# Students' Understanding of the Equal Sign when Solving Arithmetic Tasks

Forster D. Ntow[1], George Adom[2], Solomon Essel[3], & Samuel Kenney[4]

**Abstract**

Although the equal sign is considered an important symbol because it represents a fundamental concept in the structure of any algebraic equation, studies have indicated that many students do not possess a sufficient understanding of the three different meanings of this symbol. This study aimed to investigate Grades 6 to 8 basic school students' achievement, and their interpretation of the equal sign, when solving arithmetic tasks. The study adopted a descriptive cross-sectional survey design. A purposive sampling technique was used in selecting a sample of 311 students, comprising 92 sixth, 107 seventh, and 112 eighth graders. Data were collected using an achievement test and a semi-structured interview guide. The achievement test results were analysed and reported using descriptive statistics and a bar graph. The interview data on the other hand were analysed as themes and reported in the form of narratives and excerpts of students' work. The results revealed that students in the study exhibited a dominant understanding of the equal sign as an operation. Also, their achievement depended on the type of task and the operation to be performed. These findings suggest that a broader understanding of the equal sign should be taught to ensure a smoother transition to their learning of algebra-related concepts such as linear equations in the future.

Keywords equal signs; arithmetic tasks; basic mathematics operations; equivalence; identity

## Background

Literature suggests that students struggle with five algebraic concepts and the equal sign is one (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). The equal sign has two meanings: operational and relational (Knuth et al., 2008). Students with an operational understanding of an equal sign perceive it as a command *to do something.* On the other hand, the relational meaning of the equal sign entails realizing that it conveys the idea that the two sides of an equation have the same value and that this value can therefore be substituted for one another. However, a relational understanding of equal signs can be conceived of in two ways (Stephens et al., 2013). In other words, relational understanding may represent an *identity* relationship between objects (Mirin, 2019) or *equivalence* (Matthews et al., 2012).

There is broad agreement among researchers that a relational understanding of the equal sign is necessary for a conceptually developed understanding of the equal sign (Byrd et al., 2015; Matthews et al., 2012; Rittle-Johnson et al., 2011), and that the understanding of the equal sign as a representation between equal sets "opens up the power of algebra for representing

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problems and performing complex operations on mathematical expressions" (Carpenter et al., 2003, p. 21). As such students' ability to recognize the equal sign as equivalence is a requirement for mastering higher-level algebra and a crucial ability for students' mathematical development (Alibali, Knuth, Hattikudur, McNeil & Stephens, 2007).

It is argued that when teachers and learners focus only on the operational meaning of the equal sign, it affects the mathematics achievement of learners in subsequent grade levels (Özdemir, 2022). This is because the relational meaning of the equal sign is crucial and essential for algebraic equations and other higher-level concepts (Opsal & Topphol, 2023; Matthews et al., 2012). Therefore, students understanding of the equal sign is deemed to be well-developed if they have a relational understanding (Matthews et al., 2012).

However, studies indicate that students' relational understanding of equal signs are less prevalent than their operational knowledge (Özdemir, 2022; Baiduri, 2015). The study of Baiduri shows that students prefer to adopt operational processes by making solutions, comparisons, and replacements rather than paying attention to relationships in equivalence. Eichhorn, Lindsey, and Rombacher (2018) examined how well students in the first grade understood non-standard equations and the equal sign. The authors reported that equations with an equal sign appearing immediately before the solution (e.g., $a + b = c$), are routinely given to students in textbooks. Their research examined how well students performed on a number sense screener and the Early Grade Mathematics Assessment (EGMA) when solving non-standard problems. According to the survey, only 46% of students correctly recognized missing addends in the non-standard form, $18 = \square + \square$, whereas 62% of students correctly identified 2 addends in the missing addend question, $\square + \square = 7$. They held the opinion that such exposure could result in a variety of misunderstandings, such as the notion that the equal sign denotes 'do something' or 'the solution'.

