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Addressing common deficiencies of mathematics skills among chemistry student teachers

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Abstract

The need for the inclusion of a special mathematics course for students studying chemistry at the tertiary level of education has been justified by several reasons ranging from the mathematical processes involved in understanding chemistry, students' inability to handle simple mathematical manipulations required in chemistry, and the lack of transfer of mathematical skills learned in mathematics class to chemistry. At a South African university, we faced the challenge of addressing common mathematical skill deficiencies among our chemistry student teachers (N = 82), who appeared unable to cope with basic mathematical skills required in understanding certain chemistry topics. To address this problem, students' mathematical deficiency skills (DMS) were assessed using tests on various areas of mathematics relevant to some chemistry topics. The data was collected in two stages. DMS were first identified using a Semi-open Diagnostic Test, and then students who had the worst DMS were interviewed. The findings revealed that a lack of skills in these mathematics topics (e.g., algebra, curve sketching, simple differentiation and integration) causes students to struggle in producing the breadth of knowledge required to understand chemistry concepts such as kinetics, rates, and reaction mechanisms, as students lacked: (i) the ability to resolve the relational complexity among major mathematics concepts in chemistry problems, and (ii) the ability to process information and representation skill, number and pattern concept skill, and visual-spatial skill.

Keywords: Chemistry; student teachers; mathematical deficiency; mathematical deficiency skills

Introduction

Number concepts, logarithms and indices, algebraic manipulations, graphing, calculus and general competence are important mathematical skills use in chemistry. The dominant perspective for chemical calculations is that these skills are important in learning chemistry concepts, the goals of which are to show how mathematical knowledge is constructed in chemistry (Darlington & Bowyer, 2016; Royal Society of Chemistry [RSC], 2009; Shallcross et al., 2011). There is no doubt that chemistry students do need knowledge of some abstract aspects of mathematics in chemistry. For instance, they need to differentiate, but that is deductive technique and can be done systematically. They need to be able to integrate, but little more than x^n and e^x. In most cases, they need to be able to verify that a proposed solution is a reasonable solution by differentiation. They need to be familiar with complex numbers in order to interpret the chemical equations they represent. Thus, a student teacher's development as a chemistry teacher entails, in no small part, becoming increasingly competent with the applications of various mathematical skills used in chemistry. In this connection, many chemistry educators have seen their students grappling with basic mathematical skills or getting stuck when using a limited group of mathematical

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skills to perform chemical calculations (Donovan & Wheland, 2009; Scott, 2012; Shallcross & Yate, 2014). This study refers to a student's inability to apply mathematical skills in chemical calculations as a mathematical deficiency. Therefore, mathematical deficiency in chemistry is considered to have occurred if a student lack the ability to correctly apply the relevant mathematical skills to chemical calculations or "get stuck" using a limited group of mathematical skills and fail to notice that a different set of mathematical skills could quickly and easily solve a given problem.

Research background

Many first-year students accepted into a science education program at a South African university are inadequately prepared for the level of mathematics knowledge and skills required in the study of chemistry. As a result, a supplementary mathematics course was introduced to teach mathematics in chemistry as a means of assisting students in their chemistry studies. Students who were identified to attend the course were those deemed at risk by the science education department, that is, those who fell below the cut off mark required to progress to the next semester course. These student teachers enroll in (five rather than eight) of the modules normally required of first-year science education students. The course covered topics such as algebra, curve sketching, simple differentiation, and integration, with the goal of equipping student teachers with the knowledge and skills they need to succeed in their chemistry course. Despite this valiant effort, a worrying trend in student performance persists. One of the reasons cited for the high failure rate is the discrepancy in mathematical backgrounds among first-year students enrolled in university degree programs to study science-related courses. According to some educators, school mathematics does not adequately prepare students for university mathematics (Darlington & Bowyer, 2016; Setati, 2009).

The general concern at the institution where the current study was conducted was in attempting to maintain a standard in chemical mathematics course with our major students who appeared to be unable to cope with the basic mathematical skills required in learning and understanding certain chemistry topics (e.g. kinetics, rates and reaction mechanisms, nuclear chemistry and radioactivity). As part of our effort to address this issue, lecturers' workloads are being increased in order to retrain students with mathematical skills up to the level required to understand basic and complex chemical calculations. As such, a significant amount of time is spent on remedial classes for students who struggle with basic mathematical concepts. Despite this, many students give up mathematics and refuse to attend mathematics classes. Students concede that they do not understand mathematics and find it hard to acquire a feeling for the nature of this subject and its place in chemistry. A student once asked whether he could pass a chemistry examination if he avoided solving the mathematical problems in the examination paper.

