Exploring high school students' experiences in the process of solving problems of numerical functions with GeoGebra

Abdoul Massalabi Nouhou[1] & Moussa Mohamed Sagayar[2]

Abstract

This research focuses on the study of the uses of a dynamic software environment in teaching - learning numerical functions in high school mathematics. We are particularly interested in improving the learning of numerical functions in high school through the dynamic mathematics software, GeoGebra. The theoretical framework is structured around registers of semiotic representations and the theory of semiotic mediation. The experiment focused on an ordinary mathematics class in the final year of high school science students (18-19 years old) in a public school in the city of Arlit, Niger. Two sessions on the study of numerical functions with a real variable were observed and the problem solving strategies were analyzed. The results allowed us to notice in the resolution of Problem 1 that most students easily introduce the algebraic register but find themselves destabilized when it comes to using the graphic register concerned. Despite the introduction of GeoGebra, the students had difficulties mobilizing the different registers of representation of the function. The results allowed us to note during the resolution of Problem 2 that the students manage with a certain ease the introduction and use of the algebraic register but also to use the graphic register and the numerical register of the table of values. The introduction of GeoGebra as a mediation tool allowed the students to invest a little more than usual in this second session. This improvement implies a direct assumption of responsibility for carrying out unusual mathematical tasks to introduce or use the registers of semiotic representation of numerical functions.

Keywords numerical functions; registers of semiotic representations; semiotic mediation theory; GeoGebra

Introduction

Numerical functions, a mathematics notion, is used in many domains including education. In the context of high school, mastering of numerical functions is decisive for studies in mathematics, science and technology in universities. In Niger, numerical functions are introduced in high school, with the introduction of linear functions, in particular the year 10 (teenagers 15-16). The study of elementary functions (affine functions, square functions, inverse functions, square root functions, cubic functions, cosine functions and sine functions) begins in year 11 (teenagers 16-17), with the study of their general properties (domain of definition, direction of variation and graphical representation). Polynomial (degree higher or equal to 3) and rational functions are introduced in year 12 (teenagers 17-18), and the introduction of real analysis such us limits,

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continuity, differentiation) are introduced in the study of reference functions. Exponential and logarithmic functions are introduced in year 13 (teenagers 18-19). In the same year, real analysis notions are consolidated through different numerical functions (polynomial functions, homographic functions, exponential functions and logarithmic functions). The choice of numerical functions in this study is motivated by the difficulties Nigerien science students face in mathematics, and to contribute to the revitalization of mathematics learning in high school.

Niger Republic has a policy of integrating the technologies' innovations to improve the quality of mathematics teaching and learning in the high school level. The GeoGebra Institute of Niger (GIN) was set up in 2011, in the "Institut de Recherche sur l'Enseignement des Mathématiques (IREM)" of Abdou Moumouni University (Nouhou and Jaillet, 2023). GIN is involved in the training of high school mathematics teachers, educational advisors and inspectors in the use of GeoGebra. In the same university, "Ecole normale superieure (ENS)", the college of education is also interested in GeoGebra. ENS introduced GeoGebra in the curriculum of training of the future mathematics teachers, advisors and inspectors of high school. GeoGebra is a software for teaching and learning mathematics, in particular in high schools. It's also an open-source, free and accessible software which can be used on computers, tablets or smartphones. The French language is a medium through which GeoGebra can be taught in Niger. The GeoGebra environment is easy to use, and allows to work simultaneously on multiple representations of the mathematical concept (algebraic, geometric, calculus and statistics). The reason for choosing GeoGebra in this study is related to this interest of GeoGebra integration and the design of this so1ftware.

The research interest of this work is to study the uses of GeoGebra software environment in learning numerical functions in science "terminale[1] class" of high school in Niger republic.

Literature review

The notion of function and its learning in high school

The term "function" was introduced at the end of the 17th century by Leibniz to designate quantities that depend on a variable. But the first uses of the term function date back to antiquity, in the form of correspondences between two quantities. The Babylonians (circa 1800 B.C.) used the correspondence between two quantities to determine the days (the table for calculating the square of a number, or astronomical tables). Ancient Greek mathematicians such as Hipparchus of Nicaea (circa 190-120 BC) used string tables to calculate the sizes and distances of the sun and moon. The Pythagoreans used the table of values to account for the dependence between two quantities, to model natural phenomena or to schematize geometric locations. These periods marked the introduction of function correspondence tables. But it took the middle ages to the notion of the function truly emerged, with the introduction of "algebra" and the algebraic expression of the function in the form of Al-Khwarizmi (780-850) sentences. The introduction of modern algebra by Viète (1540 - 1603) and of analytic geometry by Descartes (1596 - 1650) led to the introduction of analytic expression of the function and graphical representation, but

[1] The "terminale classe" is the final class in which student at the end of academic year sit for baccalaureate exam

above all to the correlation of the different ways of representing a function: curve, correspondence table, algebraic formula and analytic expression.

