Teaching and learning circle theorems with Dynamic Autograph technology: Should we employ the social-interaction or the self-exploration strategy?

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Abstract

The use of technology as a pedagogical practice is growing at a phenomenal rate due to the availability of numerous computer mathematics software. As a result, conscientious efforts need to be made by teachers to integrate technology effectively to enhance the teaching and learning processes and consequently learning outcomes of students. This non-equivalent quasi-experimental study sought to explore students' achievements and their attitudes after using a dynamic Autograph software technology to learn circle theorems. One hundred and fourteen Senior High School students purposively selected from three different schools participated in the study. While students from two of the schools were taught using the Autograph technology in a social-interaction and self-exploration environment respectively, the group of students from the last school were taught conventionally without the use of the Autograph technology. The findings of this study showed that the three teaching strategies had a significant positive effect on students' achievements. However, the social-interaction Autograph-assisted group significantly outperformed, followed by the self-exploration Autograph-assisted group, and then the conventional group. The study also showed that though the self-exploration group showed higher positive attitudes than the social-interaction, the Autograph-assisted groups in general reported overall positive attitudes towards using the technology indicating further that the Autograph-support made lessons practical, stimulating, and provided imagery simulations and multiple representations that enhanced visualisation and understanding. Implications of the Autograph technology use in teaching, while engaging students in small groups are discussed.

Keywords   Autograph software, social-interaction, self-exploration, achievements, attitudes

Introduction

A comprehensive understanding of mathematical concepts forms the solid foundation for its application in the real world. The mode of transmission of these concepts to students is key, to ensure effective understanding and application. Learners, therefore, need to be effectively supported to gain an in-depth understanding of mathematics concepts for the appropriate application. To motivate and improve students' mathematics confidence, teachers need to be creative by using innovative explorations to create dynamic instructional pedagogies that may involve technology integration and learner-centred approaches.

Technology has brought advanced innovations into mathematics education that need to be fully maximised by teachers. In this contemporary, appropriate technology integration into mathematics education is noted as an effective approach that can facilitate effective teaching and learning processes (Agyei, 2013).

It is replete in the literature on how technology integration into the mathematics classroom positively affects the teaching practices of teachers and motivates the learning behaviours of learners (Mensah, 2019; Tay & Mensah-Wonkyi, 2018; Karnasih & Sinaga, 2014; Shadaan & Leong, 2013; Tarmizi et al., 2009). Several

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research studies have revealed that dynamic software supports cognitive processes and reduces mental load; encourages logical reasoning and hypothesis testing; offers cooperative or social learning which promotes students' high-order thinking skills; and reduces time spent on manual arithmetical computations (Karnasih & Sinaga, 2014; Tarmizi et al., 2009). The use of technology promotes pragmatic and social constructivist instructional principles; from teacher to learner-centred, individual to social-learning, and from the teacher as a knowledge source to the learner as the source of knowledge (Essel & Adjei, 2017; Agyei, 2013). In a technology-motivated lesson, learners gain an opportunity to formulate their knowledge by playing active rather than passive roles and assume responsibility for their learning by focusing on reasoning, reflection, and problem-solving (Tay & Mensah-Wonkyi, 2018). It is highlighted that technology adds new dimensions to the teaching process and provides new learning opportunities that help learners visualise and engage with different mathematical objects and concepts (Saha et al., 2010). The opportunity offered by technology to explore and visualise mathematical objects and concepts is claimed to foster understanding and improve high-order spatial abilities.

Literature Review

Conceptual difficulties of students in learning circle geometry

Geometry is the study of space and shape (Saha et al., 2010). Boyraz (2008) stated that the concept of circle geometry extends to varied fields of modern mathematics including architecture and engineering. To help students develop an in-depth conceptual understanding of geometry, they must have a deep sense of spatial reasoning, and spatial visualisation and orientation. However, many studies have reported that learners appear to experience challenges in applying or relating the required knowledge of geometric properties and relationships to

a given task even after teachers have taught the concept (Shadaan & Leong, 2013; Boyraz, 2008). According to Shadaan and Leong (2013), students lack the cognitive and process abilities to understand and apply concepts of circle geometry. The learning and understanding of circle geometry are described by students to be difficult and thus, many students are unable to develop the appropriate understanding of geometric concepts, geometric reasoning, and geometric problem-solving skills (Saha et al., 2010; Idris, 2007).

Contextually, Ghanaian SHS students experience similar geometric challenges. Secondary school students are described to experience challenges in constructing, visualising, and justifying the existing relationships that link the circle geometric properties (Tay & Mensah-Wonkyi, 2018; Mereku, 2010). According to Tay and Mensah-Wonkyi (2018), the lack of comprehension of circle geometry causes discouragement among students which leads to their poor achievements and attitudes toward geometry. Reports of national examinations contain evidence of Ghanaian SHS students' deficiencies in answering circle theorems related questions. The chief examiners' report of the West African Examinations Council (WAEC) contains evidence that students who sit for the West African Senior Secondary School Examinations (WASSCE) consistently perform abysmally in circle theorems questions (WAEC 2014; 2016). Mensah-Wonkyi and Adu (2016) also have it that Ghanaian SHS students experience high levels of difficulty in identifying and or using appropriate circle theorems in problem-solving.

Insights from literature revealed that the learning deficiencies students experience in geometry are attributed to teachers' resolute use of conventional teaching approaches which do not provide the appropriate learning platform for students to attain high geometric thinking abilities (Addae &

Agyei, 2018; Tay & Mensah-Wonkyi 2018; Mensah-Wonkyi & Adu, 2016). The conventional teaching strategy is described as not to bridge the gap of students' challenges as it focuses on what and how much students can recall rather than their extent of reasoning (Idris, 2007). This teacher-centred instructional approach is pronounced to impose "forced learning" and offers students limited opportunities to explore, analyse, and hypothesise to develop their visualisation manpower, geometric reasoning, and problem-solving skills (Fumador & Agyei, 2018; Kivkovich, 2015; Seloraji & Eu, 2017).