Mathematics is a requisite for chemistry students, and its application is made even harder by the fact that a minor error can invalidate the entire result of a chemistry experiment. One aspect which illustrates this point is that when Rutherford was confronted with the problem of interpreting his 'alpha-scattering experiments'; he had to return to school to study mathematics (Cline, 1965, p. 4). As a result, his work took another dramatic turn after discovering the nucleus. He was first to knock out a part of it and thus change one element to another by artificial means. Speaking of this feat in science, it is common practice among scientists to use mathematics to confront nature and force her to reveal her secrets (Iwuanyanwu, 2019a). For the same reason, students must have a solid foundation in mathematics prior to studying chemistry, because mathematical reasoning and critical thinking skills go hand in hand with procedural fluency in chemistry problem solving (Darlington & Bowyer, 2016; Scott, 2012; Shallcross & Yate, 2014). For example, the concept of entropy is rarely understood simply by definition without first mastering the mathematical aspect of it. Therefore, the problem of this study needs to be addressed, especially for chemistry student teachers with poor background knowledge of mathematics.

The purpose of this study was to investigate common mathematical skill deficiencies among chemistry student teachers enrolled in a four-year science education program at a South African university. The study had two focus questions. These were:

1. What deficient mathematical skills do chemistry student teachers hold?
2. What factors, in the opinion of student teachers, are the causes of their mathematical skill deficiencies?

To answer these questions, a semi-open diagnostic test, the Maths-in-Chem Deficiency open-ended Test (MCDT), was used to identify student mathematical deficiencies applicable to chemistry. The MDCT was chosen because Garderen (2006), Setati (2009), and Shallcross, et al., (2011), reported that judging a student's ability to apply or express mathematical skills to a given context is the best way to determine whether or not a student has a deficiency of mathematical skills. However, MDCT can only be used to determine whether or not the student experiences a deficiency of mathematical skills. Hence, the identified mathematical skill deficiencies and their causes could be confirmed through interviews.

Literature review

Mathematics – a tool for understanding basic sciences

Mathematics is one of the core subjects over which much emphasis is placed in popular courses such as Medicine, Engineering, Computer Science, and Pharmacy, because of its practical utility-value in understanding important aspects of these courses. Recognizing the significance of mathematics in understanding basic science subjects such as physics, chemistry, and life science is, in fact, a stepping stone to success in science careers. Mathematics has been referred to as the "backbone of science" by some scholars (Bing & Redish, 2009; Kemeny, 1959; Popper, 1965), and it is widely used by scientists because it allows them to express themselves beautifully in half a line what would otherwise take a whole book of words. Mathematics provides a language for the concise expression and application of chemistry's physical and chemical properties, as well as their relationships (Kousathana &Tsaparlis, 2002). Students who do not enjoy mathematics and/or chemistry in high school are unlikely to pursue chemistry/mathematics majors in university (Darlington & Bowyer, 2016; Setati, 2009). These disaffected students, in particular, will carry with them a negative image of chemistry/mathematics as something "very difficult," "very theoretical," or "boring" (Iwuanyanwu, 2019b, p.16).

It is known that many students enter university without the basic computation skill (Shallcross et al., 2011), information skill (Royal Society of Chemistry, 2009), number concept skill (Yuriev, Capuano, & Short, 2016), and visual-spatial skill (Garderen, 2006) required for chemistry courses, as well as the habits of mind required to successfully complete college-level work (Venezia & Jaeger, 2013). It is also known that some mathematics and science students do not possess sufficient logical reasoning skills

(Kuo et al., 2013) and are not accustomed to reading and understanding problems, organizing strategy and solving problem, and checking reasonableness of problem solution as well as processing errors (Tambychika & Meerah, 2010). The underlying assumption is that these students may have a difficult time tackling complex chemical calculations found in topics such as kinetics, rates and reaction mechanisms, nuclear chemistry and radioactivity, and might struggle for a long time to complete a chemistry course due to a lack of mathematical knowledge and skills. A general concern for many students in the latter category is that they may develop a lot of anxiety and frustration, and may become averse to chemistry and try to avoid it as much as possible, resulting in lower grades in chemistry courses.

