Effect of Problem-based Learning on High School Students' Performance in Solving Simultaneous Linear Equation Word Problems

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Abstract

Problem-based learning is a student-centered, inquiry-based instructional model in which learners engage with authentic, ill-structured problems that require further research. This study sought to find out the impact of problem-based learning approach to teaching on senior high school students' performance in solving simultaneous linear equation word problems. The study employed a quasi-experimental research design in which 103 second year science students were randomly sampled from four senior high schools and assigned to a control group (n=51) and an experimental group (n=52). Data were gathered using tests (pre-test and post-test) and analyzed using descriptive and inferential statistics. The study revealed that many SHS students performed poorly in solving word problems involving simultaneous linear equations. This was established from the fact that about ninety-eight (98) of the students representing 95.1%, scored less than half of the total score (100) in the pre-test. However, students that were exposed to Problem-based learning approach of teaching mathematics performed better in the post-test than those taught by the Traditional approach. This suggested that Problem Based Learning has positive effect on students' performance in solving word problems involving simultaneous linear equations. The study therefore concluded that Problem Based Learning is a more effective approach, which mathematics teachers as well as other subjects' area teachers need to incorporate into their teaching.

Keywords: problem-based learning; problem-solving; perceived difficulty; simultaneous linear equation; mathematics performance

Introduction

A paradigm shift in the teaching and learning practices has occurred in the educational environment in recent years with a growing focus on creative approaches that encourage critical thinking, problem-solving abilities and students' active engagement. Problem-based learning (PBL) is one of such instructional practices or strategies that is gaining international popularity. In the Ghanaian context, mathematics is a compulsory subject studied in school right from kindergarten to senior high school and academic success in mathematics bears great importance for students' future efforts. Since students need to obtain certain critical grades in mathematics before they can further their studies in many subject areas at the tertiary level. Problem-based learning prepares students to take charge of their mathematical studies and they are challenged to work to find solutions to real

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world issues. This approach does not only conform to the national curriculum but also embodies the wider objectives of Ghana's educational framework, stressing the development of such core competencies as problem-solving, critical and creative thinking and digital literacy skill.

Linear equations in two variables is a topic in the core mathematics syllabus that is taught in senior high schools in Ghana. Adu et. al., (2015) pointed out that linear equations in the curriculum are to be taught in two forms, (i) mechanical or standard linear equations problems, defined as tasks or exercises in linear equations and presented without words (or with very few words); and (ii) word (or contextual) linear equation problems, defined as tasks or exercises presented largely (or wholly) using words to describe the mathematical tasks involved. Box 1 shows an example of a standard and equivalent contextual, linear equation in one variable problem

A common difficulty observed by the WASSCE mathematics chief examiner in its reports in the past five years is the poor ability demonstrated by students in translating contextual problems into mathematical problems. The core mathematics syllabus recommends the use and application of appropriate mathematical problem-solving strategies in solving everyday mathematical problems. Besides, students are expected to learn mathematics concepts while acquiring problem-solving skills, positive attitudes and values to become efficient problem solvers (Karim, 2017).

The senior high school core mathematics syllabus is structured into content domains each of which has problem-solving as its integral part. The curriculum emphasizes the value of developing both theoretical knowledge and real-world problem-solving competencies (Ministry of Education, 2010). Though such problem-solving competencies that emphasize critical thinking are lacking and many students are finding it difficult to learn to mathematize word problems, effective strategies that could improve learning achievement, learning attitude, ability to mathematize or translating contextual problems into mathematical statements are still only partially explored.

Problem-based learning could motivate students to learn how to comprehend and mathematize problems; however, in many

Standard linear equations	Contextual linear equations
Solve for x and y. $x + y = 60$ $2x + 4y = 224$	A farmer was selling grown chickens and pigs on his farm. When he counted the animals ready for sale, he had 60 heads and 224 legs. Exactly how many chickens and pigs are on the farm?
Solve for x and y. $2x + y = 1.40$ $3x + 2y = 2.60$	Two oranges and an apple cost GH¢1.40. If a school girl paid GH¢2.60 for three oranges and two apples, what is the total cost of an orange and an apple?