When students are provided with learning opportunities that engage them in explorations and visualisation of geometric relationships, they develop their geometric conceptual ideas. It is highlighted by Seloraji and Eu (2017) that developing the spatial visualisation abilities of students directly lead to high geometric achievements due to the visual nature of geometry. Masri et al. (2016) reported that teaching materials in the form of images and simulations can address these difficulties by helping students to process, visualise and build links between geometry properties and their existing mathematics knowledge.

The integration of technological instructional tools in teaching mathematics is a noticed new dimension that has the potential of helping students explore the abstract ideas of mathematics and develop an in-depth understanding of mathematical concepts which can lead to high performance. It has also been established that supporting students' mathematics learning with geometric dynamic software can create an enjoyable teaching-learning environment, emphasizing that conceptually and pedagogically, computer-assisted lessons have provided a positive impact on mathematical learning (Saha et al., 2010). Benning and Agyei (2016) posited that when instruction is aided with dynamic technological tools, the mathematical concept gets represented in multiple forms

which reveal interconnected relationships and hence enhance understanding. Therefore, the dynamic software Autograph has been adopted in this study, to explore its effect on the students' attitudes and achievements in learning circle theorems.

Potentials of Autograph software for teaching and learning mathematics

There exists numerous computer software available in the Open Source Software (OSS) market for teaching and learning of mathematics. In this study, the OSS Autograph was used to explore the geometric spatial abilities of the students. Dynamic Autograph Software, developed by Douglas Butler in 2005 is an open-source and free Dynamic Mathematics Software (DMS) that is used for teaching and learning mathematics at all levels of education. Autograph provides a linking interface of dynamic geometry systems and computer algebra systems that can reveal the relationships of concepts in calculus, statistics, algebra, geometry, and vectors when students use it for exploring these concepts (Tarmizi et al., 2009). The use of Autograph as a teaching tool has the potential to transform the learning process by enhancing teachers' pedagogical approaches, and engaging students in interactive geometric constructions, demonstrations and explorations of relationships (Isiksal & Askar, 2005). According to Tarmizi et al. (2009), an Autograph is multiplatform that can offer students opportunities to learn mathematics in more interactive and attractive ways through independent and collaborative explorations of concepts.

Researchers explain that Autograph fosters student-centred learning, active student participation, collaborative learning, and discovery learning as it offers the platform to experiment with mathematical ideas, and theorems and engage in interactive explorations (Tarmizi et al., 2008, 2009). The research literature is replete about the significant effect of Autograph technology on students' mathematics self-efficacy and

on their ability to represent concepts in variant forms (Ragelia Sinaga et al., 2018). This highlights the potential of the Autograph technology in enhancing the mathematical confidence of students and supporting their experiences with learning alternatives such as representing contents in variant forms. The findings of cooperative learning research added that students develop effective Mathematics Problem Solving abilities (MPS) and Mathematical Connection (MC) abilities when the Autograph software is used to support the learning of mathematics (Karnasih & Sinaga, 2014). Karnasih and Sinaga (2014) emphasised that the dynamic features of Autograph support students to investigate and verify the connected relationships among mathematical concepts.

The evidence revealed by literature suggests that the use of dynamic software Autograph in learning various concepts of mathematics does have positive effects on students' cognitive and affective developments. In this study, a teaching-learning approach, the Autograph Teaching Strategy (ATS), was designed and situated in the principles of social constructivism, scaffolding, and zone of proximal development to explore the effect of the Autograph technology on students' attitudes and achievements in learning circle theorems. Autograph Teaching Strategy (ATS) offered the opportunity to explore the social constructivist teaching principles which foster learner-centredness, active student participation, collaborative learning, and discovery learning (Tarmizi et al., 2009). Based on this study, Autograph was used to model a technology integration framework for teachers, guided by social constructivist teaching theories which offered a platform for students to explore geometric ideas, theorems, and relationships to gain conceptual understanding.

Theoretical Underpinnings and Conceptual Framework

This study is underpinned by the social constructivist theory of learning and conceptualised using the concepts of Zone of Proximal Development (ZPD) (Amineh & Asl, 2015). Social constructivism concerns the construction of knowledge and understanding that is attained jointly by individuals in a social context (Amineh & Asl, 2015). This process of knowledge formation must be done by students themselves. Thus, in this social constructivist setting of learning, learners were actively engaged in active thinking and organising, exploring geometric properties in social groups, and offering peer support to make sense of the geometric relationships (Zulnaidi & Zakaria, 2012). Also, the social constructivist classroom was characterised by: direct minds-on and hands-on activities that encouraged cognitive explorations; learner-centred and autonomy where students' ideas mattered and each student was responsible for their learning; social interaction where students worked in groups by sharing ideas and more capable peers supporting other learners; and the teachers' role was more of demonstrative and consultative. These principles of social constructivism were anchored with the tenets of the Zone of Proximal Development (ZPD) to model a conceptual framework (Figure 1) based on which the research objectives were formulated. Mcleod (2012) explained that the ZPD is the gap between what a learner is capable of doing without the support and what can be achieved with support, encouragement, guidance, and motivation from a more knowledgeable and skilled partner. Also, scaffolding is the guidance and support given to a student to attain a specific ZPD (Shadaan & Leong, 2013).