Research has also shown that many students who accepted to study science related courses at universities are poorly prepared for first year mathematics. Some studies, for example, have found that undergraduate students struggle with areas of mathematics essential for the study of chemistry by investigating students' mathematical knowledge and skills in chemistry (Royal Society of Chemistry [RCS], 2008), students solving the mathematics problems in chemistry (Iwuanyanwu, 2019c; Shallcross et al., 2011), degree of mathematical fluency among chemistry students (Darlington & Bowyer, 2016), and students' understanding of mathematical expressions in physical chemistry contexts (Becker & Towns, 2012). It has also been reported that students invent their own mathematics in order to solve science problems (Bing & Redish, 2009). In South Africa, the achievement of standards in mathematics and science education remains a source of concern that must be addressed (Jojo, 2019; Petersen, 2017; Selvaratnam, 2011). Each year, the proportion of students who require remedial preparation in mathematics and science courses grows (Iwuanyanwu & Ogunniyi, 2020; Kizito, Munyakazi, & Basuayi, 2015). Departments of science and mathematics education cannot be exempt from this problem. Our student teachers face challenges. We should not be defensive about this. If we approach each challenge positively, we will be able to achieve excellence in our teaching. It seems clear that by grappling with and attempting to resolve our difficulties, we keep our subject alive and our teaching in line with contemporary needs and attitudes.

The nature of mathematical deficiency in chemistry

Some deficiencies in mathematical skills that students at various educational levels have are easily recognizable and identifiable. For example, some deficiencies of mathematical skills have been identified in a variety of chemistry topics. A few of these are chemical kinetics (Habiddin & Page, 2020), chemical equilibrium (Kousathana & Tsaparlis, 2002), redox reactions (Johnstone, 1993) and acid-base reactions (Jiménez-Liso, 2020). Many chemistry educators believe that students' mathematical deficiencies can be difficult to address (Voska & Heikinnen, 2000), because they actively influence student learning of interrelated chemistry and mathematics topics (Scott, 2012), making it difficult to improve students' understanding of fundamental chemistry concepts (RSC, 2008). A number of factors have also been identified as contributing to students' deficiencies in mathematical skills, including the nature of instructional/learning materials, students' attitudes toward learning, teachers' methods of instruction, reference books, and the relationship between these factors. The Royal Society of Chemistry (2009) alerts readers that most chemistry learning materials contain abstract concepts, and the relationships between complex mathematical concepts are elusive. As a result, students' ability to think abstractly is limited because they rely heavily on sense-based learning as well as prior knowledge because basic concepts serve as the foundation for student learning on a given topic (Devetak, Vogrine, Glazar, 2007). Teachers' methods of instruction can contribute to ineffective learning of basic mathematics or chemistry concepts due to their own deficiencies or failure to communicate effectively with students (Donovan & Wheland, 2009). Chemistry and/or mathematics textbooks and/or handbooks do not always provide complete and accurate information or explanations (Goes, Chen, Nogueira, Fernandez, & Eilks, 2020; RSC, 2009).

There is sufficient empirical research evidence to show that a lack of mathematical skills can have a negative impact on teachers' and students' abilities to solve complex scientific problems and can significantly impede a deeper understanding of many important concepts, particularly those of a scientific nature (Basson & Kriek, 2012; Selvaratnam, 2011). What appears to be required is an emphasis on key aspects of mathematics such as algebra, curve sketching, simple differentiation and integration, and general competence in students' chemistry learning. This could lead to our students gaining a better understanding of specific aspects of chemistry as well as a broader understanding of what chemistry is as a discipline. There is no doubt that mathematics plays a significant role in chemistry. In this regard, Redlich (1972) asserts that mathematics in chemistry is widely accepted and that its effectiveness in achieving educational goals is rarely questioned. This is because negating the importance of mathematics in chemistry is akin to attempting to separate "what matters" in describing a physical system – one in which the system's complex behavior is derived from combinations and elaborations of simple structures and their interactions.