Box 1

statements or mathematizng word problems (WAEC, 2019, 2018, 2017). Thus, students have difficulty in handling questions that are contextual linear equations or involve word developing countries, such as Ghana where the traditional teaching method is dominant, students are less likely to participate in problem-based learning activities if they are not interesting enough, and to date little research has been done on its educational applications in Ghana. This study, therefore, attempts to provide important insights into the possible advantages of using the BPL pedagogy, by providing opportunities for students to develop these competencies through productive learning environment settings. This study therefore sought to find out the impact of problem-based learning on senior high school students' performance in solving word problems involving simultaneous linear equations.

Research Method

Research Design

The study used a quasi-experimental inquiry. A quasi-experimental study is a type of evaluation which aims to determine whether a program or intervention has achieved the intended effect on a study's participants (NCTI, 2011). In quasi-experimental inquiry, the program or policy is viewed as an 'intervention' or treatment (White & Sabarwal, 2014), and in the present study the PBL approach constituted the treatment design.

Instrumentation

The study used achievement test (pre-test and post-test), as the instrument for data collection. The pre-test consisted of 10 word-problems involving simultaneous linear equations in two variables. These questions were based on objectives in the core mathematics teaching syllabus (MoE, 2010). The post-test also consisted of 10 word-problems involving simultaneous linear equations in two variables similar in content and structure to those of the pretest. Both tests were designed for a duration of thirty (30) minutes. The score for each of the items ranged from zero (0) to ten (10). Each test was scored out of a total score of 100.

Participants

The experiment was conducted on second-year students in a senior high school in the Central Region of Ghana. Second year students were chosen because the topic of interest, 'word problems involving simultaneous linear equations in two variables' is treated in this year. Multi-stage sampling technique was employed in the study. Convenience sampling was used to select two senior high schools (A and B) in the Central Region and purposive sampling was used to select an intact science class of elective mathematics year 2 students from each school. One intact science class in one of the schools was used as the control group and one in the other school was used as the experimental group. The sample size for the study was 103, comprising 51 in one school and 52 in the other school.

Treatment procedure

Two lesson designs were employed - PBL lesson design and the traditional lesson design. The latter refers to the teaching approach that largely involve the teacher using the chalk and board and students using pen and paper. The traditional lesson design was used by a mathematics teacher in School B to teach the control group.

In the traditional lesson design, the teacher uses largely the lecture and discussion methods. S/he gives input verbally or writes on the board and the learners strictly follow the instruction. The students in the control group were treated with lessons using the traditional design that follows the textbook presentation on how to solve simultaneous linear equation word problems, using the algebraic methods, i.e., substitution and elimination methods. The teaching strategies were largely teacher explanations of textbook examples, with no direct consideration of the students' alternative conceptions. A large proportion of the instruction time (75%-85%)

1. A farmer was selling grown chickens and pigs on his farm. When he counted the animals ready for sale, he had 60 heads and 224 legs.
   (i) Exactly how many chickens and pigs are on the farm?
   (ii) After selling off some of the animals, the number of chickens and pigs on the farm reduced to 45 heads and 160 legs. How many chickens and pigs were sold off?
2. In the table below, fill out the first two columns only with 6 possible combinations that will yield 20 60 heads. Use these to complete the animal's columns and determine the combination of heads that will give the 176 animal legs.

Heads			Animal legs		
Chickens	Pigs	Total	Chickens	Pigs	Total
24	36	60	48	144	192
		60			
		60			
		60			
		60			
		60			

Box 1

is devoted to instruction and teacher-led discussions and explanations.