In this study, Autograph software was the primary scaffold as well as the teachers' and peers' support. The ZPD of students was their developed visualisation abilities and

conceptual understanding of circle theorems and a positive attitude formed towards learning circle theorems with the Autograph software. The development of students' ZPD was conducted using two social constructivist teaching-learning strategies, described as social-interaction Autograph strategy, and self-exploration Autograph strategy. The framework (Figure 1) of this study was modelled using these social constructivist teaching-learning strategies (Social-interaction Autograph strategy and self-exploration Autograph strategy), scaffolds (teacher-student guidance and peer-to-peer support), and mediating tools (Autograph software) as means to attain the ZPD (ability to visualise and understand circle theorems, and form a positive attitude to learning with Autograph) of students. The

(S) in this study means *"students working in groups"* using the Autograph software to learn the concepts of circle theorems. The concept of self-exploration (S) is defined as, *"a student working alone or individually"* using the Autograph software and teacher facilitation to learn the concepts of circle theorems. Achievement (A), the ZPD of students, is the numerical estimate of students' performance in the circle theorems achievement test obtained as a result of their ability to visualise and understand circle theorems when answering questions. The students' attitudes (A) explored in this study was their attitude toward using Autograph software to learn circle theorems in either a social interaction or a self-explored environment. The students' perceived ease of use of Autograph and their perceived
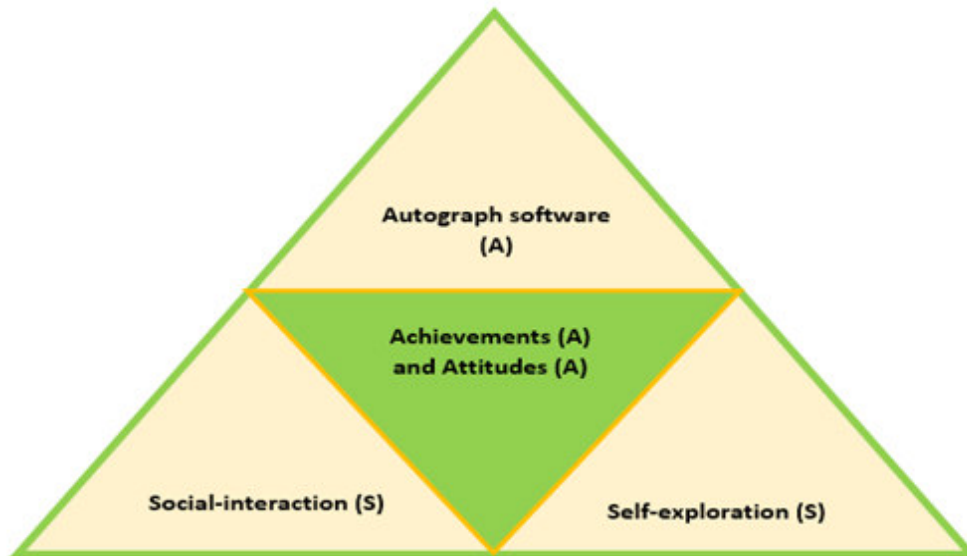

Figure 1: A S²A² Conceptual Framework

relationship between the social constructivist teaching-learning strategies (Social-interaction Autograph strategy and self-exploration Autograph strategy) and the mediating tool or scaffold (Autograph software) that leads to attaining the desired ZPD (positive attitudes and achievements) is conceptualised as the AS^2A^2 framework in this study, shown in Figure 1.

The Autograph (A) software is a *"dynamic geometric software or a computer software"* which was used in the study to teach and learn Circle Theorems. Social-interaction

usefulness of Autograph constituted or defined their attitude towards the Autograph software they used in learning circle theorems.

The underlying concepts of the framework; Technology (Autograph software), Social-interaction, and Self-exploration interplayed to develop the visualisation ability and conceptual understanding of geometry among students. Social interaction and self-exploration were concepts that guide how the research intervention activities are designed, how the research setting is

organised and the roles of participants. Autograph as a concept was a mediating tool and a scaffold in a constructivist classroom. The consequence of the interaction effect of social interaction or self-exploration and technology on the learner is visualisation and understanding of circle geometry, which is theorised as the zone of proximal development of the learner. That is, the zone of proximal development of learners, formulated from the framework enhances students' ability to visualise and understand circle geometry using Autographs in either social-interaction or self-exploration classrooms.

From the AS²A² framework, the social-interaction concept engaged students to work in groups, support one another by sharing understanding and used the Autograph as a scaffold to enable them to explore and visualise the circle theorems. More capable students helped less capable ones to attain their ZPD whiles the higher ability students also gained insights from the shared knowledge of their peers. The self-exploration concept provided a platform where individual students supported their learning of circles with the Autograph software, unlike social interaction where students learned in groups. Each student used the Autograph hand-in-hand with the facilitator to construct and explore the circle theorems and made conclusions based on their findings.

Research Questions

In this study, the consequence of integrating Autograph technology into a social interaction or self-exploration classroom was investigated. Based on the underlying concepts of the AS²A² framework, the researcher aimed to explore the effect of the Autograph software on Senior High school students' achievements and attitudes towards learning circle theorems. The main objectives of the study were to: explore the effect of Autograph technology on the achievement of students learning of circle theorems; and examine the attitude of students towards using Autograph technology to learn circle theorems. Specific research questions that were formulated to guide the study are: (1) to what extent do the teaching methods (social interaction, self-exploration, and conventional) affect students' achievements?

Methods

Research approach

To explore the effect of Dynamic Autograph software on students' achievement in learning circle theorems and ascertain their attitudes towards using the technology in either a social-interactive or self-explored classroom, both quantitative and qualitative data were used to answer the proposed research questions. Therefore, a seemingly (quasi) explanatory sequential mixed-method design was employed as a strategy of inquiry (Creswell, 2012). To determine the impact of the Autograph-assisted lessons on students' achievements, a pre-post-test quasi-experimental control group design was used.