Factors contributing to students' deficiencies of mathematical skills in chemistry

As previously stated, teachers or lecturers may be the source of students' mathematical deficiencies due to inconsistencies and inadequacies in their teaching methods or their use of unfamiliar mathematical terms or symbols. For example, the use of difficult-to-interpret terms or symbols may contribute to students' mathematical deficiencies (Gray & Tall, 1993). Textbooks may also present chemical information in symbols or terminologies that students find difficult to understand (Goes et al., 2020). The inability of science teachers to properly present abstract concepts of chemical calculations to students, either through visualization or analogy, can also contribute to students' mathematical deficiency. According to the findings of a study conducted by Basson and Kriek (2012), the majority of teachers (N = 68) tested in their study were unable to perform simple mathematical conversions or apply contextual knowledge. This was found to be the case, particularly in township and rural schools in three South African provinces: Gauteng, North West, and Western Cape. The authors argue that teachers who do not fully understand the content they teach may also be unable to detect correct or incorrect answers in textbook calculations. This concern, as well as those raised previously, may not be adequately addressed if the student teachers' mathematics history, for example, their entry qualification, is not taken into account.

Students' inability to demonstrate how mathematical knowledge is organised in specific chemistry concepts has been shown in research (Barry & Davis, 1999; Habiddin & Page, 2020; James, Montelle, & Williams, 2008). Is it possible that the teaching of these subjects is deteriorating, or that we are attempting to teach students of mathematics and science to less able students, or that it has something to do with the pedagogical issues unique to science and mathematics, as well as the widely used terminologies in these subjects? A study conducted in the United Kingdom by Mahdi (2014) found that poor mathematics background influenced students' attitudes negatively toward chemistry by 24 percent (N = 70). A study on students' understanding of chemistry concepts conducted by Ogunniyi and Mikalsen (2003) with the sole purpose of determining why the percentage entry for science students in South Africa (N = 130) and Norway (N = 121) was low, revealed that students' ideas about chemistry concepts (e.g. acids and bases) are fraught with conceptual, linguistic, and attitudinal problems. The overall performance of the students in both countries was below what was desirable.

Methodology

The current study used a survey design (Creswell & Plano Clark, 2011) to collect data using a questionnaire (semi-open diagnostic test) to investigate student teachers' mathematical deficiencies in areas such as algebra, curve sketching, simple differentiation and integration applicable to nuclear decay rates (kinetics) and half-life, decay series, radioactive dating, and so on. Although no topic is exceptionally difficult, the latter topics can present some difficulties for the average first-year chemistry student teacher with limited mathematical skills. Purposive sampling (Creswell & Plano Clark, 2011) was used to select 82 chemistry student teachers enrolled in a four-year science education program at a South African university. The entry requirements for mathematics and physical science were examined (see Table 1).

Table 1 Distribution of mathematics and physical science entry level

Description of score	Range (%)	Percentage (%) of students that passed	
		Mathematics	Physical science
Code 6	60 – 69	13.6	15.3
Code 5	50 – 59	18.2	20.9
Code 4	40 – 49	23.7	22.5
Code 3	30 – 39	34.4	39.6

In terms of mathematics enrollment, there were 133 students in the mathematics course, of which 82 (61.65%) were in developmental or remedial level mathematics and chemistry courses. As seen in Table 1, more than 31% of the students passed above 50% (i.e. codes 5 and 6) in their high school mathematics and 36.2% in their physical science, respectively and were admitted in the mathematics and chemistry courses as regular status students. However, students whose high school grade scores were below the cut-off point (e.g. code 3) but met certain defined criteria gained admission as special admit (motivational students) who are academically less prepared for university and who demonstrate the potential to succeed in developmental or remedial level mathematics and/or chemistry courses. Students admitted to the program complete a one-year course to assist them in developing basic knowledge and skills required to succeed in first-year mathematics and/or chemistry.

Instruments and Procedure

Diagnostic tests are used to identify students who are at risk of failure and to recommend remediation or support programs (Coutis, Cuthbert, & MacGillivray, 2002). In the context of the current paper, a diagnostic test was adopted from the work of Barry and Davis (1999) and modified as Maths-in-Chem Deficiency open-ended Test (MCDT). This method gives students more time to think and write about their own ideas. Sample problems are taken from the previously mentioned mathematical assumed knowledge of chemistry topics, which have links to algebra, curve sketching, simple differentiation, and integration. The MCDT is divided into two sections, A and B, which contain examination-style questions designed to identify deficiencies in chemistry students' mathematics skills. Section A is made up of 30 multiple-choice questions with four possible answers (1–4) and four options grounds (A–D). Section B consists of five quantitative problems designed to assess the nature of students' mathematical deficiencies and the consistency of their responses to multiple choice questions. This was done to distinguish between mathematical weakness and a lack of grasp of chemistry principles where inadequacies are expected. Section B of the MCDT instrument also examined important areas of mathematical skills that can reveal students' fluency or deficiency in performing chemical calculations: (i) computation skill (proficiency of subject-specific skills and algorithms, correct classification and application of theories, models, terminologies and strategies) as highlighted in the work of Bing and Redish (2009), (ii) information and representation skill (making strategic connections between relevant mathematical and chemical knowledge and skills while solving given problems) by Shallcross et al. (2011), (iii) number and pattern concept skill (i.e. modelling mathematical concepts and specific skills in chemistry, and coalescing basic concepts of maths-in-chemistry within a larger structure that enables them to function together) by Yuriev, Capuano, and Short (2016). It is known that a lack of visual-spatial skill can lead to difficulties in differentiating, relating, and organizing information meaningfully (Garderen, 2006); therefore, it was necessary to examine students' logic thinking and visual-spatial skill (i.e. proficiency of skills required for classifying data and information, understanding orientation of geometrical shape and space, and drawing the correct conclusions).