The PBL lesson design was used by a mathematics teacher in School A to teach the experimental group. PBL lessons initially designed by the researcher were reviewed with the collaboration of the teacher of the experimental class who ensured the students' previous knowledge and learning experiences in solving linear equations in one variable met the requirements of the problem tasks. The PBL lesson plans used a student-centred approach in which students were engaged in groups to analyze and solve tasks involving linear equations in two variables. The tasks were unfamiliar and/or open-ended real-world problems without defined solutions intended to challenge students to address real-world problems.

The PBL tasks and worksheets used in the lessons required students to analyze and solve pairs of simultaneous linear equations graphically first before being introduced to the algebraic method. These tasks were planned to enable them to understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs because the points of intersection satisfy both equations simultaneously.

By first solving systems of two linear equations in two variables by drawing table of values and graphing the equations, students will develop the conceptual understanding required to solve real-world or contextual mathematical problems involving two linear equations in two variables. Two examples of the PBL tasks and worksheets are presented in Box 1.

Table 1 Learning activities students experienced in the two groups

	EG – School A	CG – School B
Design	Problem-based lesson	Traditional lesson
Delivery	Student-centred approach	Teacher-led didactic interactions
Organization	Group work	Whole class
Tasks	Contextual simultaneous linear equations in two variables tasks, which are unfamiliar and/or open-ended real-world problems (see Box 1)	Standard textbook simultaneous linear equation problems (see Box 1)
Approach	Analyzing and mathematizing (i.e., writing the equations) from the problems	Identifying which of the algebraic methods will be easier to get the solution
Activities	Students are required to trial-and-improvement, and table of values/graphical methods and explain their answers	Students are required to directly do the tasks and explain their answers
Applications	Real-world problems	Textbook problems

The learning activities that students experienced in the two groups are described and shown in Table 1. The procedure spanned over a period of five weeks. Before the start of the experiment, a pre-test which consisted of ten constructed-response test items was given to the participants in each of the groups. The mean scores of the groups were calculated and the group (School B) with the highest mean was assigned the control group and the other group (School A) was assigned the experimental group. The two groups were treated with the lesson designs for three weeks following the pre-test, and the post-test was administered to all the groups in the fifth week.