Moreover, McNeil and Alibali (2004) determined fourth-grade children's errors, such as adding all and adding to the equal sign for both operation-side problems, such as $3 + 4 = \_\_\_ + 5$. Students did not perform well on tasks involving non-standard equations, with the unknown as the final part of the arithmetic task (e.g., $3 + 4 + 5 = 3 + \_\_\_$). In another study by McNeil (2007), children aged 7, 9, and 11 years were assessed using non-standard operation-both-side equations. Children aged 7 and 11 years performed better, but not well, in solving non-standard equations than 9-year-old children. This indicates a variation in performance during the elementary grades. This differential performance based on arithmetic skills indicates that elementary school children may perform better on standard equations than on non-standard equations.

The above studies reveal that students have major misunderstandings about the equal sign (Özdemir, 2022; Eichhorn et al., 2018; Baiduri, 2015; Matthews et al., 2012). This tends to affect their understanding and performance in arithmetic and algebra (Byrd, et al., 2015; Booth & Davenport, 2013). This problem may not be alien to students in Ghana. Generally, there have been concerns about Ghanaian students' performance in mathematics, as evidenced by both national and international large-scale assessment results, such as the National Education Assessment (NEA), Early Grade Mathematics Achievement (EGMA), and Trends in International Mathematics and Science Studies (TIMSS). Researchers such as Davis, Ntow, and Beccles (2022) and Adu, Assuah, and Asiedu-Addo (2015) have highlighted concerns about the mathematics achievements of Ghanaian students at the pre-tertiary levels of education. For example, in a study conducted by Adu, Assuah, and Asiedu-Addo (2015) on "students' errors in solving linear equation word problems," the findings revealed that 84% of students could not get the correct answer because of processing skill errors. Further evidence of Ghanaian students' underachievement in mathematics came from Mills and Mereku (2016). Their study investigated Junior High School students' performance on the National Minimum Standards (NMS) and reported that approximately 70% of the participants performed below minimum proficiency in mathematics.

In basic schools, concerns about Ghanaian students' lack of proficiency persist (e.g., Ntow, 2022). For example, the 2016 National Education Assessment (NEA), a nationwide examination conducted in literacy and mathematics, reported that in the operation subdomain, only 44% and 46% of the basic grades 4 and 6 learners, respectively, were deemed to be proficient. Based on the literature reviewed, we argue that Ghanaian students' lack of proficiency in mathematics may be attributed to their interpretation of the equal sign that underpins all the operations they perform at either the primary or junior high school levels. Additionally, considering that concerns about students' understanding of the equal sign persist into advanced grade levels, it is necessary to investigate whether this holds in Ghana. Studies have focused on Ghanaian students' fractional knowledge (Ntow, 2022) and linear equations (for example, Adu, Assuah & Asiedu-Addo, 2015), we have not come across any study that has specifically examined Ghanaian students' interpretation of the equal sign despite the reported challenges faced by students globally in interpreting this sign. Such an investigation will shed light on students' difficulties in performing basic operations involving the equal sign. Therefore, this study investigated Ghanaian basic school students' achievement in arithmetic tasks and whether their interpretation of the equal sign differed across grade levels. The study was guided by the following research questions (RQs):

RQ1 What are the students' achievements in arithmetic tasks (i.e., standard and non-standard) involving the use of equal signs?

RQ2 How do students interpret equal signs when presented with arithmetic tasks?

## Theoretical framework

The study that underpinned our research drew on Skemp's (1971, 1976) work on the nature of understanding. Skemp's theoretical framework posits a fundamental aspect of subjectivity inherent in the comprehension process as it entails the assimilation of knowledge within appropriate cognitive structures (Skemp, 1971). Theorists discern two distinct forms of understanding in the domain of mathematics: relational and instrumental. Relational understanding encompasses a profound and adaptable grasp of mathematical concepts that surpasses the application of formulas or procedures to obtain solutions. Conversely, instrumental understanding refers to the application of mathematical rules without understanding or inability to explain why a particular rule works (Skemp, 1971, 1976). In this study, a student is said to possess relational understanding when the three different meanings of the equal sign (that is, operation, identity, and equivalence) are applied appropriately. Instrumental understanding, however, implies the misapplication of different meanings when solving a given arithmetic task.