The content validity of the instrument (MCDT) was determined by two educators from the university's mathematics and chemistry departments. Following the completion of the consideration process, the MCDT was revised until all of the test items were declared valid. The Cronbach's Alpha coefficient (α) for grouped questions was used to assess the reliability (internal consistency) of the MCDT. The Cronbach's Alpha coefficient for the MCDT was found to be 0.80. The MCDT was administered during one of the lectures (2.5 hours) in the final week of the second semester. A standardized grading scheme was used. The diagnostic test scores of students were compared to the end-of-semester examination results. As previously stated, MCDT can only determine whether students have mathematical deficiencies or not, but it does not provide information on the cause of deficiencies and sources. The identified deficiencies in mathematical skills were further assessed through interviews with students who had these deficiencies. In this regard, 21 student teachers were identified as having deficiencies in mathematical skills and were chosen for follow-up interviews, with 11 of them having the most deficiencies in mathematical skills. They were chosen not to represent all student teachers with mathematics skills deficiencies, but rather to identify the sources of these deficiencies. Prospective interviewees were informed why they were chosen for the interview prior to the interview and were given their MCDT answer scripts to help them make references when necessary. For the convenience of the interviewees and the researcher, the time and location of the interview were also set.

Finally, the selected students were interviewed for 3.5 hours using a guide of six open-ended questions (Creswell & Plano Clark, 2011) to further examine what they thought or felt were the causes of their deficiencies of mathematical skills in solving mathematics problems in chemistry.

Data analysis

The data from the MCDT questionnaire (sections A and B) were transformed into numerical values and analyzed using the Statistical Package for Social Science version 23, and the results obtained were then interpreted statistically. Mathematical skill deficiencies were identified by checking the student teachers' answers to MCDT questions and analyzing the consistency of answers and reasons, which were confirmed in the interviews (Creswell & Plano Clark, 2011).The percentage of student teachers with mathematical skills deficiencies was calculated by comparing the number of student teachers who solved the MCDT items successfully/unsuccessfully using any of the mathematical skills typology (i.e. computation skill, information and representation skill, number and pattern concept skill, and visual-spatial skill) to the total number of participants who attempted each MCDT item. The total score of the student teachers was converted to a scale of 0 - 100 and quantitatively analyzed, as shown in Table 2. It was expected that student teachers with the necessary mathematical background knowledge and skills would be able to score between 60 – 69%, or at least 70%, on this test, which is the standard for high school students (Table 1). Two coders used open coding to summarize and analyze the interview dataset for the qualitative analysis (Corbin & Strauss, 2015). Thus, guided by the quantitative findings and open to other emerging trends, the coders independently open-coded a few randomly selected responses from the student teachers. In doing so, a detailed set of code-notes was developed and then applied to the responses of 11 student teachers who experienced the most deficiencies in mathematical skills for which the coders were unaware of their identifications.

Table 2 Number of student teachers obtaining various ranges of scores in the assessments

		Maths-in-Chem Deficiency open-ended Test (MCDT) Score Range			
		0 – 49	50 – 69	70 – 100	Total
Final Maths Examination Score Range	50 – 100	9 (6)[1]	23 (15)	31 (20)	63 (40)
	40 – 49	5 (3)	11 (7)	24 (15)	40 (26)
	0 – 39	20 (13)	18 (11)	16 (10)	54 (34)
	Total	34 (22)	52 (33)	71 (45)	157 (100)

[1] Percentages in parenthesis

Results and discussion

Table 2 shows a crosstabulation of the number of student teachers obtaining various ranges of scores in the two assessments (i.e., the maths-in-chem deficiency open-ended test or diagnostic test and the final maths examination).