For Duval (1993), understanding the function implies a coherent articulation of the different semiotic registers involved in solving the problem. For Duval (1993), the functions is first and foremost an algebraic and graphical expression. The reading of the graphical representations the function presuppose the perception of corresponding variations.

In the Niger school context, the notion of function is introduced at the end of lower secondary school and consolidated at secondary school level. Functions are introduced in a general approach, with the general properties of elementary functions, and the introduction of the notions of limits, continuities and derivatives of functions, based on the study of common functions as a problem-solving tool. At the end of the cycle (in the final year), these concepts are extended through the study of homographic functions, trigonometric functions, exponential functions and logarithmic functions. Official high school mathematics curricula recommend the promotion of a problem-solving approach based on the algebraic and graphical expressions of a function. In addition to these two registers, there's also the register of the language of instruction (French). Michel Artigue points out that the research has highlighted the difficulties many students have in detaching the function object from its representations, particularly its algebraic representations, which are the most widely used, and in playing flexibly with the different registers, to choose the most appropriate for solving a given task (Artigue, 2009).

When it comes to solving problems involving numerical functions, many registers are likely to come into play (Coppé et al., 2007, p. 160). According to Artigue (2009), we usually distinguish 6 registers for the notion of function: the natural language register, the numerical register of tables of values, the algebraic register of formulas, the graphical register of curves, the graphical register of tables of variation and the intrinsic symbolic register.

Added to this are students' difficulties in solving problems involving functions. Students are often used to associating the numerical function with its algebraic expression. This can be explained by the predominance of the algebraic register in the approach to teaching functions (Dufour, 2011). For Fernando (1998), students' difficulties are due to extrinsic obstacles of a didactic nature (approaches to teaching functions in high school) and intrinsic obstacles in the sense that students find it difficult to consistently mobilize the different registers of function representation.

The choice of GeoGebra in this study is motivated on the one hand by the fact that the use of software, visualization and simulation tools, can profoundly change the nature of learning numerical functions and on the other by the fact that dynamic mathematical software designed for learning are educational digital tools that offer didactic potential in the process of solving mathematical problems.

GeoGebra, the natural representation of registers

GeoGebra is a dynamic geometry software created by Markus Hohenwarter to enhance the mathematical learning process. The acronym GeoGebra derives from two words: Geo from Geometry and Gebra from Algebra. GeoGebra is an interactive application for geometry, algebra, statistics and formal calculus (see Figure 1), designed for learning and teaching mathematics and science from elementary school to university. This dynamic geometry software is freely available, facilitating its use in public schools in developing countries. It can be used in several languages, including French. It is available on several platforms and websites. GeoGebra is also used in other educational disciplines. GeoGebra can be installed and used on computers, digital tablets and android phones.

integral, etc.). The spreadsheet window can be used to enter antecedents, calculate direct or indirect images, determine the coordinates of a point and set up a (x,y) correspondence table. The introduction of GeoGebra dynamic

