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Problem Based Learning: Errors Minimization, Conceptual and Procedural Understanding in Trigonometry

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Abstract

This study investigated problem-based learning by utilizing mixed method approach in which 100 pre-service teachers were randomly selected from four Colleges of Education in the Volta-Oti Region of Ghana and assigned to control and experimental groups. Trigonometry Achievement Test (TAT) and semi structured interviews were used to collect quantitative and qualitative data respectively. During treatment, problem-based learning (PBL) method of instruction was used to teach the experimental group while the control group was taught by conventional instructional approach. Both pre-test (pre-TAT) and post-test (post-TAT) were administered to pre-service teachers and their responses were scored to obtain quantitative data. The results revealed that pre-service teachers in both groups had improved in their post-TAT compared to their pre-TAT. However, it was revealed that pre-service teachers in the experimental group performed better in the post TAT than those in the control group. Finally, the findings revealed that PBL instruction promotes students' motivation. In conclusion, PBL instructional approach to teaching and learning of trigonometry concepts was found to promote students' motivation and increased achievement in trigonometry than the conventional instruction. Subsequently, the implications for practice and research are discussed.

Keywords problem-based learning; errors in trigonometry; conceptual understanding; procedural understanding

Introduction

The importance of mathematics education is significant in every sphere of life. It is evident that effective mathematics instruction provides students with several advantages, including the ability to reason critically and solve problems (Mo-John & Tang, 2017). However, there are numerous problems with problem with mathematics education in Ghana. In this regard, there has been an increased emphasis on raising the mathematics proficiency of pre-service teachers in Colleges of Education in the country in recent years. Thus, the quality of mathematics teaching may likely improve pre-service teachers' performance in the subject. Studies appear to support the idea that students who get high-quality mathematics instruction demonstrate stronger and more sustained success gains than their counterparts who receive lower-quality mathematics instruction (Mo-John & Tang, 2017; Kaharuddin, 2018; Lozinski, Poon & Spano, 2017; Masitoh & Fritriyani, 2018; Mushlinhuddin, Nurafifa & Irvan, 2018).

The majority of Ghanaian Colleges of Education use lecture, demonstration and other teaching techniques in their mathematics lecture halls, but these teaching methods have not resulted in the expected learning outcomes for the pre-service teachers (Mensah, 2017; Bukari, 2019; Bukari & Asiedu-Addo, 2019). In Ghana, many pre-service teachers struggle greatly

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with mathematics problems, particularly those involving trigonometry. They commit series of errors in the process (Mensah, 2017). This shows that pre-service teachers in the Colleges of Education in Ghana find trigonometric concepts difficult and it is very imperative that mathematics tutors at these Colleges of Education find a way to present trigonometric concepts during teaching and learning period to promote conceptual and procedural understanding among the student teachers.

Trigonometry is an area of mathematics that is applied frequently in life, particularly in astronomy, architecture, and transportation (Iji, et al., 2015). It entails measurement of lines, angles, and distances. The knowledge about angle of elevation and depression from trigonometry enables aero-planes to take-off and land successfully. The knowledge acquired from trigonometry helps students to appreciate shapes and situations around their environment and develop reasoning skills that is necessary for learning mathematics.

Furthermore, Gyan et al., (2021) argue that learning trigonometry as a subject is necessary to understanding concepts associated with Architecture, Physics, Surveying, and Engineering that have a substantial impact on human existence. However, due to the significance given to the topic in the growth of the individual and society, mathematics tutors have expressed concern about the low accomplishment in trigonometry by the pre-service teachers. Further research found that compared to other subjects, trigonometry is perceived as more challenging and abstract by pre-service teachers. This is due to the fact that trigonometry tutors tend to place more emphasis on methods than conceptual comprehension (Iji, et al., 2015).

Additionally, according to Orhun (2015), students struggle to understand the fundamentals of trigonometry and frequently make mistakes. Many of these mistakes resulted from lack of conceptual understanding in trigonometry due to inappropriate teaching and learning methods by the teachers in handling the topic. This affects students' performance in trigonometry tasks therefore mathematics teachers must adopt appropriate teaching and learning strategies that help students have conceptual understanding of the topic.

Studies on instructional strategies that will improve the quality of teaching and learning in mathematics has continued to generate interest among mathematics educators and scholars since the traditional method has failed to meet the needs of learners. The result of these gaps recognized in the traditional method has to be explored which will improve students' achievement. For this reason, there is a need to use an effective alternative strategy like problem-based learning.

Recent studies conducted on mathematics topics found out that Problem Based Learning approach improved students' learning and understanding, and their achievement level in the subject. They further stated that problem-based learning motivates students to become good problem-solvers, critical thinkers, and self-directed learners and therefore, encourages mathematics teachers to adopt problem-based learning as instructional approach to improve students' conceptual understanding and academic achievement in every mathematics topic

(Kaharuddin, 2018; Lozinski, Poon & Spano, 2017; Masitoh & Fritriyani, 2018; Mushlinhuddin, et al., 2018).