The variables of this study are the teaching method (social-interaction and self-exploration Autograph Teaching Strategies, and the Conventional Teaching Strategy), the student's achievement in circle theorems, and the attitude of students towards using the Autograph technology to learn in either a social-interactive or a self-explored class. The teaching method is the independent variable with three levels; social-interaction Autograph teaching strategy, self-exploration Autograph teaching strategy, and the conventional teaching strategy which when manipulated, were hypothesised to affect the dependent variables. The dependent variables of the study were students' circle theorems achievements and their attitudes toward using dynamic Autograph software to learn circle theorems in a socially interactive environment or self-explored environment.

The social-interaction Autograph teaching strategy group was learner-centred and was

purported to observe the effect of the Autograph technology in a collaborative pedagogically oriented learning classroom. Students used the Autograph software in groups of 2 or 3 to explore the circle theorems alongside the researchers' facilitation. In the self-exploration Autograph teaching strategy group, each student was assigned a computer to use for learning the circle theorems. This second experimental group was meant to inquire about the sole effect of the Autograph technology on students' attitudes and achievements in a self-explored learning environment. The control group was taught with the Conventional Teaching Strategy where the theorems were demonstrated by the researcher, by drawing them out on the whiteboard and explaining them to the students. The research design was structured to provide the same pre-tests for the three groups before the treatments are implemented and the same post-tests for all groups after the intervention. After the post-tests were conducted, a perception questionnaire and a semi-structured interview were used on the students to ascertain the attitude of students towards using Autograph software to learn circles theorems in a social-interactive or a self-explored classroom.

Respondents

The study participants were obtained by using a multi-stage sampling procedure. The study used three Senior High Schools within the Bono East Region, Ghana. For purposes of group equivalence, the three schools were of the same category according to the Ghana Education Service (GES) categorization of SHSs. The specific schools were purposively selected based on ICT resource availability and the random number probabilistic generation procedure was employed to obtain an intact class from each selected school. Two of the intact classes were considered as the Autograph Teaching Strategy (ATS) groups, thus, social-interaction and self-exploration (Experimental groups) and the other as the

Conventional Teaching Strategy (CTS) group. The total sampled students who participated in the study were 114 (70 males and 44 females). There were 41 students in the social-interaction group, 42 students in the self-exploration group, and 31 students in the control group.

Measures

Three types of instruments were used to obtain data from the respondents; the Circle Theorems Achievement Tests (CTAT), Students' Attitude Questionnaires (SAQ), and Interview guide. The CTAT were used to measure students' conceptual knowledge of circle theorems before and after each circle theorem lesson. Also, SAQ and the interview guide assessed students' perceptions towards using Autograph to learn in either a social-interactive or self-explored classroom. CTAT is a pre-post teacher-made test with a reliability index of 0.786 which is within acceptable standards (Ary, Jacobs, Sorensen, & Razavieh, 2010). The Students' Attitude Questionnaires (SAQ) were adapted from Technology Acceptance Model (TAM) attitude questionnaires (Venkatesh & Davis, 2000; Ma & Liu, 2004; Lee et al., 2003). SAQ was made of two sub-scales: Perceived Ease of Use of Autograph (PEOU) and Perceived Usefulness of Autograph (PU), with Cronbach Alpha coefficients of 0.81 and 0.74 respectively. Students of the Autograph-assisted groups were to indicate the extent of their agreement to each statement, on a five-point Likert scale ranged from strongly disagree to strongly agree (scored from 1 to 5). The interview guide was made up of two items that were geared towards obtaining information about how easy and useful students perceived the use of the Autograph technology in learning circle theorems.

Treatments

This study used three different treatments as the intervention; Autograph Teaching Strategies (ATS) were used for the experimental groups (social interaction and

self-exploration) whilst the Conventional Teaching Strategy (CTS) was for the control group. With the social-interaction Autograph teaching strategy, students used the Autograph software to learn by working in small groups of two or three whilst with the self-exploration Autograph teaching strategy, the students used the Autograph software individually (one student to one computer) to learn the circle theorems. The conventional teaching strategy was whole-class activity-based and teacher-directed instructions from the whiteboard.

All lessons of the experimental groups were conducted in the schools' computer laboratories where students readily had access to the computers in which the Autograph software was installed. Students of ATS were first introduced to the Autograph software to make them familiar with the features and functions of the software. In the second phase, students of each group were introduced to the concept of circle theorems. This was followed by Autograph-assisted teaching in the respective experimental groups and conventional teaching strategy in the control group.

The instructional materials for this study were lesson plans and students' worksheets. For consistency, similar instructional materials were used for all three groups. The interventions were carried out in four weeks and six similar lesson plans and students' worksheets were developed and used for each group to learn the concepts of circle theorems. The lessons were structured in the form: an introduction to the circle theorem; technology exploration of the theorem (for social-interaction and self-exploration groups) and whiteboard illustration for the control group; paper-pencil worksheet activities; and lesson evaluation. All groups were taught the same content to reach the same objectives in the cognitive domain.

There were three intermittent pre-tests and their corresponding post-tests for each group. The same pre-tests and post-tests

were administered to all three groups and each test lasted a duration of 10 minutes. The three different pre-tests were combined to form one pre-test and the three different post-tests formed one post-test. At the end of the respective treatments, the students' attitude questionnaire was administered to students of the social-interaction and self-exploration groups to elicit their perceptions about using Autograph to learn circle theorems. Also, six volunteered participants, three from each experimental group were interviewed to seek their in-depth views about how easy and useful the Autograph supported their learning of circle theorems.

Data analyses

In the data analyses, descriptive statistics, independent sample t-test, paired sample t-test, and analysis of covariance (ANCOVA) were used to describe the performance of students in all groups in their pre-test and post-test. Descriptive statistics and independent sampled t-test were also used to analyse the students' attitude questionnaires to describe their attitudes towards using Autograph to learn circle theorems in either social interaction or a self-explored learning environment. The interview data were transcribed and analysed based on the emerged themes.

Results

The results and discussion of this study are presented in the following section based on the research questions and hypotheses.