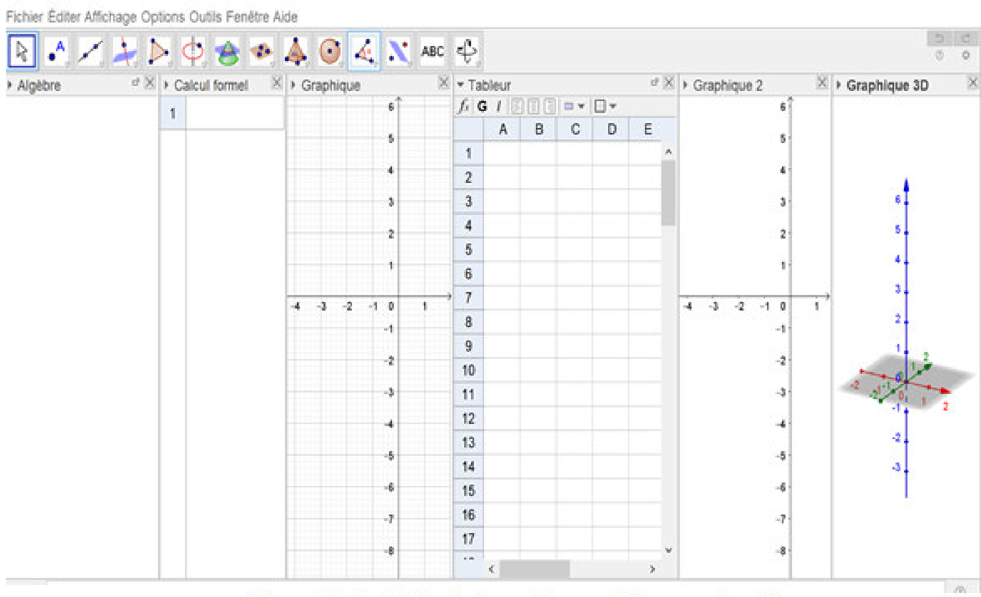

Figure 1 GeoGebra's input bar, windows and toolbar

When it comes to teaching and learning functions, GeoGebra can enable the user to work in several windows (algebraic, graphical, formal calculation, spreadsheet). It can be used as an environment for active exploration of the numerical function as a mathematical object (Freiman et al., 2009, p. 39). The input bar is used to write algebraic and symbolic expressions of the function object. The algebraic window displays algebraic entries. The graphics window simultaneously displays the graphical representation of the function (curve and graph) once the numerical function has been correctly entered. The formal calculation window lets you enter symbolic expressions for the function and perform the usual functional analysis calculations (solving an equation or inequality, calculating bounds, calculating the derivative, calculating the geometry software can offer students the opportunity to consistently mobilize several registers of semiotic representations of the mathematical function during the learning process.

On the basis of these various observations, the present research will attempt to study the introduction of dynamic geometry and formal calculus software to improve the process of learning numerical functions in high school. We will seek to answer the following research question: Does the introduction of GeoGebra dynamic geometry software require high school students to revise their problem-solving strategies for numerical functions?

Theoretical framework

Theory about movement within and between semiotic representation systems

Semiotics comes from the Greek word "semiosis", meaning the action of marking with a sign. It is therefore concerned with the relationships between signs (syntax), the relationships between signs and signified (semantics) and the use of signs (pragmatics). Semiotics enables the study of sign systems. Duval (1993) defines semiotic representation registers as modes of representing mathematical objects. Students' conceptualization of the function passes through three stages: the formation of representations of the function in different registers, the processing of these representations in each register and the conversion between these representations from one register to another (Duval, 2002).

In this process, everyday functions can acquire the status of objects, then gradually become abstract objects or function concepts (Anderson, 2015). According to Coppé, Dorier and Yavuz (2007), there are 6 main registers likely to be involved in learning numerical functions: the natural language register (Rl), the algebraic register of formulas (Ra), the graphical register of curves (Rg), the numerical register of tables of values (Rn), the graphical register of tables of variation (Rtv) and the intrinsic symbolic register (Rs). (Bloch, 2005) has pointed out that learning the notion of function in high school requires students to familiarize themselves with the registers of function representations. Knowledge of several registers facilitates processing activities within registers and register conversions by students. For Duval (1993), conversions between registers of representation of the function object enrich mental representations of the concept and its conceptualization, i.e. the student's understanding of the notion.

Semiotic mediation theory

Semiotic mediation theory (SMT) was introduced by Bussi & Mariotti (2008). It aims to transpose the concept of semiotic mediation into the field of mathematics didactics. Semiotic mediation theory offers a framework for studying mathematical activity as a mediated activity. The mediating function is exercised by artifacts (tools) and signs (Falcade, 2006). The process of constructing mathematical knowledge is then seen as a consequence of mathematical activity instrumented by different types of signs (mathematical, symbolic, linguistic, gestural, technical, etc.). From this perspective, the process of knowledge construction is twofold, highlighting the links between artefact/task and artefact/mathematical knowledge. The artefact enables specific tasks to be accomplished, fostering the emergence of personal meanings through the production of signs linked to the activity with the artefact. Since the aim is to learn, the use of the artifact is linked to specific mathematical knowledge.

Some elements of the instrumental approach in mathematics didactics (Rabardel, 1999) have been adapted by semiotic mediation theory, notably the difference between artifact and instrument, or the notion of instrumental genesis. The artifact is a material or symbolic object in itself, whereas the instrument is a mixed entity made up of the artifact and its schemas of use. The process of instrumental genesis enables the user (the student) to transform the artifact into an instrument. GeoGebra in the hands of students, its use can influence the process of building knowledge about the registers of semiotic representations of numerical functions.

Methodology

The methodological approach of this paper concerns experimental research in a high school in Niger. To begin with, we contacted the school administration of the public school named CES Sanda Maigari of Arlit city (Niger) where we intended to carry out this research. We then conferred with the mathematics teachers about the real difficulties high-school science students have in learning numerical functions during the high-school level, and about their relationship with ICT in teaching-learning mathematics. The mathematics teachers are not experts in techno-pedagogy, but they all have experience of using software to teach mathematics.

the process of learning numerical functions and enabling students to mobilize the different registers of semiotic representation of functions during problem solving.