The results of this study suggest that student teachers' mathematical skills are inadequate to fully understand important chemical topics including kinetics, rates, and reaction processes, nuclear chemistry, and radioactivity. For example, the results in Table 2 clearly show that 16% of the student teachers who scored less than 50% in the final examination also scored less than 50% in the diagnostic test demonstrating they lacked numerous mathematics skills, including information and representation, number and pattern idea, and visual-spatial skill. This shows that the student teachers were unable to coordinate a number of concepts and skills (Bing & Redish, 2009), including the integration of auxiliary information with major mathematics ideas in chemistry problems (Shallcross & Yate, 2014). The absence of all of these fundamental mathematical skills frequently leads to confusion and difficulty in drawing meaningful connections in the problems to be solved, and could lead to algorithmic errors (Tambychika & Meerah, 2010). Also, from Table 2, it is clear that only 20% of the student teachers who scored above 50% in the examination were able to score at least 70% on diagnostic test (MDCT). This implies that passing the examination does not guarantee that student teachers will do well in a chemistry course.

Table 3 and 4 show samples of the questions tested, the students' average scores for each item, the most popular wrong options selected or offered and their inferences. Due to space constraints, only a few examples are provided to demonstrate the patterns of solutions provided by students, as well as the common areas of weakness and the severity of such weakness.

It can be seen from Table 3 that out of the students (93.8%) who completed the diagnostic test and the final examination for the chemistry course, only 31.8% obtained the correct option for Item 1. Students (46.9%) provided the most popular wrong option for the same item. The former were able to demonstrate basic computation skill, as well as applied number concepts and representation skill, whereas the latter were unable to coordinate a variety of those skills and were thus stuck with algorithmic errors. This is a clear indication that these students grappled to relate each mathematical skill to the chemical concepts where it was applied (Scott, 2012). Similarly, of those (65.6%) who attempted Item 2, about 42% of them provided the most popular wrong option, while only 29.4 percent chose the correct option. It seems that the student teachers did not understand indices and differentiation.

Another intriguing finding from Table 4 is the students' difficulty to demonstrate the essential maths-in-chemistry skills needed to solve one of the open-ended items in section B (Item 3).The problem-solution procedures for each step that students took while attempting to solve the item have been

Table 3 Common areas of students' weakness in Section A

Sample Question - (Section A)	Percentage of students' responses		Inference
	Correct option	Most popular wrong option	
1. If $123_y = 83_{10}$, the value of y is: (a) $y = -4 \text{ or } 8$ (b) $y = -10 \text{ or } 8$ (c) $y = 12 \text{ or } 5$ (d) $y = 20 \text{ or } -4$	(b) 31.8%	(a) 46.9%	Indication of mathematical deficiency
2. If $y = \frac{x^2+3x}{x^2} + \frac{\sin x}{\cos x}$, then $\frac{dy}{dx}$ is: (a) $\sec^2 x + 3x^2$ (b) $x^2 - 2\sec^2 x + 3$ (c) $2\frac{\sec^2 x}{3x}$ (d) $\frac{-3}{x^2} + \sec^2 x$	(d) 29.4%	(b) 41.6%	Indices and differentiation were not understood

condensed into episodes or clusters of responses A to D. The percentage of attempts to item 3 was 75%; however, only 21.3 percent of those who attempted it provided the correct solution. Thirty-three percent provided the incorrect solution and were able to mobilize some conceptual mathematical skills but were unable to apply those skills in chemical calculations. Some (17%) had a tendency to confuse unrelated mathematics concepts by believing that they were the same. They also lacked the ability to process algorithmic errors or recognize that they have made these errors. These findings further reinforce the notion that chemistry students need to be able to relate their mathematical skills to real applications in chemistry (Darlington & Bowyer, 2016; RSC, 2009).