Effect of Social-interaction Autograph Teaching Strategy, Self-exploration Autograph Teaching Strategy, and Conventional Teaching Strategy on Achievement

A paired sample t-test was conducted to determine the effect of each teaching strategy on the achievements of students. The extent of the effect on the achievement of students by each teaching strategy was determined by comparing students' pre-post-tests mean scores of each group. Results of the paired sample t-test between

the pre-test and post-test of each group are shown in Table 1.

19.26, SD = 3.81). A paired samples t-test comparative analysis of the pre-post-test scores of the self-exploration group

Table 1 Paired Samples t-test of Social-interaction, Self-exploration and Conventional Groups

	Pre-test		Post-test					Effect Size
	Mean	SD	Mean	SD	T	df	Sig.	
Social-interaction	7.76	3.72	22.34	3.53	20.96	40	.0001	.917
Self-exploration	12.02	3.87	19.26	3.81	10.14	41	.0001	.715
Conventional	10.52	5.42	14.13	3.51	-5.08	30	.0001	.462

Statistically significant at $p < \alpha = .001$

With regard to the effect of social-interaction Autograph teaching strategy on achievements of students of the social-interaction group, the pre-test mean score (M = 7.76, SD = 3.72) and post-test mean score (M = 22.34, SD = 3.53) suggests an improvement in their achievement. Results of the paired sample t-test revealed a statistically significant difference between pre-post-tests mean scores of students; $t(40) = 20.96$, $p < 0.001$. The extent of impact of the teaching method revealed a statistic of 0.917 which is a large effect size between the pre-post-tests mean scores (Pallant, 2016; Field, 2018). This meant that the extent of the difference in achievements between the pre-test and post-test mean scores of the students was very large. This is an indication that the students had progressed significantly in their understanding of circle theorems after the lessons. The significant improvement realised in achievements is suggested to be as a result of engaging the students in small groups as well as supporting their learning processes with the Autograph software.

The results of the effect of the self-exploration Autograph teaching strategy on the achievements of students of the self-exploration group revealed that there is a difference in the achievements of students' pre-test and post-test mean scores. Improvements in achievement were realised in the students' post-test scores, from their pre-test mean score of (M = 12.02, SD = 3.87) to a post-test mean score of (M =

indicated that the difference in mean scores was statistically significant; $t(41) = 10.14$, $p < 0.001$. The self-exploration teaching approach had a large effect size of 0.715 on the achievements of the students. This suggests that 71.5% of the improvement realised in achievement from pre-test to post-test could be explained by the self-exploration Autograph teaching approach. Thus, students of the self-exploration group who learned circle theorems by using the Autograph software individually had improved their understanding of the theorems and also performed significantly better after going through this learning process.

Concerning the effect of the conventional teaching approach on students' achievements, the results revealed that the pre-test mean score increased from (M = 10.52, SD = 5.42) to (M = 14.13, SD = 3.51) in the post-test. This suggests improvement of students' achievements in the post-test after going through the Conventional Teaching Strategy. The paired sample t-test comparison of the pre-post-test mean scores also showed that the mean difference between the pre-post-test mean scores of students of the control group was statistically significant; $t(30) = -5.075$, $p < 0.001$. There was a large effect size of 0.462 between the pre-post-tests mean scores, suggesting that 46.2% of the improved variation in the post-test scores was explained by the Conventional Teaching Strategy. These results indicate

that when the students were taught using the conventional teaching approach, they also gained a significant understanding of the circle theorems concepts. Therefore, the Conventional Teaching Strategy of this study also helped improve students' understanding and achievements in circle theorems though not as much compared to the social-interaction group (effect size = 0.917) and the self-exploration group (effect size = 0.715).

Generally, each of the teaching strategies had a significant effect on the achievements of the students in each group. The students experienced various improvements in their achievements and understanding of circle theorems. The paired samples t-test analyses revealed very large effect sizes on the achievements of students from their pre-test to post-test mean scores. The teaching strategies revealed different effects of 0.917, 0.715, and 0.462 on the achievements of students of the social-interaction, self-exploration, and the control groups respectively.

Comparing achievements of students taught with the social interaction, self-exploration, and conventional teaching strategies

The effect of each teaching strategy showed a different impact on the students' understanding and achievements across the groups. The social-interaction Autograph teaching strategy revealed a greater impact (effect size = 0.917), followed by the self-exploration Autograph teaching strategy (effect size = 0.715), and then the conventional teaching strategy (effect size =

0.462). The significance of these differences in achievements across the groups was ascertained through hypotheses tests using ANCOVA, by respectively comparing the social-interaction group with the conventional, the self-exploration group with the conventional, and the social-interaction group with the self-exploration group. The analyses of covariance are reported in Table 2.

Differences in students' achievement between the social-interaction and the conventional groups

The comparative analysis of students' post-test scores of the social-interaction group and the conventional group after the intervention revealed that the effect of the social-interaction Autograph teaching strategy on the social group students' achievement is statistically significantly different from students of the conventional group who learned circles by the conventional teaching strategy, $[F(1,69) = 138.874; p < 0.001]$. This implies that students of the social-interaction group (Mean = 22.34, SD = 3.53) achieved significantly more in their post-test mean scores than students of the conventional group (Mean = 14.13, SD = 3.51). A very large effect of 0.668 caused by the social-interaction Autograph teaching strategy on students' post-test achievements was realised. This indicated that 66.8% of the improvement realised in favour of social-interaction students was as a result of the teaching method (integration of Autograph software in a social setting) used in the

Table 2 ANCOVA results comparing social-interaction, self-exploration, and conventional groups

	Mean	SD	df	F	Sig.	Partial Eta Squared
Social-interaction &	22.34	3.53	70	138.874	.0001*	.668
Conventional	14.13	3.51				
Self-exploration &	19.26	3.81	71	33.533	.0001	.324
Conventional	14.13	3.51				
Social-interaction &	22.34	3.53	81	21.237	.0001	.210
Self-exploration	19.26	3.81				

*Statistically significant at $p < \alpha = .001$

group. The high effect of the social-interaction Autograph teaching strategy on the achievement of students is noted in the pre-post-tests within-group paired samples analyses of the social-interaction and the control groups. From Table 1, the social-interaction Autograph teaching strategy explained 91.7% (effect size = 0.917) improvement in post-test scores against 46.2% (effect size = 0.462) improvement in post-test scores explained by the conventional teaching strategy for the control group. This finding suggested that students who learned through the social-interaction Autograph teaching approach achieved significantly better compared to those of the conventional teaching strategy.