Moreover, studies conducted in Ghana about the effect of problem-based learning approach in Mathematics revealed that problem-based learning approach improved students' achievement and attitudes towards mathematics (Bukari, 2019; Bukari & Asiedu-Addo, 2019). They argued that problem-based learning develops students' life-long memory of mathematical concepts. Despite the effectiveness of problem-based learning approach in mathematics, it seems a little or no work has be done to investigate the effect of problem-based learning approach in trigonometry especially in Colleges of Education in Ghana. Therefore, the researchers have decided to explore the effect of problem-based learning approach to address mathematics pre-service teachers' difficulties in learning and understanding trigonometric concepts and improve on their achievement in the topic.

Though problem-based learning approach has been practiced in health education for some years to facilitate students learning, it has now been used in other educational areas including mathematics. It is one of the constructivist approaches to engage students in concept development. According to Bukari (2019), problem-based learning is a constructivist pedagogical approach to learning in which students work in smaller groups to find solutions to complex problems. It is one of the best strategies that allow learners to construct their own knowledge in solving a given problem. Problem-based learning is an instructional learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem. For the success of this approach, the teacher guides the learning process and conducts a thorough debriefing at the conclusion of the learning experience.

Hmelo-Silver (2004) describes problem-based learning as an instructional method in which students learn through facilitated problem -solving that centers on a complex problem that does not have a single correct answer. Hmelo-Silver (2004) further noted that students work in collaborative groups to identify what they need to learn in order to solve a problem, engage in self-directed learning, apply their new knowledge to the problem, and reflect on what they learned and the effectiveness of the strategies employed. Thus, Problem-Based Learning approach involves students in the use of their three domains namely, the cognitive, affective and psychomotor domain. Students will not be learning only facts and methods given by the teacher, rather they will investigate mathematics problems for themselves. This makes learning becomes meaningful at the end of the lesson.

One effective way of implementing problem-based learning in the classroom is to teach the students not only to remember the mathematical concepts but to know why and how it has been developed (Iji, et al., 2015). Thus, providing a trigonometry problem to students does not mean that they will solve it. But the process the students will undergo to arrive at the solution whether it is correct or not implied that learning of the intended concepts and theories have taken place. Also, problem-based learning makes transition from traditional learning practices to students learning practices. Students learn how and why concepts are being developed which is the philosophy of problem-based learning as instructional approach. This helps students develop good positive attitudes toward the trigonometry and promote more active involvement in the learning process and this could increase effective learning outcomes. For this reason, mathematics teachers have to

demonstrate appropriate scaffolding, which is necessary for students to develop abilities in taking the role and responsibilities required in problem-based learning activities.

Meanwhile, problem-based learning is ideal for students working in small groups to brainstorm and come up with their ideas and thought before they solve to get the possible solution. Through collaborated activities, students take responsibility of their own learning. Students learn solving problems independently by conducting a scientific research and reasoning. Problem-Based Learning focuses on whether students can articulate the critical points of the problems, procedure for solving the problems, proposed solution and defend the solution, rather than the solution to match proposed answers. Problem-Based Learning approach has been one of the most popular curriculum innovations in education. This is because the approach helps students to have a clearer mind, been flexible with diversified ways of thinking and is considered as the paradigm of multidisciplinary studies (Johari, et al., 2013).

The following empirical studies investigated errors that students make in study of trigonometry. Fahrudin and Pramudya (2019) examined errors and their causes in solving trigonometric equation problems. Data collection was done through tests, interviews, and observations. The research test instrument consists of 3 trigonometric equation questions. From 203 students of SMA Batik 1 Surakarta, there were 34% of data errors, 32.35% of concept errors 45.15% of strategy errors, 34.81% of calculation error and 30.38% of conclusion error. The common cause of these errors is that students do not understand the concept of trigonometric periods or angles in various quadrants, students forget the trigonometric values at special angles and students are not careful enough in doing or solving the problem.

Mensah (2017) explored students' error in learning trigonometry. The samples were 100 final year students preparing to write the 2017 West African Senior School Certificate Examination (WASSCE) in the Central region of Ghana. The Mathematics Achievement Tests (MAT) and Trigonometric Diagnostic Test (TDT) were used as the instruments of this study that included two components: the use of formula and right-angled method. Diagnostic interview was also used to identify the level at which students' errors occur in solving problems involving trigonometry. The type of errors is based on Newman Error Hierarchy Model that includes reading type error, comprehension, transformation, process skill, encoding error and carelessness. The findings of the study showed that most students make errors in process skill and transformation irrespective of the method used in solving trigonometry problems. There was no error found in students' reading skills. The number of students who made encoding error and carelessness was relatively low. The students' error in solving trigonometry problems was due to their weaknesses in basic arithmetical operations. Moreover, Chigonga (2016) examined problematic concept(s) and errors that learners have in solving trigonometry equations. The article gave some valuable suggestions for possible treatment of learners' errors when solving trigonometric equations. Particular types of errors that learners made when solving

trigonometric equations, the possible causes, and ways in which information could be used to structure instructional interventions, were described from the teachers' perspectives. Data were gleaned using an open-ended questionnaire and interviews of 10 Grade 12 Mathematics teachers. The teachers' responses to the questionnaire and interviews were analyzed thematically. The most common errors that the learners made as outlined by the teachers were selected. Data revealed that learners misinterpreted sine, cosine and tangent of an angle when their values were negative; failed to identify relevant quadrant and; made invalid inference (failed to recognize the reference angle). The study found learners commit errors in solving trigonometric equations and teachers have difficulties in teaching the same content.