Problem 1
Consider the function *f* defined by: $f(x) = \frac{1}{2}x^2 + x$ 1) Using GeoGebra, represent the curve C_f of *f* on an orthonormal frame $(O;\ \vec{\imath};\vec{\jmath})$ unit 1 cm. 2) Use this graphical representation to: a) Specify the value of the derivative f' of the function f in $x = -1$? b) Draw up the table of variation of *f* c) Graphically solve the equations $f(x) = 0$ and $f(x) = 4$ d) Graphically solve the inequality $f(x) \leq 0$ 3) Let $(T): y = -(x + 2)$ be the equation of the tangent to C_f at abscissa -2. Represent (T) in the same reference frame and then determine the position of C_f relative to(T).

Box 1

Secondly, these mathematics teachers met together in their teaching unit (UP) to propose two mathematics problems, taking into account the students' difficulties with numerical functions, whose solutions require the mobilization of several registers of semiotic representations. Finally, the teacher of the science class of year 13 adapted the content to the teaching-learning conditions of his mathematics class.

A mathematics class of 21 science students of final year of the high school (18 - 19 years old) in Niger was observed during our experiment. Two mathematics lessons were observed a naturalistic approach of teacher’s activities and student activities in mathematics classroom with technology (Caliskan-Dedeoglu, 2006; Lagrange & Caliskan-dedeoglu, 2009; Ruthven & Hennessy, 2002). GeoGebra software was integrated into digital tablets. Before starting the lesson, the teacher divided the class into several groups, each of which was equipped with a tablet.

Our approach is constructivist, integrating GeoGebra dynamic geometry software into

Resultants and discussions

Problem 1 (see Box 1) was proposed to study a polynomial function from its graphical representation.

To answer Problem 1, students are asked to mobilize the algebraic register of formulas. Possible strategies are:

- algebraic formula strategy, which consists in determining the algebraic expression of the function *f* adapted to GeoGebra's input tool (algebraic processing); and
- writing the algorithmic expression adapted to the input bar, $f(x) = (1/2)x\text{^}2 + x$ of the polynomial function $f(x) = \frac{1}{2}x^2 + x$ to obtain algebraic, and graphical, representations of the function on the interface (i.e., conversion from algebraic to graphical) as can be seen in Figure 2.

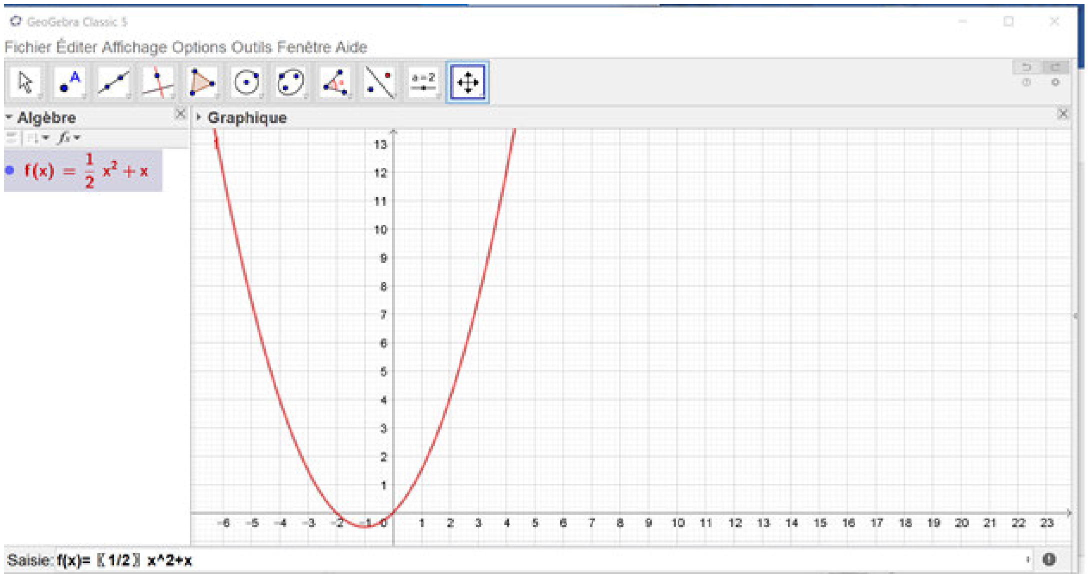

Figure 2 Entering the polynomial function and its representations on the interface

The algebraic strategy of formulas, which consists in starting from an algebraic expression of the function *f*, tracing the curve point by point on paper/pencil.