Differences in students' achievement between the self-exploration and the conventional groups

The ANCOVA results in Table 2 indicated that the effect of the self-exploration Autograph teaching strategy on self-exploration students' achievement was statistically significantly different from students of the control group, who learned circle theorems through the conventional teaching strategy $[F(1, 70) = 363.225; p < 0.001]$. The impact of the difference in achievements between the groups was 0.324. This suggests that 32.4% of the significant improvements in achievements in favour of the self-exploration group was explained by the teaching method (self-exploration Autograph Teaching Strategy). Thus, students of the self-exploration group (Mean = 19.26, SD = 3.81) demonstrated a significantly better understanding and performance in their post-test scores than students of the control group (Mean = 14.13, SD = 3.51). Within-group paired samples analyses revealed that the self-exploration Autograph teaching strategy yielded an improvement effect of 71.5% (effect size =0.715) on the students' achievement as compared to an improvement effect of 46.2% (effect size =0.462) caused by the conventional teaching strategy of the control

group. The significant difference in effect sizes in favour of the self-exploration group indicates that students' exposure to learning the concepts of circle theorems with the Autograph software contributed to their better understanding and hence, caused a high significant effect on their achievements.

Differences in students' achievement between the social-interaction and the self-exploration groups

The comparative analyses of covariance results of the Autograph-assisted groups illustrated that there is a statistically significant difference in achievement between students of the social-interaction and self-exploration groups, $[F(1, 80) = 21.237; p < 0.001]$. A large beta square of 0.210 (Pallant, 2016; Field, 2018) revealed that students of the social-interaction group advanced in achievements of about 21.0% more than students of the self-exploration group due to the impact of the social-interaction teaching approach. The teamwork effect, resulting in high achievement among social-interaction students was also realised in the pre-post-tests within-group paired samples comparison of the social-interaction and self-exploration groups as reported in Table 1. The results showed that the students' teamwork or peer-to-peer support and the Autograph software factor caused a 91.7% (effect size = 0.917) improvement in post-test scores for the social-interaction group as compared to an improvement effect of 71.5% (effect size = 0.715) caused by integrating only the Autograph software to individual students' learning activities in the self-exploration group. The large difference in effect size $(0.917 - 0.715 \cong 0.210)$ between the groups suggests that the greater achievements realised by students of the social-interaction group could be accounted for by the social interactive nature or teamwork activities and peer-to-peer support experienced by the students. These results have confirmed that both social-interaction and self-exploration teaching

approaches are effective methods that can significantly support students learning of mathematics in computer-supported environments. However, the social-interaction Autograph teaching strategy has revealed to be a relatively more effective pedagogical approach with varying potentials that can support students learning and improve their experiences.

Attitude of students towards using Autograph to learn circle theorems

The students' experiences about how they perceived learning with the Autograph technology were assessed using the students' attitude questionnaires and interview guide. Table 3 shows results of the

0.33) indicated the students had positive attitudes towards using Autograph technology to learn circle theorems. The overall attitudinal mean scores of both groups also inform that students of the self-exploration group (M = 4.73, SD = 0.33) seem to demonstrate greater positive attitudes towards learning circle theorems with Autograph technology as compared to students of the social-interaction (M = 4.54, SD = 0.56) group. Generally, students of the Autograph-assisted groups positively perceived the Autograph technology to be easy to use and useful in their learning processes of circle theorems, with students of self-exploration demonstrating greater positive attitudes as compared to their

Table 3: Students' attitude about learning with Autograph

Sub-scale	Social-interaction (N=41)		Self-exploration (N=42)	
	M	SD	M	SD
Ease of Use	4.55*	0.67	4.79	0.33
Usefulness	4.53	0.45	4.67	0.34
Overall	4.54	0.56	4.73	0.33

*Positive attitude at Mean ≥ 3.0

question, "What are the attitudes of students towards using Autograph technology to learn circle theorems?" among students of the social-interaction and self-exploration groups.

These results suggest that students of the experimental groups gave positive feedback about using the Autograph software to learn circle theorems. The reported mean scores of 'Ease of Use' for the social-interaction (M = 4.55, SD = 0.67) and self-exploration (M = 4.79, SD = 0.33) showed that the students perceived learning with the software to be easy to use. Also, the mean scores of the 'Usefulness' for the social-interaction (M = 4.53, SD = 0.45) and self-exploration (M = 4.67, SD = 0.34) indicated that the students perceived using the software was beneficial to their learning of circle theorems. The overall attitudinal mean scores of both groups; social-interaction (M = 4.54, SD = 0.51) and self-exploration (M = 4.73, SD =

social-interaction counterparts.

Reports from the students' interviews equally portray that the students demonstrated positive attitudes when lessons were supported with the Autograph technology. This supports the outcome of the attitude scale results which also depicted positive attitudes of each group towards learning with the software. The students' interview responses revealed that features of the software were readily usable which made Autograph easy to use for the circle theorems lessons. The students also reported that supporting classroom interaction with Autograph made lessons practical, interactive, and stimulating. And finally, the software offered the students multiple representations of the concepts and simulations that enhanced the visualisation and imagination of geometric properties and relationships.