## Methods

### *Research Design*

This study used a descriptive cross-sectional survey. Descriptive studies are not only restricted to factual findings but may also lead to the formulation of important principles of knowledge and solutions to significant problems (Kombo & Tromp, 2006). This study used a descriptive design because the researchers intended to present how students understood and interpreted equal signs when solving an arithmetic task. It is also cross-sectional because we sampled students across three grades, namely, grades 6, 7, and 8.

*Sample and Sampling procedure*

Five schools in Assin Central Municipality in the Central Region of Ghana were randomly selected for this study. Students in basic six (grade 6), Junior High School 1 (grade 7), and Junior High School 2 (grade 8) of the selected schools were purposively selected because grade 6 marks the end of primary school; hence, a transition from arithmetic to algebra. Grades 7 and 8 were selected to ascertain whether the algebra learned at this level influenced their understanding of the equal sign. The census method was then used to select students from various grades. The sample comprised 311 students, of which 92 were in grade six, 107 in grade seven, and 112 in grade eight.

*Data Collection Instruments*

The main instruments used for the data collection were tests and semi-structured interviews. The researchers also developed a semi-structured interview that was used to follow up on the participants' responses to gain an in-depth understanding of the thinking behind their responses. Interview participants were selected when there appeared to be a trend in their responses to an item that may lead to a misunderstanding of the equal sign.

The test instrument comprised 30 items that were adopted from Powell et al. (2016) and was used to measure students' achievement in various types of standard and non-standard equations. The researchers classified all standard equations that required students to interpret the equal sign as a command to do something under the "equal sign as an operation." For example, $7 + 3 = \_\_\_$. Items classified as "equal sign as an identity" were all items that required students to interpret the equal sign as balancing or repeating the number on one side of the equal sign on the other side. For example, $4 = \_\_\_$. However, all the non-standard equations were identified as "equal signs as equivalence." Some items required students to operate (either addition or subtraction) on the right side of the equal sign and to provide results in a blank space. For example, $2 = 7 - \_\_\_$ and

$9 = \_\_\_ + 4$. Other non-standard equations also require students to operate (either addition or subtraction) on both sides of the equal sign before equivalence is attained.

For example, $\_\_ - 3 = 8 - 2$ and $3 + 5 = 4 + \_\_\_$.