The students were able to mobilize the algebraic register of formulas on the GeoGebra interface.

To answer Problem 2, and more specifically, the four sub-questions in the problem, students are asked to mobilize the algebraic register of formulas and the graphical register of the curve. Possible strategies are:

- Graphical strategies for curves consist in starting from a curve of the polynomial function, to identify that the minimum of *f* is reached for the value of $x = -1$ so $f'(-1) = 0$. From the curve of the polynomial function, draw up a table of variations. (conversion). From the curve of the polynomial function, find the solutions of the equations $f(x) = 0$ and $f(x) = 4$. From the curve of the polynomial function, find the solutions of the inequality $f(x) \leq 0$.
- The algebraic strategies of the formulas which consist in starting from the algebraic expression of the polynomial function, determining the first derivative of the function (conversion) then calculating $f'(-1)$ (processing). From the algebraic expression of the polynomial function, draw up a table of variations. (conversion). Algebraically solve the equations $f(x) = 0$ and $f(x) = 4$ (processing). Algebraically solve the inequality $f(x) \leq 0$ (processing).

The strategies mobilized by the students in terms of the registers of semiotic representation of the polynomial function during the resolution of the question was the algebraic strategy of formulas. To answer Question 3), students were called upon to mobilize the algebraic register of formulas, the graphical register of curves or the graphical register of tables of variations. Possible strategies are:

- The algebraic strategy of the classic formulas, which consists of algebraically calculating the polynomial $f(x) - y$ (processing). From the algebraic expression of $f(x) - y$, draw up a sign table (conversion) and then algebraically interpret the relative positions of the curve of f with respect to the tangent (T) (conversion).

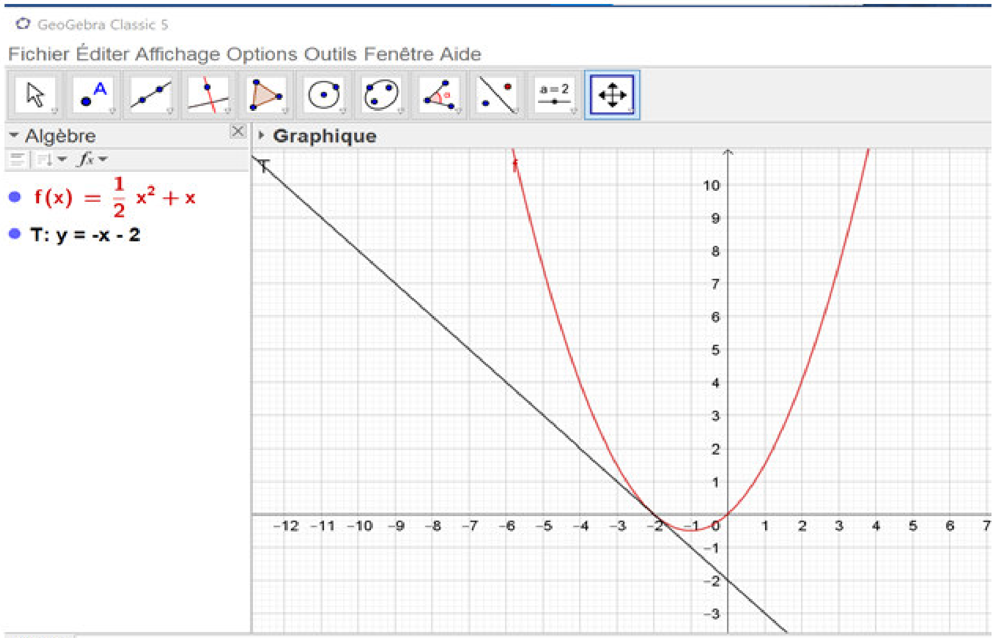

Figure 3 Drawing the tangent (T) with GeoGebra

- The algebraic strategy of the classic formulas, which consists in algebraically calculating the polynomial $f(x) - y$ (treatment). From the algebraic expression of $f(x) - y$, draw up a sign table (conversion) then graphically interpret the relative positions of the curve of f with respect to the tangent (T) (conversion).

- The algebraic strategy for formulas using GeoGebra tools consists in determining the algebraic expression for the tangent (T), adapted to the GeoGebra input tool (algebraic processing), then tracing its representative curve (conversion from algebraic to graphical). Then, algebraically interpret the relative positions of the curve of f with respect to the tangent (T) (processing).