Several researchers have suggested various techniques for mathematics instruction but it seems a lot has not been done to enrich learning experiences in all students through discussion, problem- solving and mathematical investigation. Therefore, there is the need to explore other strategies which is student centered and has the process of discussion, problem-solving and mathematical investigation such as problem-based learning that may enhance students' performance in mathematics. Hence this study examines the effect of problem-based learning on student teachers' performance in trigonometry.

Purpose of the Study

The purpose of this research was to analyze pre-service teachers' errors in trigonometry and the effect of Problem-Based Learning (PBL) on conceptual, procedural understanding and error minimization in trigonometry.

Research Questions

The following research questions underpinned the study:

1. What errors emerge from pre-service teachers' responses when solving trigonometry tasks?
2. What is the effect of problem-based Learning (PBL) on pre-service teachers' conceptual and procedural understanding in trigonometry?
3. How do pre-service teachers perceive the effectiveness of problem-based learning (PBL) in learning trigonometry?

Methodology

Research Design

This study adopted the pragmatic or Mixed Method approach which combines both quantitative and qualitative methods to provide a rich understanding of a research problem.

xed method approach which is a combination of both quantitative and qualitative data. The study employed experimental design which used mixed method to collect data from the investigation. This method is justified on the basis that the researchers collected both quantitative and qualitative data within the study period to address the aim of this study. The reason for combining both quantitative and qualitative data was to bring together the strengths of both forms of data for this research work and better understanding of the results (Sileyew., 2019; Handley, et al., 2018; Turner, et al., 2017).

In this study, quantitative data were collected to determine whether the independent variable (PBL) would influence positively or negatively on pre-service teachers' achievement in trigonometry (dependent variable). The choice of this approach is in line with the designs of earlier studies (Abdullah, Tarmizi & Abu, 2010; Kaharuddin, 2018; Masitoh & Fitriyani, 2018) that investigated the effect of PBL on students' achievement in Mathematics. On

the other hand, qualitative data was also collected for this study. This approach enabled the researcher to use interview to get an in-depth understanding of the pre-service teachers understanding of trigonometry during teaching and learning processes and the way PBL approach motivates students to learn from their perspectives.

The PBL group was put into smaller group of five and the researcher team took turns each week to teach using Problem-Based Learning Cycle approach (Othman, Salleh and Sulaiman, 2013). The PBL cycle involves learners working in small groups to confront problem(s); analyze the problem(s); discover or determine the mathematical model to solve the problem(s); do a report/presentation of the solution; and share reflection/evaluation of the process. This PBL cycle approach was chosen because it encourages students to engage themselves and work collaboratively in construction of knowledge provides opportunities for learners to see many ways that problems can be solved. A brief description of stages of problem-based learning instruction and activities undertaken during the treatment can be seen in Appendix A. The concepts taught were (1) Trigonometric identities, (2) Compound Angles, (3) Trigonometric Equation, and (4) Graph of Trigonometric Function.

The control group was taught by the use of the traditional method which emphasized the fluency of process in solving problems of trigonometry. Pre-service teachers in this class were taught the steps involved in solving trigonometric concepts. Sample questions were discussed, solutions were found and the researchers summarized by highlighting the pattern of steps involved in each question for students to memorize. During the instructional period of control group, pre-service teachers discussed difficulties they encountered when solving problems involving trigonometry with the researcher. The researcher addressed the difficulties of students by emphasizing the various steps of the concepts for them to memorize.

Quasi-experimental design employed for the study is justified on the reason that it sought to manipulate the independent variable and measure its effects on the dependent (trigonometric achievement) variable. The quasi-experimental design is more appropriate when working in a real-life settings to identify statistical relationships between two variables rather than true-experiments. This design is justified on the basis that the study was carried out in an educational setting.

Population

The study was carried out in Colleges of Education in the Volta-Oti Region of Ghana, during the second semester of the 2021 - 2022 academic year. Therefore, the population of the study comprised all pre-service teachers in the Colleges of Education in the Volta-Oti Region of Ghana. A total of two thousand five hundred and twenty-three (2,523) pre-service teachers were enrolled in the seven Colleges of Education in the Volta-Oti Region of Ghana (National Council for Tertiary Education -NCTE, 2022). The four out of the seven Colleges of Education in the two regions were purposively selected and coded as College A, College B, College C and College D for the study.

Sample and Sampling Technique

The sampling for this study was done in phases. In the first phases**,** the four out of the seven Colleges of Education in the two regions were purposively selected and coded as College A, College B, College C and College D for the study. In the second phase, the stratified sampling technique was used to randomly select twenty-five (25) pre-service teachers each from College A, College B, College C and College D. A total of one hundred (100) second year mathematics pre-service teachers were randomly sampled from the population for the study. In furtherance, a systematic random sampling was used to select fifty (50) pre-service teachers out of the total sample of hundred, these pre-service teachers constituted the experimental group while the remaining fifty (50) pre-service teachers constituted the control group of the study.