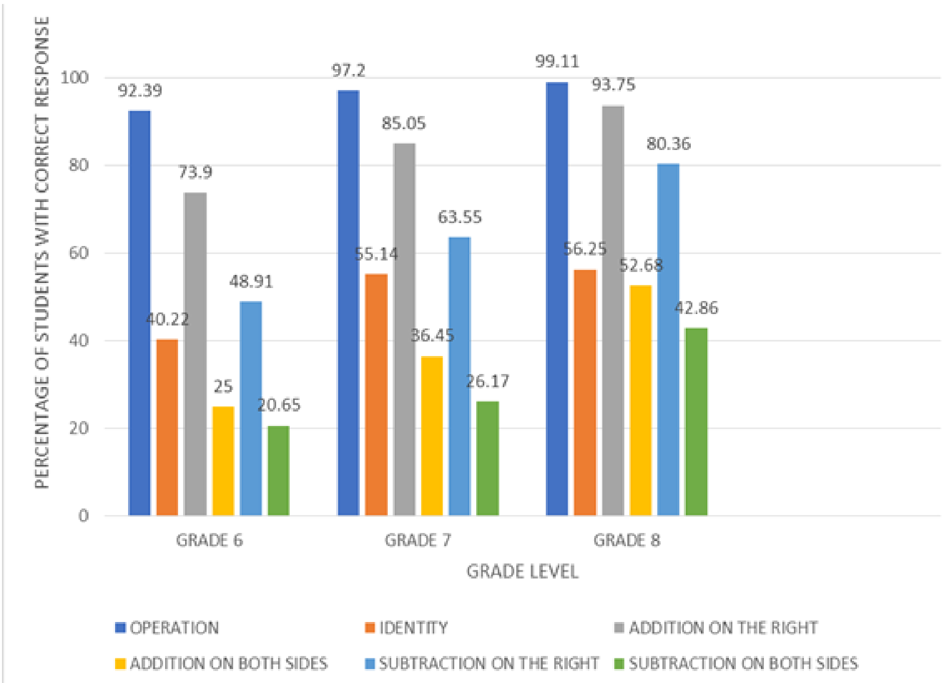
*Data Analyses*

The first research question sought to determine the achievement of students in arithmetic tasks involving the use of an equal sign based on three different types of interpretation (operation, identity, and equivalence). Each student's response to each item was correctly or incorrectly scored. The data were then analysed using descriptive statistics and presented using a bar graph. In answering the second research question, 'How do students interpret the equal sign when presented with arithmetic tasks?' The researchers conducted a follow-up interview to seek in-depth knowledge of how students at different grade levels interpreted equal signs when presented with arithmetic tasks. The data collected from students were first sorted into piles of Correct and Incorrect responses. Following this, the responses were coded and organized into themes using the three meanings of equal signs. Additionally, the excerpts of students' work are presented in the form of narratives.

*Ethical Clearance*

We sought ethical clearance from the relevant educational and school authorities in the school district where the study was conducted. That is, the school authorities and the students were informed about the rationale for the study and the potential benefits of the findings to them. While in the case of the students, they assented to be part of the study, the school leadership provided consent for the school to be part of the study.

**Fig.1 Proportion of students' correct response in each of the item types**




All the participants were assured of the confidentiality of their responses. Also, unique identifiers known only to the third and fourth researchers were used so that no other individual could match any response to any of the study's participants and the data were stored securely. Additionally, the participants were reminded that their participation in the study was voluntary and could be withdrawn at any point. Also, the data collected were stored in a safe place with only the Principal Investigator having access to the keys.

## Results

The researchers assessed the students' achievement on arithmetic tasks involving the use of equal signs. Three representations of equal signs were explored: equal signs as an operation, identity, and equivalence. We present the results for the research questions that underpin this study.

*What is students' achievement in arithmetic tasks involving the use of equal signs? (RQ1)*

The results of students' achievement in arithmetic tasks that required the interpretation of the equal sign as an operation (a command to do something, for example, $4 + 5 =$ ___), identity (for example, $7 =$ ___), and equivalence. Non-standard equations were classified under equal signs as equivalent equations. Four types of data are used in this study. Some items required students to operate (either addition or subtraction) on the right side of the equal sign and provide results in a blank space. For example, $2 = 7 -$ ___ and $9 =$ ____ $+ 4$. Other non-standard equations also require that students operate (either addition or subtraction) on both sides of the equal sign before equivalence is attained. For example, ___ $- 3 = 8 - 2$ and $3 + 5 = 4 +$ ___. A summary of students' achievements in arithmetic tasks involving the use of equal signs is presented in Fig. 1.

The results revealed that the majority of students across all three grades correctly answered items that required the interpretation of the equal sign as an operation. Notably, at least 92% of the students in all three classes (grades) correctly answered questions on equal signs. This indicates students' strong understanding of the equal sign as a command to do something. However, there was a sharp decline in students' performance on arithmetic tasks that required the interpretation of the equal sign as an identity. For instance, whereas less than 8% of the students across all three grade levels gave incorrect answers on the equal sign as an operation task, this percentage increased sharply to 59.78%, 44.86%, and 43.75% in grades 6, 7, and 8, respectively.

Regarding students' achievement of tasks assessing them on the equal sign as equivalent, the results were mixed. Students performed better on arithmetic tasks that required them to perform (addition or subtraction) on the right side. For example, $2 = 7 - \_\_\_$ and $9 = \_\_\_\_ + 4$. In contrast, they performed relatively poorly on tasks that required operations (addition or subtraction) on both sides of the equal sign. For example, $\_\_\_\_ - 3 = 8 - 2$ and $3 + 5 = 4 + \_\_\_$. It was also observed that the type of operation contributed to the level of achievement. For example, 75%, 63.55%, and 47.32% of students in Grades 6, 7, and 8, respectively, provided incorrect responses to items that required equivalence (addition on both sides). The situation worsened when the participants were required to perform tasks involving subtraction on both sides of the equal sign. This is because Fig. 1 shows that the percentage of incorrect responses increased to 79.35%, 73.83%, and 57.14% for grades 6, 7, and 8, respectively.