- The algebraic strategy for formulas using GeoGebra tools consists in determining the algebraic expression of the tangent (T) adapted to GeoGebra's input tool (algebraic processing), then obtaining the plot of its representative curve (conversion from algebraic to graphical). Then graphically interpret the relative positions of the curve of in relation to the tangent (T) (processing).

The strategy mobilized by the students in terms of the registers of semiotic representation of the polynomial function during the resolution of the question is the algebraic register of formulas to calculate the limit to infinity of $(f(x) - y)$ and observe that the result was equal to zero. In all cases, they used the interface to plot the oblique asymptote and check the result using the calculus command.

These results show that, in solving Problem 1, the students favored algebraic strategies over graphical strategies in terms of the registers of semiotic representation of the polynomial function. Observations of the activities showed that the students easily introduced the algebraic register, adapted algebraic treatments using the input tool, and performed simple algebraic calculations. Use of the graphics tool remained basic, despite the presence of the visualization and construction tools available in the GeoGebra environment. This testifies to the difficulties in mobilizing graphical strategies when solving mathematical tasks with GeoGebra. To interpret the relative positions of the curve and the tangent, students used the algebraic strategy. We observed that students did not use GeoGebra's formal calculation and spreadsheet tools. Despite the introduction of GeoGebra, with its potential for learning numerical functions mobilizing several registers, students had difficulty detaching the function from its algebraic representation (Artigue, 2009).

They were unable to mobilize all the didactic potential of this tool, in particular that linked to learning functions by moving within and between different registers of semiotic representation systems.

But despite this, this first experiment shows that the students in our experiment can turn GeoGebra into a tool for learning numerical functions. To this end, it can influence the process of building students' knowledge of the registers of semiotic representations of numerical functions.

Problem situation 2 was proposed to study a homographic function and its graphical representation.

Problem 2
Let f be the function defined by: $f(x) = 2x + 3 + \frac{4}{x-5}$
1) Give the domain of definition of the function f
2) Study the variation of the function f (direction of variation and table of variation)
3) Show that $y = 2x + 3$ is an oblique asymptote to the curve C_f at ∞.
4) Draw the curve of the function f and these asymptotes in a reference frame $(O; \vec{\imath}; \vec{\jmath})$.
5) Show that $I(5; 13)$ is a center of symmetry at C_f
6) Solve the equation $(x) = 0$, graphically and by calculation.
7) Solve the inequalities $f(x) > 0$, graphically and by calculation.

Box 2

To answer Question 1, students are asked to mobilize the algebraic register of formulas. Possible strategies are:

- The algebraic strategy of formulas, which involves starting with the algebraic expression of the function $(x) = 2x + 3 + \frac{4}{x-5}$, writing the algorithmic expression adapted to GeoGebra's input bar $f(x) = 2x + 3 + 4/(x - 5)$ to obtain the algebraic and graphical representations on the interface, then determining the domain of definition of f (conversion from the algebraic register to the register).

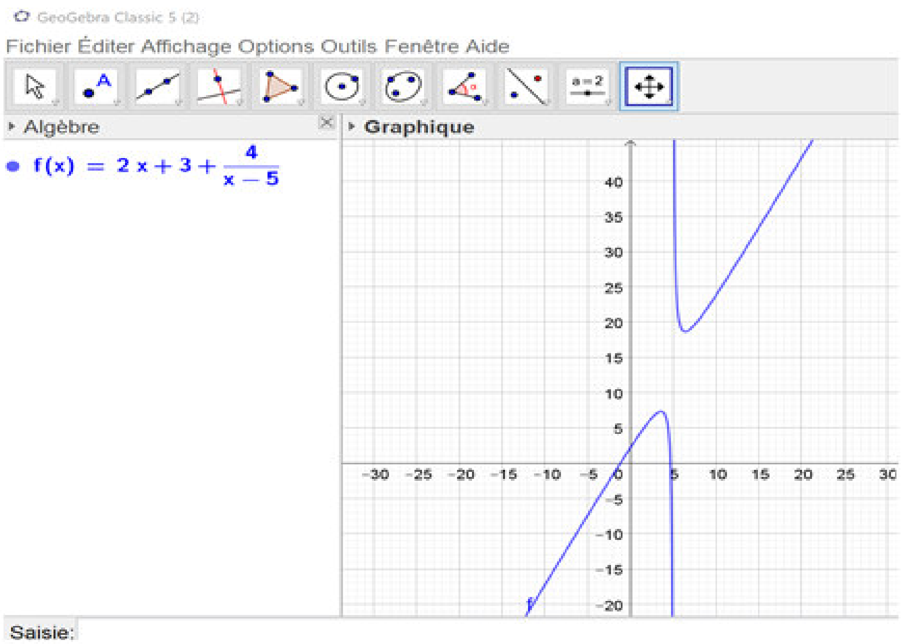

Figure 4 Algebraic and graphical representation of the homographic function

- The algebraic strategy using GeoGebra, which consists in determining the algebraic expression of the function f adapted to GeoGebra's input tool (processing). Then, from an algebraic expression of the function f, draw a curve of f(conversion). Finally, from the representative curve of *f*, determine the domain of definition of this function.