Instruments

Trigonometry Achievement Test (TAT)

Downie (1961, p. 651) as cited in Rani and Aisha (2017) defined mathematics achievement test as any written activity "that measures the attainments or accomplishments of an individual after a period of training or learning". According to the literature, achievement test is frequently used in mathematics to assess performance of students in numerous geometry effect studies (Tay & Mensah-Wonkyi, 2018; Armah, et al., 2017). The trigonometry achievement test (TAT) was used to collect quantitative data from the participants. The TAT provided the opportunity to measure the level of attainment of participants in Trigonometry (Trigonometric identities, Compound Angles, Trigonometric Equation and Graph of Trigonometric Function). In order to remove bias from the scores, two copies of the TAT were created, one to be used as a pre-test and the other as a post-test. The same component or construct was assessed using both TAT versions. The TAT was created by the researchers themselves using the concept of trigonometry in order to guarantee that the items in the TAT are in the Ghanaian context and measure the curricular objectives. TAT was divided into two sections, A and B, containing 25 items measuring two profile dimensions or aspects of knowledge and understanding and application of knowledge as outlined in the teaching syllabus.

More so, a grading system (marking scheme) that the researchers created was used to score the TAT. One (1) mark was awarded for each correct response on the multiple-choice test's 25 times. The adequacy of formulas and graphs, appropriateness of the methods used, and accuracy of the final solution were taken into account when evaluating each written response to the subjective portion of the TAT. Methods received an M mark, a B mark for accuracy that was followed by method mark, and an A mark for accuracy of the final solution or answer that was accompanied by the method mark. The written response portion received a total of 40 points in the grade. As a result, a participant can get between zero and forty (40) points for each component of the TAT.

Semi-structured interview guide

The researchers also used interview to better understand and explore views of participants taught with problem-based learning (PBL). The semi-structured interview guide contained six topical items intended to solicit views of pre-service teachers on how PBL instruction motivated them during treatment (increased participation in class activities, improved concentration in class, enjoyment during learning times, self-confidence, content mastery and ultimately recommendation of this teaching and learning method) phase of the study. The open-ended semi-structured interview guide enabled the

researchers to collect qualitative data which detailed the questions covered with participants in order to elicit their opinions on how problem-based learning motivates and aroused their interests throughout treatment phase of the study. Two experts in the field of mathematics from the mathematics department verified the appropriateness of the questions on the interview guide.

Validity and Reliability of Instruments

The Split-Half method examined students' pre-and post-test scores' reliability. First, the pre-test and post-test were divided into two halves using the odd-even items, and the scores were associated or correlated. Based on Pearson's Product Moment Correlation, this resulted in an internal consistency of 0.88. This result was then compared to the tabulated dependability or reliability coefficient, which was found to be satisfactory at 0.8 by Vakili and Jahangiri (2018).

According to Vakili and Jahangiri (2018), validity establishes if the study tools accurately measure what they are designed to measure. The study instrument was submitted to two lecturers, who patiently went over it and made the required suggestions and revisions. Some lecturers from the Department of Mathematics Education of the University of Education, Winneba were consulted to validate the test items and determine the content and face validity of the items.

Data Collection and Treatment Procedures

A pre-test examination was given to the sampled pre-service teachers to assess their knowledge of trigonometry with specific interest in Trigonometric identities, Compound Angles, Trigonometric Equation and Graph of Trigonometric Function. Errors the pre-service teachers made in solving the pre-test items were noted. These errors were used by the researchers to respond to the first research question and, by extension, the study's initial objective. The purpose of the pre-test was to determine pre-service teachers' challenges with essential question involving trigonometric identities, compound angles, trigonometric equation and graph of trigonometric function and their entering behaviour and basic knowledge of the desired learning areas of the study.

The treatment phase took a total of ten weeks. During these ten weeks, both the experimental and control groups were taken through four lessons in trigonometry. The conventional method of teaching, where marker board and textbook examples are used in the teaching process, was applied to the control group while problem-based learning (PBL) approach was applied to the experimental group. The PBL lessons for the experimental group were prepared by the team of researchers. The post-test was administered to determine whether the problem-based learning strategy was an efficient methodology employed by the researchers in trigonometric instruction.

After the intervention, the interviews were conducted. A minimum of 15 minutes and maximum of 20 minutes were spent on each interview. Electronic recording equipment was used to capture the interviews, and backup notes were also taken. Member checking was carried out to make sure the researchers accurately documented the views of interviewees in order to ensure the reliability of the interview data. To lessen the risk of respondent bias, participants were

contacted for clarification on some of the comments they made before drawing any conclusions. More so, to provide an accurate representation of the participants' voices, the researchers also kept and used interview transcripts.

Data Analysis

The study yielded both quantitative and qualitative data. Statistical Package for Social Sciences (SPSS) version 25 was used to evaluate quantitative data gathered from the TAT scores using both descriptive and inferential statistics. Measures of central tendency (mean) and measures of dispersion (standard deviation) were used to compare the scores, and an independent samples t-test was conducted to see whether there was a 95 percent confidence level of difference between the scores of the experimental and control groups. Thematic analysis was used to examine the interview data that was recorded. This ensured that the topical areas covered in the interview guide were used to evaluate the recorded audio from the interviews that participants were given. The researchers described and documented every occurrence that resulted from the interviews.

Results

Errors emerging from pre-service teachers' responses on trigonometry tasks

Based on the pre-service teachers' presentation of solution to the trigonometric questions, the researchers' examiners pre-service teachers' errors under following; conceptual, procedural and technical errors.