Table 4 Common areas of students' weakness in Section B

Sample Question - (Sections B)	Percentage of students' responses: Correct option	Percentage of students' responses: Most popular wrong option	Cluster of students' responses (A-D)
3. A *35g sample* of unknown radioactive material decays until only *5g* is left. How long did it take for this decay to take place if the half-life of the unknown compound is 25 hours? *Half-life equation* $\left(\frac{1}{2}\right)^x = \frac{mass\ of\ decaying\ material}{mass\ of\ sample}$ Step 1 $\left(\frac{1}{2}\right)^x = \frac{5}{35} \rightarrow \{Stuck\ zone\} \rightarrow$ Cluster A Step 2 $\left(\frac{1}{2}\right)^x = \frac{1}{7} \rightarrow \{Apply\ Exponential\ skills\} \rightarrow$ Cluster B Step 3 $2^{-x} = 7^{-1} \rightarrow \{Apply\ \log\ skills\}$ Cluster C Step 4 $-x\ log\ 2 = -1 \log 7 \rightarrow \{Set\ up\ algebraic\ fraction\} \rightarrow$ $x = \frac{\log 7}{\log 2}$ Cluster D	Correct option: 21.3%	Most popular wrong option: 33.3% Percentage of students' attempt: 75%	a). Students could not proceed where they needed exponential skills and logarithm skills. b). Some students who managed to apply the exponential skills could not go on with the next mathematical skills logarithms. c). Some who managed to use logarithms could not finish the calculations. d). And those who managed to process all the above mathematical skills could not deal with algebraic terms such as signs

Furthermore, analyzing the written solutions of the student teachers to item 3 provides a powerful window into how they demonstrated their mathematical skills (or lack thereof) while solving the item. In responding to the item, students' perceptions of the type of knowledge they believe is appropriate for the given context appear to have aided their chemical calculation framing, ignoring broader and more direct mathematical skills and approaches to the problem. This confirms previous findings about students' inability to deal with multiple forms of mathematics concepts and representations in chemistry problems (Habiddin & Page, 2020; Scott, 2012; Shallcross et al., 2011). The inability of these students to use maths-in-chemistry ideas effectively to solve problems reveals their difficulties in understanding mathematics concepts and their applications in chemical calculations. The results of the tests conducted in this study, as well as the popular incorrect option/solution, speak volumes about the degree of need of these student teachers. According to the findings of this study, deficiencies are widespread and are seen to be more prevalent in topics such as logarithms and indices, simple proportion, algebra, and calculus. The percentage of student teachers who provided the correct option/solution versus those who did not in the MDCT test and final examination shows a significant difference (see Tables 3 and 4, respectively).

Furthermore, the problem of student teachers who provided the incorrect option/solution is exacerbated by irregular attendance in mathematics classes (see Table 5). Students' attendance scores revealed that regular class attendance had a significant impact on student performance and resulted in improved final examination results.

Table 5 Attendance and Performance of student teachers

Group	Student Group	*N*	MDCT score range (%)	Final Exam Mean
A	Regular course attenders	31	50 – 75	65
B	Average attenders	17	25 – 40	40
C	Occasional attenders	34	10	20

The proportion of low-achieving student teachers who lacked many mathematics skills is higher than the proportion of high-achieving student teachers who showed interest in both mathematics and chemistry classes (Table 5). This finding appears to indicate that high-achieving student teachers, whether in mathematics or chemistry, are noticeably more confident in their learning of these subjects. One possible explanation for this is that, all else being equal, a student performs better in any discipline in which he or she is favourably inclined. It is also worth noting that student teachers who regularly attend mathematics class perform well in applying the mathematical processes and skills required to understand chemistry. On that note, research has shown that students with higher math achievement have the best chances of success in chemistry (Darlington & Bowyer, 2016; Redlich, 1972; Scott, 2012). Those who avoided class or were dissatisfied with the subject because they saw mathematics as irrelevant to the study of chemistry performed poorly.

When asked about their reasons for poor class attendance and mathematics anxiety in relation to their performance on the diagnostic test and the final exam, approximately 46 percent provided inconsistent statements or inappropriate examples ranging from *"I just don't get along with mathematics since my high school days and have been avoiding its application in chemistry...I really lacked confidence in my ability to do mathematics… it is a difficult subject."* Another student teacher added, *"mathematics is important in chemistry, but maths class is not a fun place to be, so I get notes from friends."* How should we address these issues in the classroom? Being aware of these challenges, at the very least, highlights an alternative cause of student teachers' difficulties, other than a simple inability or lack of mathematical skills.