The students were able to use the algebraic register of formulas on the GeoGebra interface to represent the homographic function. But they had difficulty using GeoGebra's tools to observe and graphically interpret the graphical representation of the function f and determine the domain of definition of this homographic function. This forced them to go back and use the classical algebraic strategy to determine the domain of definition of the function f.

To answer Question 2, students are asked to use the algebraic register of formulas or the graphical register of curves. Possible solution strategies are:

- The algebraic strategy of formulas, which consists in establishing a table of variations from the algebraic expression of the homographic function. (conversion).
- The graphical strategy of curves, which consists in drawing a table of variations from the curve of the homographic function. (conversion).

The strategy mobilized by the students in terms of the semiotic registers of representation of the homographic function during the resolution of the question was the algebraic strategy, but they also used the graphical interface to observe the variations of the curve and visualize the coordinates of remarkable points.

To answer Question 3, students are asked to use the algebraic register of formulae, the graphical register of curves or the graphical register of tables of variations. Possible strategies are:

- The algebraic strategy of formulas, which consists in determining the algebraic expression of the equation of the line adapted to GeoGebra's input tool (processing). Then, from the algebraic expression of the equation of the straight line, plot this representative curve (conversion from algebraic to graphical). Finally, study the tendencies of the curve and the line when x goes to infinity (processing), then interpret the result (conversion).
- The algebraic strategy of calculating the infinite limit of f(x)-y algebraically (processing) and then interpreting the result (conversion).

The strategy mobilized by the students in terms of the registers of semiotic representation of the homographic function during the resolution of the question is the algebraic register of formulas to calculate the limit to infinity of (f(x) - y) and observe that the result was equal to zero. In all cases, they used the interface to plot the oblique asymptote and check the result using the calculus command.

To answer Question 4, students are called upon to mobilize the algebraic register of formulas, the graphical register of curves or the graphical register of tables of variations. Possible strategies are:

- The algebraic strategy of the formulas, which consists in determining the algebraic expressions of the function f and the equations of the lines adapted to GeoGebra's input tool (processing). From the algebraic expressions of the function, the oblique asymptote and the vertical asymptote, draw the representative curve of the function and its two asymptotes (conversion from algebraic to graphical).
- The algebraic strategy of formulas, which consists in using the algebraic expressions of the function, the oblique asymptote and the vertical asymptote, to draw the curve and its two asymptotes point by point.

Although they had already drawn the curve of the function f and the oblique asymptote, the students chose to mobilize the same algebraic strategy of formulas to draw the vertical

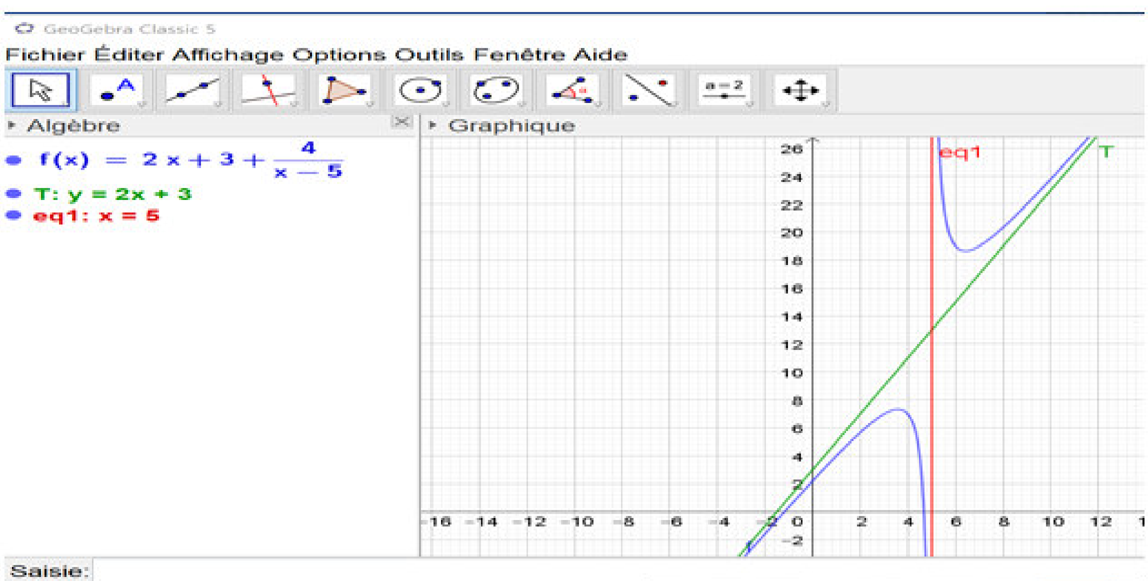

Figure 5 Algebraic and graphical representations of function f and asymptotes

asymptote at x=5 in the process of solving this question.