1. Conceptual Errors
2. The pre-service teachers had the following difficulties from the test item, which were classified as conceptual errors in trigonometry.
3. Unable to draw trigonometry table in radian measure of $y = 4 + 2cosx$, from $0°$ to 2π at interval of $\frac{\pi}{4}$.
   - Unable to state correct trigonometric identity for $tan^2x = sec^2x - 1$.
   - Unable to state the correct compound angle formulae for $\sin(A + B)$ and $\cos(A + B)$.
   - Unable to establish the fact that $cos3A = 4cos^3A + 3cosA$.
4. Procedural Errors

The pre-service teachers had the following difficulties from the test item, which were classified as procedural errors in trigonometry.

- Unable to carry out manipulation of $secx = \frac{-1}{3}\ and\ secx = 2$ to be $cosx = -3\ and\ cosx = \frac{1}{2}$ respectively.
- Unable to simplify the equation of $\cos(2A) = cos^2A - sin^2A$.
- Unable to simplify the equation of $\sin(2A) = 2sinAcosA$.
- Unable to plots the required points correctly, which contain radian measure on the graph sheet.

5. Technical Errors

Some of the basic mathematical skills errors discovered by the researchers were:

- Additive errors.
- Reducing by common-factor errors.
- Exponent errors.
- Carelessness errors (substitution, arithmetic, and missing brackets).

The distribution of pre-service teachers' committing the three types of errors in their solutions of the trigonometrical tasks is presented in Table 1.

Table 1 Distribution of the three types of errors emerging in the pre-service teachers' solutions of the trigonometrical tasks

Aspect	Conceptual Error	Procedural Errors	Technical Errors
Trigonometric Identities	32 (0.43) [1]	22 (0.29)	21 (0.28)
Compound Angle	40 (0.5)	26 (0.33)	14 (0.18)
Trigonometric Equation	38 (0.49)	29 (0.37)	11 (0.14)
Graph of Trigonometric Function	43 (0.59)	20 (0.27)	10 (0.14)
Average Percent Error	50.07	31.6	18.33

[1]Percentages in parenthesis

Table 1 reveals the categories of errors committed by pre-service teachers in their solutions of the trigonometrical tasks. As evidently presented regarding trigonometric identities, out of the 75 pre-service teachers who attempted the questions 32(42.6%) of pre-service teachers made conceptual errors, 22(29.33%) committed procedural errors while 21(28%) made technical errors. However, 25(25%) of the pre-service teachers did not attempt questions related to trigonometric identities. Moreover, out of 80 pre-service teachers who responded to questions relating to compound angles, 40(50%) committed conceptual errors, 26(32.5%) made procedural errors while 14 pre-service teachers representing 17.5% committed technical errors. Interestingly, 20(20%) of the pre-service teachers did not respond to items relating to compound angles. In furtherance, out of the 78 pre-service teachers who responded to trigonometric equation items, 38(48.72%) made conceptual errors, 29(37.18%) made procedural errors while 11(14.10%) committed technical errors. However, 22(22%) of the pre-service teachers did not attempt trigonometric equation items. Finally, out of the 73 pre-service teachers who attempted items relating graph of trigonometric function 43(58.90%) committed conceptual errors, 20(27.40%) committed procedural errors while 10(13.70%) made technical errors. However, 27(27%) of the pre-service teachers did not attempt items relating to graph of trigonometric function. Overall, it was revealed in the study that averagely pre-service teachers have committed conceptual errors (50.07%) more than procedural errors (31.60%) and technical error (18.33%). From this research, clearly there was high percentage of conceptual errors against procedural errors and technical errors, which were classified as low percentages.

Effect of the Problem-Based Learning Cycle approach on pre-service teachers' conceptual and procedural understanding of trigonometry

Table 2 Descriptive Statistics of TAT Pre-test Scores (N = 100)

Group	N	Mean	Std. Dev.	Maximum	Minimum
Experimental	50	13.73	6.77	29.00	12.00
Control	50	13.37	6.26	27.00	11.00

The effectiveness of problem-based learning (PBL) approach in teaching and learning as compared to traditional instruction on pre-service teachers' achievement in trigonometry was the main emphasis of this research question. Before starting treatment, the pre-test results for both groups were examined to see whether there was any discernible achievement difference between them. Table 2 displays the descriptive statistics of the two groups' pre-test scores.

The results from the independent samples t-test presented in Table 3 performed on the pre-test scores of the two independent groups revealed that there was no statistically significant difference between the experimental group and control group conditions. This result suggests that both the experimental and control groups were at the same level in terms of conceptual understanding of the concept of trigonometry before treatment.

Table 3 Independent Samples T-test of Pre-test Results (N=100)

Group	N	Mean	Std. Dev.	t-value	Df	p-value
Experimental	50	13.73	6.77	0.277	98	0.787
Control	50	13.37	6.26			

From Table 2, the results showed a mean score of 13.73 and 13.37 respectively for the experimental and control groups with a mean difference of 0.36. The minimum score for the experimental group was 12 while that of the control group was 11. Also, the experimental and control groups scored a maximum mark of 29 and 27 respectively. To test whether the difference in the mean scores was statistically significant, independent samples t-test was performed at 95% confidence interval. Table 3 presents the results.

To determine whether there was a statistically significant difference in the post-test scores between the group taught using the problem-based learning approach (experimental group) and the group taught using the conventional method (control group) an independent samples t-test was carried out at a 95 percent confidence interval to test the null hypothesis that there is no statistically significant difference between the mean post-test scores of the control and experimental groups (see results in Table 4).