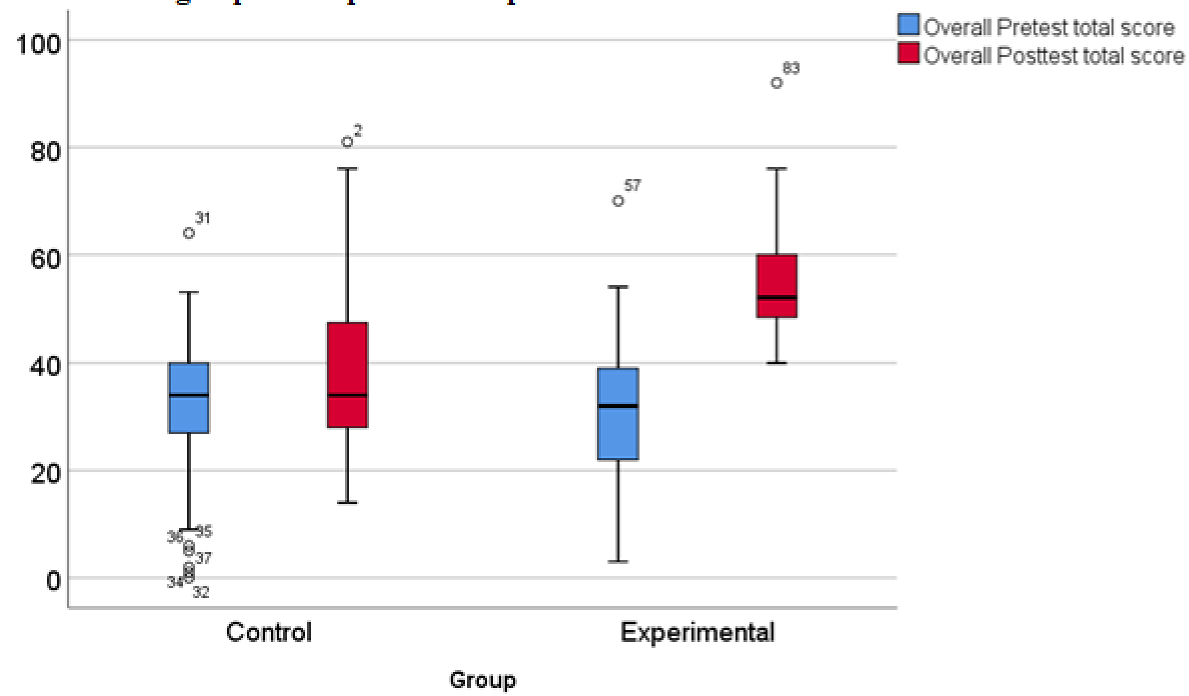
Results and Discussion

This chapter presents and discusses the results of the study. It presents the analysis of the data gathered from the senior high school students.

What is the effect of Problem-based learning on Senior high school students' performance in solving word problems involving simultaneous linear equation in two variables?

This research sought to analyze the effect of problem-based learning approach on Senior High Students' performance in solving word problems involving simultaneous linear equation in two variables. Students in the experimental group were taken through the activities in teaching contextualized problems using the problem-based learning approach. They were made to answer a set of questions on word problems involving simultaneous linear equations to see the effect of problem-

Figure 1 **Boxplots of the scores obtained by the control and experimental groups in the pre-test and post-test**



based learning approach on their performance in solving simultaneous linear equations. Table 2 shows the descriptive statistics of the participants in the control and experimental groups.

Table 2 shows the performance of the two groups prior to the experiment, that is, in the pre-test was comparable as the means and standard deviations of the control and experimental groups were [M = 31.6, SD = 13.44] and [M = 29.3, SD = 15.10] respectively. The post-test results of both groups however improved with the experimental group demonstrating a better performance in the test. The lowest score obtained in the experimental group was 40% and the highest score was 92% while the lowest and highest scores obtained by the control group were 14% and 81%.

Figure 1 is a boxplot of the results showing the spread of the scores obtained by the control and experimental groups in the pre-test and post-test. As can be seen in the figure, the two groups were around the same median score of

Table 2 **Descriptive statistics of the pre-test scores of the control and experimental groups**

Test	Group	N	Min	Max	Mean	Std. Dev.
Pre-Test	Control	51	0	64	31.6	13.44
	Experimental	51	3	70	29.3	15.10
Post-Test	Control	51	14	81	37.9	15.46
	Experimental	52	40	92	54.7	10.59

about 35% in the pre-test but in the post-test most of the students in the experimental group scored over 50% with the majority of the control group not reaching this mid score.

Table 3 **Independent t-test results of the scores of the control and experimental groups in the pre-test and post-test**

Test	Equal Variances	Group	N	Mean	Std. Dev.	Std. Error Mean	t	df	Sig. (2-tailed)	Mean Diff.
Pre-test	Assumed	Control	51	31.6	13.44	1.882	0.803	100	0.424	2.3
		Exp.	51	29.3	15.10	2.115				
Post-test	Not assumed	Control	51	37.9	15.46	2.165	-6.426	88	0.000	-16.8
		Exp.	52	54.7	10.59	1.469				