Comparing attitudes of students towards using Autograph to learn circle theorems

Descriptive statistics in Table 3 showed that students of both social-interaction and self-exploration had an overwhelming positive attitude towards learning circle theorems using the Autograph technology. Results of the students' attitude scale suggested that students of the self-exploration (M = 4.73, SD = 0.33) group appeared to report higher positive means than that of social-interaction (M = 4.54, SD = 0.56) students. An analysis of independent samples t-test was conducted to test the hypothesis of a significant difference between attitudinal means of the students as shown in Table 4.

using the Autograph technology for learning mathematics.

Discussion

This study explored the effect of Autograph as a technological instructional tool on student's achievements and attitudes in learning circle theorems. The study employed two social constructivist teaching approaches (social-interaction Autograph teaching strategy, and self-exploration Autograph teaching strategy) as well as the conventional teaching approach to determine the impact of Autograph technology on students' outcomes and attitudes in learning circle theorems. Whiles the social constructivist teaching approaches

Table 4: Independent samples t-test of students' attitudes

	Mean	SD	Mean Difference	t	df	Sig.	Effect Size
Social-interaction & Self-exploration	4.54 4.73	0.56 0.33	0.189	-2.103	81	0.038*	0.051

*Statistically significant at $p < \alpha = .05$

The independent samples test of the students' attitudes about learning with Autograph shows that the differences between their overall attitudes were statistically significant, t (81) = -2.103, P = 0.038 < 0.05. Thus, it was concluded that students of the social-interaction and self-exploration groups had significantly different positive perceptions towards using the Autograph technology to learn circle theorems in favour of students of the self-exploration group. The differences in overall attitudes revealed a small effect size of 0.051. This suggests that about 5.1% of the differences in the overall attitudes could be as a result of the self-exploration of students' access to the Autograph technology during the circle theorems lessons processes. Therefore, assessing the difference in attitudes of the two groups has shown a significant difference between them. Students of the self-exploration group demonstrate more positive attitudes towards

used dynamic Autograph and interactive learner-centred illustrations in learning, the conventional strategy employed the talk and markerboard demonstrations without technology support.

The results of the study revealed that the students experienced a positive significant impact on their achievements in learning circle theorems for each of the three teaching methods. The Autograph-assisted teaching approaches also had a positive significant effect on the attitudes of students towards using Autograph technology to learn. Students of the social-interaction Autograph teaching approach demonstrated greater significant growth in achievement (Effect size = 0.917), followed by students of the self-exploration Autograph teaching method (Effect size = 0.715), and then the conventional teaching approach (Effect size = 0.462) with the least impact on the students' performance. Results of the hypotheses revealed significant differences in achievements of students between social interaction and conventional, in favour of

social interaction; self-exploration and conventional, in favour of self-exploration; and social interaction and self-exploration, in favour of social interaction. Also, the self-exploration Autograph-assisted students (M = 4.73, SD = 0.33) demonstrated greater significant positive attitudes about learning with Autograph technology than their social-interaction Autograph-assisted counterparts (M = 4.54, SD = 0.51). Therefore, these findings confirm that the two different methods of dynamic Autograph-assisted instruction have significantly impacted students learning outcomes compared to conventional teaching.

The improvement in the quality of work of students in Autograph-assisted lessons suggests that Autograph technology can be used as an enabling tool to empower the teaching-learning processes of circle theorems. The dynamic features of Autograph enthused the students to explore and visualise the geometric relationship as compared to static whiteboard illustrations. The collaborative and student-centred learning setting engaged students in group discussions, peer support, and sharing of ideas that increased the attention of students to the task and promoted group participation which led to enhanced knowledge formulation. These classroom engagements were evidence that Autograph is a worthy pedagogical tool that can supplement the social constructivist teaching-learning approaches, and hence, provide an excellent vehicle that fosters a conceptual understanding of geometry. These realisations reflect in the conclusions of Lee et al. (2013) that integrating technology into mathematics learning to improve cognitive and affective domains can be effective by making the learners collaborate in small groups. Thus, exploring circle theorems using Autograph provided weaker students with learning alternatives to make links between relationships, develop mathematics confidence, and support students attain high-order thinking skills. The geometric affluence of Autograph-assisted students

also aligns with the views of Mercier et al. (2016). Mercier et al. (2016) expressed that when social constructivist teaching-learning approaches are supported with dynamic software, students experience a reduction in their mental efforts, share cognitive capacities, and motivate weaker learners to increase their learning confidence to explore abstract concepts initially beyond their comprehension. This reflects and confirms the effect of the adopted principles of the social constructivist teaching-learning approaches employed in this study which motivated students to progress in their experiences to a zone of affective and cognitive mathematical confidence.

The results revealed that Autograph-assisted teaching-learning strategies challenged the students to become responsible independent learners through testing and exploration of mathematical ideas, and self-discovery of geometry relationships which fostered self-learning. The students became active participants in the learning process by using their diagrams to explore and verify geometric relationships. The work of Aydogan (2007) supports this assertion when he explained that learner-centred and technology-supported engagements have the potential to motivate experiential learning and improve the geometric cognitive capacities of students. The involvement and participation of students increased in the Autograph-assisted classrooms. Mensah (2019) shares a common belief when he posited that students get motivated to learn when their learning activities are organised and delivered in a dynamic hands-on engaging manner.