*How do students interpret equal signs when presented with arithmetic tasks? (RQ2)*

This section focuses on how students interpreted the equal sign as an operation, identity, or equivalence. A semi-structured interview was conducted to determine students' reasoning and how it influenced their interpretation of the equal sign when arithmetic tasks were presented. Here, the interpretation of the equal sign is categorized into operation, identity, and equivalence. We begin with the results of students' interpretation of the equal sign as an operation.

Interpretation of the equal sign as an operation

This section focuses on the findings of the student's interpretation of the equal sign as an operation. This is the interpretation of an equal sign as the command to do something. For example, $2 + 8 = \_\_\_$. Consistent with the high achievement recorded in these types of tasks, the majority of participants (n = 300, 96.46%) correctly interpreted the equal sign as an operation.

*Interpretation of the equal sign as an identity*

This section focuses on findings regarding students' interpretations of an equal sign as an identity. For example, $\_\_\_ = 7$. An indication of the equal sign as an identity would be that these students wrote 4 and 7 as answers to questions 3 and 26, respectively. A slim majority of students, 159 (51.12%) out of 306, correctly interpreted the equal sign as an identity. Of the 147 respondents who provided incorrect responses, 96 interpreted the equal sign as an operation, further highlighting the dominant understanding of the equal sign.

Our analysis revealed two forms of incorrect interpretation of the equal sign as an identity: (1) identifying two numbers that, when operated on, give the number indicated in the question. (2) Something wrong with the question. We present excerpts of students' work for each interpretation. Fig. 2 shows an excerpt from the student's responses.

**Fig.2 Identifying two numbers which add to give the number indicated in the question.**

Grade 6 | Grade 7 | Grade 8

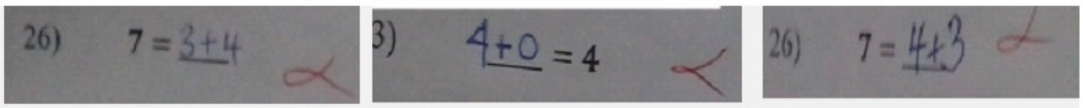



Fig. 2a | Fig. 2b | Fig. 2c

We provide an illustrative example during a follow-up interview between one of the research assistants and a grade 6 student named Patrick (pseudonym).

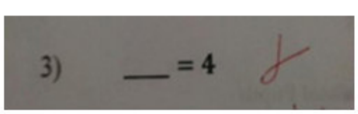
Assistant 2: "Why did you answer this question?" (Refer to Fig. 2a).

Patrick: "*They forgot to bring the operation so I wrote it for them.*"

Like several other students who got these tasks wrong, Patrick had to introduce mathematical operations and statements in an attempt to arrive at an answer. Patrick's reasoning reveals that he still sees the equal sign as acting to arrive at the answer, say 7.

However, a small minority of four students in grade 6 and one in grade 7 (1.61%) did not respond to this item. During the interviews, these students indicated that there was something wrong with the questions, hence, the no response. We present an illustrative example of a follow-up interview between one of the research assistants and a Grade 6 student named Agnes (pseudonym). See Fig. 3.

***Fig.3*: *Interpreting the task as wrongly constructed***




A follow-up interview revealed the following.

Assistant 1: Why did you not answer this question? (Figure 3)

Agnes: *The question is incorrect.*

Assistant 1: How wrong is it?

Agnes: *There is no addition or subtraction sign, so I cannot do so.*

The conversation revealed Agnes' struggle to answer questions requiring the interpretation of the equal sign as an identity, possibly because there was only one number and no operation.

*Interpretation of the equal sign as equivalence*

Non-standard equations were classified under equal signs as equivalent. Some items required students to operate (either addition or subtraction) on the right side of the equal sign and provide the results in a blank space. For example, $2 = 7 - \_\_\_$ and $9 = \_\_ + 4$. Other non-standard equations also require that students operate (either addition or subtraction) on both sides of the equal sign before equivalence is attained. For example, $\_\_\_ - 3 = 8 - 2$ and $3 + 5 = 4 + \_\_\_$.

We begin with students' interpretation of the equal sign as equivalent on tasks involving addition and subtraction.