To ascertain if the differences observed between means of the control and experimental groups in the pre-test and post-test are statistically significant, the results were subjected to further analysis using independent t-test to test the null hypothesis that “there are no statistically significant differences between means of the control and experimental groups in the pre-test and the post-test”. The Independent t-test results are presented in Table 3. The independent t-test analysis results (i.e., t-value (t (100)) = 0.803, at p > 0.05), indicate at an alpha level of 0.05 in the pre-test, the mean performance score of the control group [M = 31.6, SD = 13.44] was not statistically significantly higher than the mean performance score of the experimental group [M = 29.3, SD = 15.10]. This suggests there is no statistically significant difference in the two groups performance prior to the experiment. However, the independent t-test analysis results (i.e., t-value (t (88) = -6.426, at $p < 0.05$), indicate at an alpha level of 0.05, the mean achievement score of the experimental group [M = 54.7, SD = 10.59] was statistically significantly higher than in the mean achievement score of the control group [M = 37.9, SD = 15.46]. This suggests there is a statistically significant difference in the achievement of students taught simultaneous linear equations using problem-based learning and those taught using the traditional method. This implies that the students taught using problem-based learning performed better than those taught using traditional method, and as such, the null hypothesis is rejected. Thus, the students in the experimental group outperformed their counterparts in control group who taught using traditional method.

It is clear from the results in Table 3 and the boxplot that the students in the control group taught simultaneous linear equations using traditional method also improved upon their pre-test performance. The results were further analyzed using traditional method to determine the extent of gain made by each group. Table 4 shows the results of the paired samples t-test on differences between the post-test and pre-test scores of the control and experimental groups and the effect size of the gains made.

Table 4 **Paired Samples t-test results of the scores of the control and experimental groups**

	Group	Paired Differences Mean	Paired Differences Std. Dev.	Std. Error Mean	t	df	Sig. (2-tailed)	Effect size
Post-test – Pre-test	Control	6.27	18.30	2.563	2.448	50	0.018	0.34
	Experimental	25.55	16.25	2.275	11.230	50	0.000	1.57

The paired samples t-test (Table 4) results of the control group (i.e., t-value ($t(50) = 2.448$, at $p < 0.05$) and that of the experimental group (i.e., t-value ($t(50) = 11.230$, at $p < 0.001$), indicate at an alpha level of 0.05, the mean differences between the post-test and pre-test scores of both control and experimental groups [M = 6.27, SD = 18.30] and [M = 25.55, SD = 16.25] were statistically significantly different.

In addition, the effect size (or gains made) in the achievement of the experimental group taught simultaneous linear equations using problem-based learning and those control group taught using the traditional method were 1.57 and 0.34 respectively (see Table 4). A commonly used interpretation of these values is to refer to effect sizes as small ($d \leq 0.2$), medium ($0.2 \geq d \geq 0.5$), and large ($d \geq 0.8$) based on benchmarks suggested by Cohen (1988). In this light, it can be concluded from the calculated effect sizes of approximately 1.57 and 0.34 that the effect of the PBL method was large while the effect of traditional method was medium. These imply that the effect of the experiment on the students' ability to solve simultaneous linear equations was higher among the problem-based learning group than the traditional method group.

Earlier studies in the Mathematics Education Department, UEW, have shown similar results, example, (Issaku, 2012). Booth (1984), also concluded that students have little or no knowledge when it comes to word problems involving linear equation.

Conclusion

The main purpose of the study was to investigate the effect of problem-based learning on senior high school students' performance in solving simultaneous linear equations word-problems. It was revealed that many senior high school students initially perform poorly in solving word problems involving simultaneous linear equations as ninety-eight (98) out of one hundred and three (103) students, representing 95%, scored less than half the total mark of the pre-test. Consequently, the students that were exposed to problem-based learning approach performed better in the post-test as against taught with the traditional method suggesting the use of problem-based learning technique was more effective for teaching learners how to solve simultaneous linear equations word-problems than the traditional method. An advantage of problem-based learning technique over the traditional method could be attributed to the fact that students are free to share ideas in a group. students who are exposed to problem-based learning approach are more likely to possess a meaningful in-depth knowledge of the content area.

Recommendations

Based on the findings of the study the following recommendations are made:

1. Teachers should create opportunities for students to relate what they learn in school to the real-world situation outside the classroom.
2. There should be workshops organized by the Ghana Education Service to train teachers with the requisite knowledge and skills to teach mathematics, especially word problems.

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