Autograph software has proven to be an effective technological tool in supporting and enhancing mathematics teaching and learning especially in circle theorems. The Autograph-assisted groups were able to experience a practical, hands-on, and or collaborative-oriented method of learning which had a positive impact and helped them to comprehend geometric concepts better

rather than being passive learners. The Autograph software allowed the teacher and students to walk through the concepts together through exploration and visualisation. This encouraged a more interactive teacher-student interactional environment where everyone worked as a team member to guide, help and assist one another to reach the required goals. The dynamic nature of the Autograph technology enabled students to represent the theorems in multiple displays or orientations which provided them with the opportunity to realise the changeable and unchangeable relationships of the theorems. Inasmuch students of the conventional group made significant gains in their understanding of circle theorems after the intervention, there were recorded difficulties with theorems that were not directly displayed in questions. Such difficulties were noted to be associated with insufficient multiple representations of the theorems when using conventional methods which offer students limited opportunities to explore and observe existing variations of the same theorem. In contrast to the experiences of students of the conventional group, interview responses indicated that students of Autograph-assisted lessons were able to observe, visualise, recognise, and comprehend geometric shapes with variant and invariant properties which provide them with spatial reasoning abilities to make conjectures and generalisations. These findings corroborate with the conclusions of Akgül (2014), and Çekmez (2020) that dynamic geometric software provides an interactive interface which offers students multiple opportunities to explore and realise the existence and preserve of geometric relationships in different figures.

The findings also revealed that Autograph technology supported the imagination and visualisation abilities of students. Students' interview comments revealed emphatic statements about how the visual simulations and images of Autograph enhanced their ability to imagine and visualise the theorems. Thus, the positive effect of Autograph-based instruction on students' achievements could be stemmed from the simulations and visual representations the software provided. These visual representations were likened to movies, video games, and mental pictures that helped students easily reflect on required theorems as compared to static whiteboard diagrams. It could then be stated that supporting students learning with dynamic software Autograph offered them different dimensions of classroom engagements that improved their spatial reasoning. This support why Hollebrands (2007) and Saha et al. (2010) claimed that visualization and exploration of mathematical objects and concepts in multimedia environments can foster understanding in new ways. Students also expressed excitement and curiosity about the automaticity of the Autograph software which motivated them to engage in testing out various theorems. The students constructed multiple geometric figures to satisfy their curiosity about the figures' conformance to and preservation of the geometric properties. Such stimulating and simulating classroom interactions encouraged active learning and developed the students' self-confidence from their self-initiated investigations and explorations. This agrees with Sinaga et al. (2018) who realised Autograph-assisted instruction impacted students' self-efficacy in mathematics and improved their ability to represent geometric properties in variant forms.

The results of the study have projected the impact of the social-interaction Autograph teaching approach on the students' achievements over the self-exploration teaching strategy. The evidence of significant achievements of social-interaction students could be associated with the collaborative nature of the learning process which fostered students' peer-to-peer support, enhanced learning confidence, motivated critical thinking skills, and improved problem-solving abilities. The learning experiences of students in the social-interaction class are confirmed by the

conclusions of several research studies (Boyraz, 2008; Lee et al., 2013; Mercier & Higgins, 2013; Higgins & Joyce-Gibbons, 2016) that when technology is employed in a collaborative setting to explore mathematical concepts, it increases the attention of students to their learning and cause equity of active participation among learners.

The results of the study have generally highlighted the consequence of the social constructivist teaching-learning principles on the students' learning of circle geometry as evident in their attitudes and achievements. The demonstrated positive attitudes and high achievements can be attributed to the designed social constructivist learning activities which were supported by the twin concepts of scaffolds and zone of proximal development. These results support the conclusions of (Dogan & Icel, 2011; Shadaan & Leong, 2013) that when technology-rich lessons are situated in the social constructivist twin concepts of scaffolding and zone of proximal development, students gain high-order thinking skills through shared experiences, enabling students to become active and independent learners, create visual opportunities that develop students' spatial abilities and redefine the role of teachers as facilitators, where students were guided to discover geometric properties and relationships. Thus, this study advocates that the social-interaction Autograph teaching strategy is considered in designing mathematics lessons with technology, especially the Autograph software. Based on Benning and Agyei (2016) conclusions, such instructional strategies engage students in self-learning that leads to concept discovery, and motivate students to engage in scientific enquiries such as questioning, argumentations, investigations, hypothesising, and verification of ideas.

Results of the attitude scale indicate that students of the Autograph-assisted groups demonstrated high positive attitudes about learning with the Autograph technology. According to Boyraz (2008) and Masri et al (2016), when learning is supported by technology, students do not avoid but enjoy using the technology to explore mathematical concepts through designed activities which motivate and affect their attitudes positively. The results showed that the students could easily use the technology to explore geometric properties. It was also revealed that the Autograph technology was perceived to be useful in exploring and visualising geometric relationships. Insights from the students' interview responses suggest that Autograph-based activities made lessons student-centred and practical. The Autograph was said to provide multiple representations of geometric properties that enhanced visualisation. The lessons were described to be interesting, exciting, and enjoyable. The simulative and imagery features of Autograph were also described to support students in developing their visualisation abilities which facilitated geometric reasoning and understanding. Berman et al. (2001) and Boyraz (2008) noted that when mathematics learning is supported with dynamic software, the students get provided with various stimulating experiences such as excitements, interests, and other enjoyable ways that motivate and encourage contextual and experiential learning. This implied that Autograph technology has high effectiveness index as an instructional tool for teaching and learning geometry. It is an effective alternative and supplement learning tool that can improve teachers' instructional practices, motivate mathematics learning, support students' spatial reasoning, and mediate students' geometric learning deficiencies.

Conclusion

It is concluded that Autograph technology is an effective instructional tool that motivates and develops positive learning attitudes, and enhances active hands-on learning that improves teaching practices and learning

outcomes of students in mathematics, especially circle theorems. Reflection on the social-interaction teaching approach encouraged that a more interactive, student-centred, peer-to-peer support, and student-teacher interactional setting is necessary to unearth the potentials of the Autograph technology to support teaching-learning processes. The learning behaviours and outcomes of students have explained that when Autograph technology learning activities are situated in the concepts of the AS²A² conceptual framework, the social constructivist principles of teaching and learning mathematics can be achieved.

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