Table 4 Independent Samples t-Test of TAT Post-test Scores

Group	N	Mean	Std. Dev.	t-value	df	p-value	Eta Squared
Experimental	50	29.71	12.37	2.788	98	0.005	0.096
Control	50	25.78	11.78				

There was a statistically significant difference between the experimental group and control group circumstances, according to the independent samples t-test results shown in Table 4. With regard to this finding, the experimental group which received instruction using a problem-based learning approach performed better than the control group, which received instruction using a conventional approach. According to Cohen (1988), the eta squared value of 0.096 signifies a medium effect size, meaning that the teaching strategy was responsible for 9.6% of the variance in the TAT post-test results. Thus, when compared to the traditional educational method, the results indicated that the problem-based learning strategy was beneficial.

Pre-service teachers' perception of the effectiveness of PBL approach in their learning of trigonometry

This aspect presents the results from the semi-structured interview administered to 5 pre-service teachers selected purposively from the experimental group. To ensure anonymity, the participants of the interview were assigned the codes Student 1, Student 2, … Student 5. The interview data was transcribed, analyzed and yielded the results that follow.

Some of their responses on their views about trigonometry as a topic in the mathematics syllabus when it was first taught were:

> *"Trigonometry is a difficult topic and I do not like it at all. I did not understand the concepts because the mathematics teacher rushed through the trigonometry claiming that we have been taught Geometry at previous lesson which serves as a foundation and this makes the topic difficult for me to understand."* (Student 1)

> *"With regards to trigonometry, when the teacher came to class, he only talked about the concepts without involving us, put examples on the board and solved them for us. So, I found it difficult to learn and understand the topic."* (Student 2)

> "*I used to have the feeling that trigonometry is difficult but now I have a change of mind because of the current understanding I have about it."* (Student 3).

Based on the responses, it is evident that Project-Based Learning (PBL) instructional strategy significantly helps students learn and solve trigonometric problems with ease. PBL enables students to concentrate and enjoy the

learning period, leading to a better understanding of trigonometry concepts. The interesting and problem-solving nature of PBL trigonometric activities further enhances students' understanding and confidence in the subject. The alignment of the PBL model with real-life aspects in trigonometric learning makes it a promising alternative for mathematics teachers. Therefore, PBL can be a beneficial approach, especially for teaching trigonometry.

When asked whether PBL approach made any difference in their learning process of trigonometry as compared to traditional method, some of their responses were:

> "*When I was involved in doing something, it was easy to remember as compared to you telling me how to do it. Therefore, being part of PBL activities helped me to have better understanding than just listening to the concepts which used to be the case.*" (Student 3).

> "*PBL is practical therefore; all of us are involved in the teaching and learning process."* (Student 5).

> *"It improved my understanding of the trigonometric concepts than the previous method."* (Student 1).

Students' feedback reveals that active involvement in learning tasks makes it easier to remember and comprehend compared to passive listening. Engaging in Project-Based Learning (PBL) activities had a substantial positive impact on their understanding of trigonometry concepts. The practicality of PBL, involving all students in the teaching and learning process, fostering inclusivity and enhancing engagement. Moreover, the effectiveness of PBL in improving their grasp of trigonometric concepts, surpassing the results of the previous traditional teaching method.

Discussion

The findings indicated that the pre-service teachers commit errors in their quest to solve problems relating to trigonometry. These errors were classified as conceptual errors, procedural errors and technical errors. The conceptual errors comprised the following: pre-service teachers were unable to draw trigonometry table in radian measure for $y = 4 + 2cosx$, from $0°$ to 2π at interval of $\frac{\pi}{4}$, pre-service teachers were unable to state correct trigonometric identity for $tan^2x = sec^2x - 1$, they were not able to state the correct compound angle formulae for $\sin(A + B)$ and $\cos(A + B)$ and finally, they were not able to establish the fact that $cos3A = 4cos^3A + 3cosA$. These findings are in line with the studies of (Fahrudin & Pramudya, 2019; Mensah, 2017; Chigonga, 2016) who in separate research observed that learners commit errors in solving trigonometric equations and teachers have difficulties in teaching the same content. Therefore, conscious effort must be made to reduce the errors committed by learners in order to improve their performance in mathematics.

Furthermore, findings from the independent samples t-test showed a statistically significant difference between the performance in the post-tests of pre-service teachers who were exposed to PBL and those exposed to the conventional approach. However, the statistic of eta squared value of 0.096 indicated a moderately medium effect size (Cohen 1988). This may be due to the similarities that exist between the control and the experimental groups of the study. The findings show that PBL in teaching mathematics is more effective than the conventional instruction. PBL instruction was found to be more effective than the

conventional instruction in this study because the teaching process was student-centered; linked the concept to real-life situations and provided a serene and conducive teaching and learning environment as well as provided many ways of communicating concepts to pre-service teachers. These finding strongly agree with the studies by (Kaharuddin, 2018; Lozinski, et al., 2017; Masitoh & Fritriyani, 2018; Mushlinhuddin, et al., 2018) who in separate studies found that PBL as a teaching strategy improved significantly in the post-test as compared to those exposed to the conventional method.