To answer Question 5, students are asked to mobilize the algebraic register of formulas, the graphical register of curves or the graphical register of tables of variations. Possible strategies are:

- Graphical curve strategy: determine the point of intersection of the asymptotes (processing), then establish a conjunction (conversion).
- The algebraic strategy of formulas, which consists in using the coordinates of point I (a;b) to determine f(2a-x)+f(x)=2b (processing), then deducing that I is a center of symmetry (conversion).

The strategy mobilized by the students in terms of the registers of semiotic representation of the homographic function during the resolution of the question was the algebraic strategy of formulas.

To answer Question 6, students are asked to use the algebraic register of formulae, the graphical register of curves or the graphical register of tables of variations. Possible strategies are:

- The graphical strategy of curves, which consists in finding the solutions of the equations f(x)=0 from the curve of the polynomial function.
- The algebraic strategy of formulas, which consists in algebraically solving the equations f(x)=0 and f(x)=4 (treatment).

The strategy mobilized by the students in terms of the registers of semiotic representation of the homographic function during the resolution of the question was the algebraic strategy.

To answer Question 7, students are asked to use the algebraic register of formulae, the graphical register of curves or the graphical register of tables of variations. Possible strategies are:

- The graphical strategy of curves, which consists in finding the solutions of the inequality f(x)>0 from the curve of the polynomial function.
- The algebraic strategy of formulas, which consists in algebraically solving the inequality f(x)>0 (treatment).

The strategy mobilized by the students in terms of the registers of semiotic representation of the homographic function during the resolution of the question was the algebraic strategy.

These results on the possible strategies and the strategies used by the students in terms of the registers of semiotic representation of the homographic function show that algebraic strategies predominate among the students during the problem-solving process, compared with graphical strategies for curves and the table of variations. Observations of the tasks carried out during the problem-solving process showed that the students were able to introduce the algebraic register with ease during the resolution of the tasks. They also found it easier to identify and use the relevant graphical and table of variations registers.

The introduction of GeoGebra dynamic geometry software helped students to overcome the difficulties of consistently mobilizing the registers involved in solving the problem Fernando (1998), and to assume direct responsibility for performing unusual mathematical tasks on the function object (Bloch, 2005). As Duval (1993) points out, such strategies can enrich students' mental representations of the notion of function.

Conclusion

This research focuses on the introduction of dynamic mathematics software GeoGebra to improve the process of learning numerical functions in high school. Initial results showed that students were able to introduce the algebraic register with ease, in particular writing the algorithmic algebraic expression with GeoGebra and performing algebraic calculations. The presence of the graphical register alongside the algebraic register, the students mobilized the algebraic register more. Despite GeoGebra's dynamic potential to mobilize several registers, students found it difficult to detach themselves from the algebraic register when solving Problem 1 (Artigue, 2009). However, this activity enabled the students to distinguish between the two registers of representation of this polynomial function. The second set of results also showed that students found it easier to mobilize the algebraic and graphical registers to solve Problem 2.

Thus, the presence of the graphical register alongside the algebraic register, as well as other registers in the GeoGebra environment, forced students to review their strategies for solving problems involving numerical functions in high school. Such an approach, combining the use of the GeoGebra computer tool with mathematics learning, real-variable numerical functions in high school, can enable students to overcome difficulties in learning the numerical function (Fernando, 1998) and mathematics teachers to change the predominance of the algebraic register in the teaching of functions (Dufour, 2011).

The present research has identified the difficulties students have in using all the GeoGebra tools that are essential for using several registers of semiotic representation during the process of solving problems involving numerical functions. This is why the question of high school students' mastery of the functionalities of GeoGebra tools and its effect on the mobilization of possible strategies also seem important to explore in this research. Still, there is much work to be done in this domain. Further research can be carried out on the important notions such as limits; continuity and derivability of the numerical function in the class of high school science to offer possibilities for overcoming extrinsic obstacles of the didactic type and intrinsic obstacles in the sense that students have difficulty in consistently mobilizing the different registers of representation of functions.

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