Student teachers' views on the causes of deficiencies in mathematical skills

The following are the main causes of DMS identified by student teachers: (i) students' long history of lack of interest in mathematics and poor background knowledge of mathematics; (ii) mathematics anxiety; (iii) the fast pace and change in difficulty of the work assigned to them; and (iv) students' perception of the type of knowledge they believe is appropriate to a given context. During the interview, student teachers were asked to comment on their ability to demonstrate the four important mathematics skills identified in the literature: (a) information and representation skill, (b) computation skill, (c) number and pattern concept skill, and (iv) visual-spatial skill. To condense the episodes of interview data (Table 6), a system was used to indicate the student teachers' agreement on common areas of weakness and the severity of such weakness, whether minor (○), major (⊙), or severe (◉) as well as whether common areas of weakness are inconsistent statement or inappropriate example (☯). Table 6 shows the interview results of the eleven student teachers who had the most mathematical skill deficiencies in the diagnostic test (MDCT) and final examination.

Table 6 Student teachers' perceived ability to demonstrate key mathematics skills in working with Logarithm and Indices, Graph and Calculus

		Perceived ability[1] to demonstrate key mathematics skills			
	Test type	Information and representation skills	Computation skills	Number and pattern concept skills	Visual-spatial skills
a. Reading and understanding problem	Diagnostic	○	◉ ⊙	◉	◉
	Exam	○ ⊙	○	○ ⊙	◉ ⊙
b. Organizing strategy and solving problem	Diagnostic	◉ ⊙	◉	○ ⊙	○ ⊙
	Exam	◉	☯	○ ⊙	☯
c. Checking reasonableness of problem solution and processing errors	Diagnostic	○ ⊙	◉	☯	◉⊙
	Exam	◉ ⊙	◉ ⊙	◉	◉

[1] Student teachers' perceived ability rating codes: *Minor* → ○; *Major* → ⊙; *Severe* → ◉; and *Inconsistent Statement or Inappropriate Example* → ☯.

As seen in Table 6, some of the students agreed that when it comes to reading and understanding problem in the diagnostic test, they had minor weaknesses in applying 'information and representation skills' but had major and severe weaknesses in applying the remaining three mathematics skills required to do well in the chemistry course. Similarly, some of the students agreed that when it comes to 'organizing strategy and solving problem' in mathematics examination, they had severe weaknesses in applying information and representation skills but had major and severe weaknesses in applying 'information and representation skills' and have difficulties in selecting appropriate examples of computation skills and visual-spatial skills.

Regardless of the students' feelings about mathematics and/or its application in chemistry, they all agreed that a solid mathematical foundation was required for a better understanding of chemical concepts. In the case of mathematics being regarded as a difficult subject by students and a boring class to be in, it is necessary to devote the first few topics to the use of logarithms, unit selection, ratios, proportions, and statistics at the start of the chemistry course when designing a remedial program. To lay the groundwork for introductory calculus, the mathematics topics could be followed by simple algebra and graphs. This could have an impact on transfer and relevance by demonstrating what mathematics does or can do in chemistry. It may also enable student teachers to recognize the importance of mathematics in chemistry from the start, potentially alleviating some of their mathematics anxieties. Based on the findings of this study, course coordinators may offer bridging programs to students with poor mathematics background knowledge and skills when teaching chemistry.

Conclusion

The findings of this study suggest that we should not assume that students have the basic mathematical skills required to perform chemistry tasks without checking to see if they do and are able to apply these acquired skills to the given tasks. Mathematics, unlike the sciences, is hierarchical, and what happens in primary school mathematics classrooms is critical to secondary school, college, and/or university mathematics performance. For the same reason, higher education institutions should identify areas, including mathematical skills that should be taught to university science students from the start in order to help bridge the current gap between schools and tertiary institutions (Onwu, 2009; Setati, 2009). The underlying assumption is that students' chemistry and mathematics problems begin earlier than is commonly recognized. It has been demonstrated in this study and supported elsewhere (Scott, 2012) that mathematical skills are required to learn chemistry effectively. There is a need to encourage student teachers to be realistic about their mathematical skills and background so that they are fully prepared for university chemistry courses. For students with poor mathematics skills and backgrounds, tackling complicated chemical calculations will always be a challenge. Therefore, identifying the mathematical skills, dividing them into units, and then structuring them in a way that runs parallel to the chemistry taught is one strategy to help students who have major or severe inadequacies in mathematics abilities essential for studying chemistry. Finally, chemistry educators should identify potential deficiencies in students' mathematical skills and backgrounds early in their teacher education in order to inform their teaching, and once identified, provide students with a pathway into successful chemistry study.

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