Meanwhile, on how pre-service teachers perceive the effectiveness of problem-based learning approach, the findings indicated the approach motivated them to study trigonometry through its multi-modal capabilities and practical nature. These findings resonate strongly with the research conducted by (Iji, Emiakwa et al., 2015; Bukari, 2019; Bukari & Asiedu-Addo, 2019; Masitoh & Fritriyani, 2018; Mushlinhuddin, et al., 2018) in separate studies found that PBL in mathematics education promotes motivation of students through its practical nature and visual presentation of concepts.

Conclusion

The results of this research showed that tutors dominated classroom interaction activities during teaching and learning of trigonometry. This gave student teachers' limited opportunities to take active part in the lesson such as sharing ideas on the concepts among themselves. Mathematics tutors gave much information to the pre-service teachers to absorb without much contribution from the students which did not encourage conceptual understanding. Mathematics tutors should have allowed pre-service teachers to do much of the work during teaching and learning of trigonometric concepts for better understanding of the topic.

It was noted that pre-service teachers had difficulty in analyzing trigonometric graphs, they had difficulty in the use of trigonometric equations, they had difficulty in performing algebraic calculation to arrive at final solution and difficulty in solving trigonometric problems concerning real life situations. These difficulties were as a result of mathematics tutors' use of instructional activities that did not encourage students to have understanding of the concepts, their failure to relate trigonometric concepts to real life situations for students to be familiar with and their inability to encourage students to solve enough problems in Mathematics.

Furthermore, pre-service teachers in the experimental group performed better in understanding of the trigonometric concepts than those of the control group. It can be stated that PBL instruction of trigonometric concepts had a positive effect on pre-service teachers' achievement. The involvement of the student teachers in the lesson by PBL led to the understanding of the concepts and therefore, improved student teachers' performance in the post-test.

Finally, findings from the interview revealed that pre-service teachers enjoyed the lessons with PBL approach through group activities, investigation and presentation than traditional approach. According to the interviewees, PBL approach has increased their learning abilities through participation in classroom activities, concentration in the lesson, self-confidence and content mastery. The

respondents also wished PBL approach is made compulsory for all mathematics topics.

Recommendations

Based on the findings of this study, the following recommendations are offered:

- Mathematics tutors in College of Education in Ghana should use problem-based learning approach in the teaching and learning of trigonometry since it improves pre-service teachers' conceptual understanding and have positive effect in their achievement in trigonometry.
- Ghana Tertiary Education Commission (GTEC), Transforming Teaching, Education and Learning (T-TEL), and policy makers should promote professional development and in-service training for mathematics tutors in the use of PBL to promote students' participation in trigonometry instruction.

References

Armah, R. B, Cofie, P. O & Okpoti, C. A. (2017). The Geometric Thinking Levels of Pre-service Teachers in Ghana. Higher Education Research, 2(3), 98-106

Bukari, H. I. (2019). Using problem-based learning to enhance junior high school students' understanding and attitudes toawards Linear equations word problems. Journal of Education and Practice, 10(3), 126-135. doi:10.7176/JEP

Bukari, H. I., & Asiedu-Addo, S. K. (2019). Effect of problem-based learning on College of Education students' achievement in probability and atitude towards solving probability related problems. Africans Journal of Education Studies in Mathematics and Sciences, 15(2), 155-168. doi:org/10.4314/ajesms.v/5ii2.13

Chigonga, B. (2016). Learners 'errors when solving trigonometric equations and suggested interventions from grade 12 mathematics teachers, OISTE Conference, Mathematics, Science and Technology Education, 1-10

Cohen, J. (1988). Statistical power analysis for the behavioural sciences (2. Auflage). Hillsdale, NJ: Erlbaum.

Fahrudin, D., & Pramudya, I. (2019). Profile of students' errors in trigonometry equations. In Journal of Physics: Conference Series (Vol. 1188, No. 1, p. 012044). IOP Publishing.

Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn? Educational Psychology Review, 16, 235- 266.

Iji, C. O., Emiakwu, S. O., & Utubaku, R. U. (2015). Effect of problem-based learning on senior secondary school students' achievement in trigonometry in Northern Educational Zone of Crosss River State, Nigeria. IOSR Journal of Mathematics, 11(1), 16-25. doi:10.9790/5728-11/5/625

Johari, S., Nor-Husniza, I., & Mohktar, M. (2013). Implementation of problem-based learning in Higher Education Institution and Its Ipmact on Students' Learning. 4th International Research Symposium on PBL. Aalborg: Aalborg University Denmark.

Kaharuddin, A. (2018). Effect of problem-based learning model on mathematics learning outcomes of 6th Grade students of Elementary School Accreduted B in Kendari City.

Internatinal Journal of Trends in Mathematics Education Research, 1(2), 43-46. doi:10.33122/ijtmer.v/i2.14

Lozinski, D., Poon, A., & Spano, M. (2017). Effectiveness of problem-based learning prior to lectures on learning and retention, discussion on university science teaching. Proceedings of the Western Conference on Science on Education, 1. Retrieved from www.iri/ib.uwo.ca/wcsedust/vol1/iss1/12

Masitoh, L. F., & Fitriyani, H. (2018). Improving students mathematics self-efficacy through problem-based learning. Malikussaleh Journal of Mathematics Learning, 1(1), 26-30. doi:org/10.29103/mjm/.v/i1.679

Mensah, S. F. (2017). Ghanaian senior high school students' error in learning of trigonometry. International Journal of Environmental and Science Education, 12(8), 1709- 1717.