*Interpretation of the equal sign as an equivalence (Addition and subtraction on the right side of the equal sign)*

In these types of tasks, students are expected to interpret the equal sign as conveying the idea that the two sides of an equation should have the same answer. Therefore, the task is to find a value that will give both sides the same answer. Therefore, a correct interpretation of equal signs in this context requires relational understanding. The students' performance across the three grades was good. The percentages of correct answers for the addition tasks in grades six, seven, and eight were 73.90%, 85.05%, and 93.75%, respectively. For the subtraction tasks, the following percentages of correct answers were recorded for the three grades: 48.91%, 63.55%, and 80.36% respectively (see Fig. 1).

**Fig. 6 Excerpts of student work involving subtraction on the right side of the equal sign.**

Grade 6 Grade 7 Grade 8

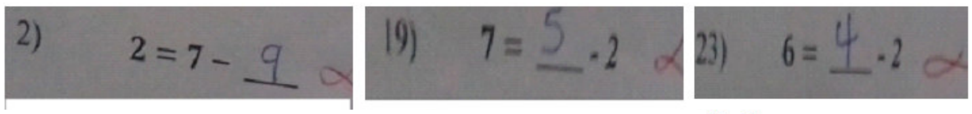



Fig.6c

**Fig.5. Students' conditioning to standard equations**


Concerning the addition tasks, an analysis of the incorrect responses and interview data revealed a common trend in the way those who gave wrong answers interpreted the equal sign. For this group of students, they simply added the given numbers and wrote the sum obtained as the answer instead of finding a value which will help 'balance' both sides of the equal sign. See Fig. 4.

Ken's reasoning was prevalent among almost all students who responded incorrectly to arithmetic tasks under this construct.

*Ken's reasoning was prevalent among almost all students who responded incorrectly to arithmetic tasks under this construct.*

Interestingly, two students in Grade 6 decided to introduce another equal sign on the right side of the operation. For example, in Fig. 5, Sarah and Fred (both in Grade 6) neglect the space provided to them and introduce another equal sign. Although this is seen among only two students in grade 6, it appears that their minds are conditioned by the idea that the equal sign must always precede the final answer ($a + b = c$).

An inquiry into the reasons revealed the following.

Assistant 1: Why do you write this answer?
Sarah: *because the question is wrong.*

**Fig.4 Misinterpretation of the equal sign as adding all numbers instead of as an equivalence**

Grade 6 Grade 7 Grade 8

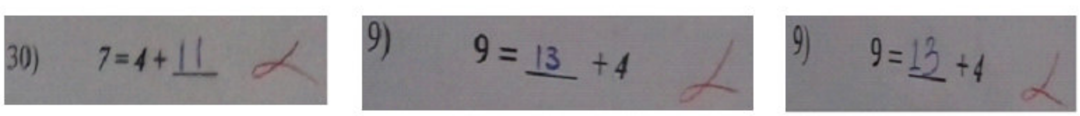



Fig. 4a Fig. 4b Fig. 4c

A follow-up interview reveals the following.

**Assistant 1**: How did you arrive at your answer? (Fig. 4b)
**Ken:** *I have added all of the numbers and wrote the answers (blank space).*

Assistant 1: *how is it wrong?*
Sarah: *the equal sign must come after the* $\mathbf{2 + 6}$.

Concerning the subtraction tasks, the analysis revealed two ways in which the students misinterpreted the equal sign as equivalent. Students either added the given digits and wrote the answer in the space provided, or subtracted digits on opposite sides of the equal sign. See Fig. 6.

A follow-up interview reveals the following.

Assistant 2: How did you arrive at your answer? (Fig. 6a)

John: *I added two to seven and wrote the answers here.*

Assistant 2: The operation is subtraction. Why did you add?

John: *It is not possible to subtract 7 from 2 so I added.*

*Interpretation of the equal sign as an equivalence (Addition and subtraction on both sides of the equal sign)*

Regarding these types of tasks, students were expected to interpret the equal sign as conveying the idea that the two sides of an equation should have the same answer. Therefore, the task is to first find either the sum or the difference on one side of the equal sign, and then determine the numeral that should either be added or subtracted to balance the equation. As such, these types of tasks require a relational understanding of equal signs.

