. It specifically sought to find out the stages of the Van Hiele levels of understanding Ghanaian students reach in the study of geometry before entering Senior High School (SHS). In all, 188 SHS Form 1 students from two schools were involved in this study. These students were given the Van Hiele Geometry Test (VHGT) adapted from the 'Cognitive Development and Achievement in Secondary School Geometry Test' items. The results show that the stage of the Van Hiele level of understanding reached by most (i.e. over 90%) Ghanaian students before entering SHS is lower than what most students at this stage (or age) reach in other countries in the study of geometry.

A major caveat to the interpretation of the results of the study however was the use of only two schools SHS in the Winneba Municipality in the country for the sample. Though this was compensated for by the strategic location of the schools to attract students from several regions of the country, it is still difficult to generalize the findings for the whole country.

Recommendations

The following recommendations are made for the improvement of the junior high school curriculum:

1. The teaching and learning of geometry should involve more hands-on activities that will actively engage the students. This will enhance students' conceptual understanding of geometric concepts. When teaching about geometric concepts, teachers should ensure that students understand and know the properties of all geometric shapes. By knowing the properties of the geometric shapes, students will be able to establish class inclusion, which according to this study is sorely lacking. Students can only recognize, describe and distinguish geometric shapes from each other by knowing their properties.

2. When teaching about geometric shapes and concepts, teachers should ensure that the proper geometric terminologies are used by both the teachers and students. This will address language barriers in students who use English as a second language. This involves correct spelling of the concepts, proper pronunciations and using the correct names of the geometric shapes.

References

Anamuah-Mensah, J., & Mereku, D. K. (2005). Ghanaian JSS 2 Students' Abysmal Mathematics Achievement in TIMSS-2003; A Consequence of the Basic School Mathematics Curriculum. *Mathematics Connection Volume 5* , 1-13.

Anamuah-Mensah, J., Mereku, D. K., & Asabere-Ameyaw, A. (2004). *Ghanaian Junior Secondary School Students' Achievement in Mathematics and Science: Results from Ghana's participation in the 2003 Trends in International Mathematics and Science study.* Accra: Ministry of Education Youth and Sports.

Atebe, H. U. (2008). *Students' Van Hiele levels of Geometric Thought and Conception in Plane Geometry: A collective case study of Nigeria and South Africa.* Unpublished (Ph.D)Thesis.

Atebe, H. U., & Schafer, M. (2008). Van Hiele levels of geometric thinking of Nigerian and South African mathematics learners. In M. C. Polaki, T. Mokulu, & T. Nyabanyala (Eds.), *Proceedings of the 16th Annual Conference of the Southern Africa Association for Research in Mathematics,Science and Technology.* Maseru: SAARMSTE.

Burger, E. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17(1)* , 31–48.

De Villiers, M. D. (1997). The future of secondary school geometry. *Pythagoras,44* , 37-54.

French, D. (2004). *Teaching and learning geometry.* London: Continuum.

Fuys, D., Geddes, D., & Tischler, R. (1988). The Van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education Monograph,3.Reston:NCTM* .

Hoffer, A. (1983). Acquistion of mathematics concepts and processes. In A. Hoffer, *Van Hiele-based research. In R. Lesh & M. Landau (Eds)* (pp. 205-227). New York: Academic Press.

Hoffer, A. (1983). Van Hiele based research. In R. Lesh, & M. Landau (Eds.), *Acquistion of mathematics concepts and processes* (pp. 205-207). New York: Academic Press.

Hoffer, A. (1988). Geometry and visual thinking. In T. R. Post (Eds.), *Teaching mathematics in grades K-8: Research based methods* (pp. 233-261). Newton,MA: Allyn & Bacon.

Mayberry, J. (1983). The Van Hiele levels of geometric thought in undergraduate pre-service teachers. *Journal for Research in Mathematics Education, 14(1)* , 58-69.

Ministry of Environment Science and Technology(MEST). (2009, December 11). *Ghana Government Official Portal.* Retrieved June 11, 2010, from Ghana Government Official Web site: http://www.ghana.gov.gh

Pegg, J. (1995). Learning and teaching geometry. In L. Grimison, & Pegg. L (Eds.), *Teaching secondary school mathematics* (pp. 87-103). London: Harcourt Brace.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry:Cognitive development and achievement in secondary school geometry project.* Chicago: University of Chicago Press.

Van Hiele, P. (1999). *Developing geometric thinking through activities that begin with play. Teaching Children Mathemtics.*

Van Hiele, P. (1986). *Structure and insight: A theory of mathematics education.* Orlando: Academic Press.

West Africa Examination Council. (2003). *Senior Secondary School Certificate Examination, Chief Examiners' Report.* Accra: West African Examination Council.

West Africa Examination Council. (2004). *Senior Secondary School Certificate Examination, Chief Examiners' Report.* Accra: West African Examination Council.

West Africa Examination Council. (2005). *Senior Secondary School Certificate Examination, Chief Examiners' Report.* Accra: West African Examination Council.

West Africa Examination Council. (2006). *Senior Secondary School Certificate Examination, Chief Examiners' Report.* Accra: West African Examination Council.