Mushlihuddin, R., Nurafifa, & Irvan. (2018). The effectiveness of problem-based learning on students' problem solving ability in vector analysis course. Journal of Physics: Conference Series 948 012028. doi:10.1088/1742-6596/948/1/012028

Nurmeidina, R., & Rafidiyah, D. (2019, August 7). Analysis of students' difficulties in solving trigonometry problems. Profunedu. Surakarta, Indonesia: EAI. doi:10.4108/eai.7-8-2019.2288422

Orhun, N. (2015). Students' Mistakes and Misconceptions on Teaching of Trigonometry (1 ed.). Retrieved January 14, 2015

Rohimah, S. M., & Prabawanto, S. (2019). Student's difficulty identification in completing the problem of equation and trigonometry identities. International Journal of Trends in Mathematics Education Research, 2(1), 34- 36. doi:org/ 10.33122/ijtmer.v2i/.50

Siagian, M. V., Saragih, S., & Sinaga, B. (2019). Development of learning materials oriented on problem-based learning model to inprove students' mathematical problem solving ability and metacognition ability. International Electronic Journal of mathematics education, 331-340. doi:org/10.29333/iejme/5717

Sileyew, K. J. (2019). Research design and methodology . Rijeka: IntechOpen.

Tay, M. K, & Mensah-Wonkyi, T. (2018). Effect of using Geogebra on senior high school students' performance in circle theorems. African Journal of Educational Studies in Mathematics and Sciences, 14, 1-17.

Turner, S. F., Cardinal, L. B., & Burton, R. M. (2017). Research design for mixed methods: A triangulation-based framework and roadmap. *Organizational Research Methods*, 20(2), 243-267.

Vakili, M. M., & Jahangiri, N. (2018). Content validity and reliability of the measurement tools in educational,

behavioral, and health sciences research. *Journal of Medical Education Development*, 10(28), 106-118.

Appendix A.

1. Meet the problem:

   The researchers posed sample trigonometric tasks such as

   $\frac{sim30^0+tan45^0-cosec60^0}{sec30^0+cos60^0+cos45^0}$ and $tan48^0tan23^0.tan42^0tan67^0$

   to pre-service teachers to evaluate in their groups. Questions such as "*what type of tasks in trigonometric are these", what ideas or understanding do your group have about the tasks"* were asked the groups for them to identify facts from the tasks posed. The objective of this stage is for all group members to establish a common understanding of the tasks posed, and to generate ideas that may lead to plausible solutions. The groups become more engaged and curious, and this compelled them to present more answers for evaluation.

2. Problem analysis:

   In this stage, the researchers performed learning activities for the groups.

   The tasks $\frac{sim30^0+tan45^0-cosec60^0}{sec30^0+cos60^0+cos45^0}$ and $tan48^0tan23^0.tan42^0tan67^0$

   was broken to component $(sim30^0, tan45^0, cosec60^0, sec30^0, cos60^0\ and\ cos45^0)$ and $(tan48^0, tan23^0, tan42^0\ and\ tan67^0)$. Each member in the group was given a component of the broken tasks to evaluate by providing trigonometric values to trigonometric angels. The researchers guide the groups to use their previous knowledge to solve the problems. After a few minutes, the groups then discuss the assign tasks of each member. The researchers moved from one group to another to supervise their work and encouraged every student to take part in the group discussion.

3. Discovery and reporting:

   Pre-service teachers reported their finding as

   $(\frac{sim30^0+tan45^0-cosec60^0}{sec30^0+cos60^0+cos45^0} = \frac{\frac{1}{2}+\frac{1}{2}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}-\frac{1}{1}})$ and $(tan48^0tan23^0.tan42^0tan67^0 = \cot(90^0-48^0)\cot(90^0-23^0).tan42^0\tan 67^0)$.

   Groups were then encouraged to continue the process to final solution by teaching and explaining the process to their peers. Members of various groups were also encouraged to shared information they have gathered with their groups and discussed to generate possible solutions. Pre-service teachers decided on the solution which best fit the problem and

wrote their findings clearly showing step by step of their solution on the problem and prepared for the presentation to the class.

4. Solution Presentation:

   In this stage, groups presented their solutions to the class to show their conceptual understanding and methods of solving the problems. Pre-service teachers from other groups assessed their colleagues' presentation and probed the solution. Through these experiences, pre-service teachers developed deeper understanding and adequate methods in solving trigonometry.

5. Reflection and Evaluation:

   This is the final stage of the cycle. The researchers did the final briefing of the concept learnt to the class

   $(\frac{sim30^0+tan45^0-cosec60^0}{sec30^0+cos60^0+cos45^0} = \frac{\frac{1}{2}+\frac{1}{2}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}-\frac{1}{1}} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4} = \frac{43-24\sqrt{3}}{11})$ and
   $(tan48^0 tan23^0 . tan42^0 tan67^0 = \cot(90^0 - 48^0)\cot(90^0 - 23^0) . tan42^0 \tan = \cot 42^0 cot67^0 . tan42^0 tan67^0 = 1).$

   The pre-service teachers reflected on their presentations. The researchers assessed the pre-service teachers understanding in the trigonometry concepts and their abilities to know the progress of pre-service teachers towards achieving the teaching objectives. This was done through oral questioning, quiz, and project work on trigonometric concepts.