**Fig.7 Excerpts of student work involving addition on both sides of the equal sign.**

Grade 6 | Grade 7 | Grade 8

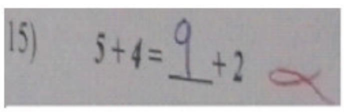



*Fig.7a*

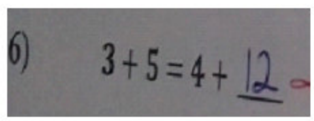



*Fig.7b*

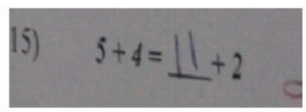



*Fig.7c*

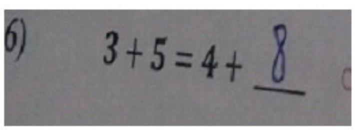



*Fig.7d*

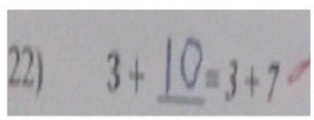



*Fig.7e*

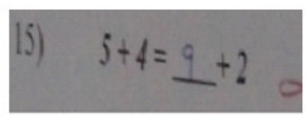



*Fig.7f*

Assistant 2: But the task is $2 = 7 - \_\_$.

John: *Yes, that is what I have done. Thus, nine minus seven is now two.*

An excerpt of the second type of misinterpretation is as follows.

Assistant 2: How did you arrive at your answer? (Fig. 6c)

Mary: *I subtracted two from six. The answer is 4 so I wrote it here (blank space)*

Assistant 1: How did you arrive at your answer? (Fig. 6b)

Ken: *I subtracted two from seven. That is five.*

In the next subsection, we present the results of the students' interpretation of the equal sign as equivalence involving addition and subtraction on both sides.

Students' performance across the three grades was quite low compared to the other types of tasks. The percentages of correct answers for the addition tasks in grades six, seven, and eight were 25.00%, 36.45%, and 52.68%, respectively (see Fig. 1). Apart from the students' computational errors that resulted in incorrect responses, a large number added all the given digits and wrote the sum in the space provided in the case of the addition tasks. Others added only two of the given digits and wrote the sum in the provided space. See Fig. 7 for excerpts from the students' work

A follow-up interview further revealed participants' misapplication of the equal sign. For example, in Fig. 7, Mabel, a grade seven student, explained her reasoning as follows.

Assistant 1: How did you arrive at this answer? (Fig. 7b)

Mabel: *The question says we should add. Therefore, I have added all of the numbers and wrote the answers in a blank space.*

An example of how some students added some of the given numbers is provided in the following narrative.

Assistant 1: How did you arrive at your answer? (Fig. 7f)

Caleb: *We have added the numbers on the left side of the equal sign and wrote the answer in space.*

The results from the students' performance on subtraction on both sides of the equal sign were the lowest compared to all the other types of tasks they were presented with. The percentages of correct answers were recorded for three grades: 20.65%, 26.17%, and 42.86% (Fig. 1).

The results indicated that at least 57% of students across all grades were unable to complete the given tasks. Those who provided incorrect responses either subtracted the digits on the left of the equal sign and wrote the difference in the space provided, or subtracted all the digits on both sides of the equal sign and wrote the result in the space provided. See Fig. 8 for excerpts of students' work.

**Fig.8: Excerpts of student work involving su**

Grade 6 Grade 7

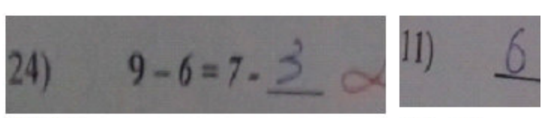



*Fig.8a* ***Fig.8b***

For example, in Fig. 8a, the student subtracted the values on the left side of the equal sign and wrote the value in the space provided. A follow-up interview revealed the following.

Assistant 2: Explain how you received your answers. (Fig. 8a)

Robert *Nine minus six is three, so the answer is here (pointing to a blank space).*

Assistant 2: What is about seven on the right-hand side of the equal sign?

Robert: *Yes, we would also subtract three from it to obtain four. However, there is no equal sign or space to write that one, so I have left it.*

It is evident from the analysis of the interview data that students predominantly interpreted the sign as a command to "provide an answer" or "do something." This finding reveals the students' over-generalization of the equal sign as an operation. As such, the notion of the equal sign signifying equivalence does not appear to be well understood by the majority.

**Discussion**

This study explored the students' achievement and interpretation of equal signs when solving arithmetic tasks. Three meanings of the equal sign, namely, operation, identity, and equivalence (Mirin, 2019; Matthews et al., 2012; Rittle-Johnson et al., 2011) were investigated. We observed that the students performed very well on tasks that required interpretation of the equal sign as an operation. However, their achievement was progressively lower in tasks that required them to interpret the equal sign as either an identity or an equivalence. The majority of the students' responses revealed that they interpreted the equal sign as a command to "provide an answer" or "do something". This corroborates the findings of previous research, which revealed that students' operational knowledge of the equal sign may be more prevalent than that of other forms of interpretation (Wahyuni & Herman, 2019; Machaba, 2016; Baiduri, 2015).

A second finding of this study was that approximately 50% of all students across the three grade levels responded incorrectly to the identity and equivalence items.

Considering that students in Grades 7 and 8 are taught algebra-related concepts such as linear equations and algebraic expressions, we expected that they would have a deeper meaning of the equal sign. This is because solving linear equation tasks requires the notion of identity and the formation of a series of equivalent expressions. However, despite the grades 7 and 8 students' exposure to algebra-related concepts, their achievements and interpretations of the tasks showed the dominant nature of their understanding of the equal sign as an operation. Literature suggests that as students age, they are expected to develop a deeper understanding of concepts (Asquith et al., 2007). However, Blanton et al. (2018) argued that students' interpretation of the equal sign as an operation is dominant in lower and upper grades and even extends to the university. However, the picture emerging from this study is that spending more years in school and being taught additional concepts in algebra, which is a progression from arithmetic, cannot strengthen students' understanding of the use of the equal sign in its varied meanings. This may be because students are rarely taught to interpret the meaning of an equal sign, as revealed by Li et al. (2008).

A third finding of this study is that the nature of the tasks (standard or non-standard) also determines students' achievement and interpretation of the equal sign. Concerning standard tasks (i.e., addition or subtraction on the right side of the equal sign), the study revealed that students performed better than non-standard tasks (addition or subtraction on both sides of the equal sign). For example, Essien and Setati (2006) reported that the majority of students consider the equal sign as a 'command' for executing the operation appearing on its right side. Additionally, their achievement declined sharply when the arithmetic task involving equivalence was subtracted from the right. This affirms a study by Meyvis and Yoon (2021), which revealed that subtraction is considered more difficult than addition because individuals mostly consider solutions that add features to solutions that subtract them. It was not surprising that the situation worsened when students had to operate (subtraction) on both sides of the equal sign.

The results from this study indicate that the majority of the respondents across the three grade levels have an instrumental understanding of the equal sign (Skemp, 1971, 1976). This assertion is based on the fact that despite the varying demands of the arithmetic tasks requiring different meanings of the equal sign (that is, operation, identity, and equivalence), the students in this study mostly relied on only one idea of the equal sign, that is, as an operation. They therefore misapplied the meaning of the equal sign. A relational understanding requires that students know when to draw on a particular meaning of the equal sign and apply it appropriately based on the given task.

## Conclusion

We hypothesize that the students in this study's predominant interpretation of the equal sign as an operation might be a result of teachers' and textbooks' frequent presentation of the equal sign in standard types of the equation, as identified by Eichhorn, Lindsey, and Bombacher (2018) (in which the equal sign immediately precedes the answer). Studies indicate that mathematics knowledge needs to be developed extensively in basic schools (Meyer, 2016), and one of the major concepts that facilitate understanding instead of rote learning is the equal sign (Prediger, 2010). Therefore, stakeholders in mathematics education should include different interpretations of equal signs in their mathematics curriculum. The authors of mathematics textbooks must present several examples that exhibit different meanings and interpretations of equal signs for students to practice. Teachers must also be encouraged to consciously and intentionally guide students to identify different meanings of equal signs. This is essential because an understanding of the different meanings of the equal sign enhances the understanding of other mathematical concepts at higher levels. We argue that these findings have implications for how the equal sign is introduced to students and their learning of algebra-related concepts such as linear equations in the future.

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