Use of Maple Software to Reduce Senior High School Students' Errors in Integral Calculus

Emmanuel Kwadzo Sallah[1]

Abstract

A quasi-experimental non-equivalent group design research was conducted to use Maple software to reduce students' errors in integral calculus in a senior high school in the Oti region of Ghana. Convenience and simple random sampling techniques were employed to obtain a sample of 80 students, which comprised 40 students in the control group and 40 in the experimental group. Tests in integral calculus and questionnaire were used for data gathering. The analysis revealed that students committed many conceptual, procedural and technical errors when solving integral calculus tasks. The results also indicated that the students of the experimental group exposed to the use of Maple software in learning integral calculus significantly outperformed their counterparts in the control group exposed to traditional method. The researcher recommends the Maple assisted instruction in the teaching and learning of integral calculus and also the need to employed blended teaching approach using the Maple software to complement the traditional teaching strategy.

Keywords Maple software; student errors; integral calculus;

Introduction

Calculus is a branch of mathematics concerned with the calculation of instantaneous rates of change, known as Differential Calculus, and the summation of infinitely many small factors to determine some whole, known as Integral Calculus (Berggren, 2016). Calculus is an important concept for SHS students as it is the gateway to studies in Engineering, Medicine, Business and in fields of higher mathematics such as differential equations, vector analysis and complex analysis. SHS students' performance in calculus is a concern to mathematics educators and policy makers, however it still remains challenging and problematic concepts, despite its wide usage in daily life and many other fields (Berggren, 2016; Salleh & Zakaria, 2015; Usman, 2012; Mahir, 2009; Abdul Rahman, 2005). For this reason, SHS mathematics teachers should focus on the development of students' understanding in calculus concepts and provide a better teaching and learning educational environment.

Poor understanding of pre-requisite concepts; the use of inappropriate teaching methods; poor attitudes and perceptions towards the study of calculus are some of the contributing factors to low performance in calculus (Salazar, 2014; Yee & Lam, 2008; Kiat, 2005). However, irrespective of the measures put by stakeholders to promote paradigm shift in the mode of teaching from teacher centered to learner centered method of teaching mathematical concepts to enhanced students' academic gains, our classrooms are still dominated by traditional method. Very little has been done to investigate students' errors in integral calculus to boost their performance in integral calculus.

[1]Emmanuel Kwadzo Sallah, Department of Mathematic & ICT, Evangelical Presbyterian College of Education, Amedzofe, Volta Region, Ghana. Email: emmanuelsallah100@gmail.com

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The use of technology in education is still relatively rare in Ghana, as few working in mathematics education today are unaware of the growth in recent years of computer technologies for teaching, learning and research in mathematics. Notwithstanding there are growing body of research that suggests its extended use is imminent (Salleh & Zakaria, 2015; Usman, 2012; Mahir, 2009; Abdul Rahman, 2005). The teaching method has been enhanced with the integration of technology through exploring integral calculus using mathematics software known as Maple. Maple software can be used to solve general-purpose mathematical problems. Problems in the area of mathematics, science, and engineering can be investigated using Maple software and it is well suited to aid students to learn mathematics through verifying, calculating, manipulating of mathematical expressions and graphical visualization of 2D, 3D complicated graphs. Maple system uses only a procedural language of 4th generation (4GL), similar to the C language, FORTRAN, BASIC and Pascal. Tedious computations are performed by Maple software by featuring systematic solution of the problem as obtained when done manually. Maple software was chosen because it is suitable for a variety of uses including solving very difficult calculus problems. Furthermore, it requires minimum programming as compared to other mathematical software's. Considering the importance of this subject in academic undertaking, the researcher was prompted to use Maple software to reduce students' errors in integral calculus.

Purpose of the Study

The purpose of this research was to examine students' errors in integral calculus and the impact of Maple integration in teaching and learning integral calculus. The following research questions were formulated to guide the study:

1. What errors emerge from students' responses when solving integral calculus tasks?
2. What is the impact of Maple software on students' understanding of integral calculus?
3. How do students perceive the effectiveness of Maple software in learning?

Methodology

The study was designed in line with Fraenkel and Wallen's (2006) description of non-equivalent quasi-experiment group design.

Groups	Pretest	Treatment	Posttest
Experimental	O_1	X	O_2
Control	O_3	C	O_4

Figure 1

The pretests O_1 and O_3 were done to determine the initial entry points and compare difference between groups before treatment. The posttests O_2 and O_4 were administered to examine the treatment effect after experimental group received integral calculus tuition through Maple instruction ***(X)*** and the control group received integral calculus tuition through the conventional instruction ***(C)***.

The study was carried out at Bueman Senior High School located in the Oti Region of Ghana during the first semester of the 2019 - 2020 academic year. Convenience sampling technique was used in selecting a sample of 80 students from accessible population of all 405 third-year students, geographical proximity, availability and willingness to participate in the study. Simple random sampling technique was also used in categorizing them into control group of 40 students and experimental group of 40 students respectively.

The instrument used to measure students' understanding in integral calculus are tests and questionnaire. The instruments were developed by the researcher and has been carefully piloted to ensure its reliability and validity. By using Cronbach Alpha, the reliability of the tests instrument was proven high with item reliability of 0.90. The spearman- Brown reliability was also proven good for questionnaire, which is 0.81. Integral Calculus achievement pretest was administered to both groups and the scores were analyzed using descriptive error analysis and independent samples t-test. An intervention was carried out for six weeks, where the experimental group was exposed to integral calculus learning using Maple instruction and the control group exposed to traditional method integral calculus instruction. Integral Calculus achievement posttest was administered to both groups after the intervention. A paired samples t-test was further used on pretest posttest of the groups.

A Likert-scale questionnaire was administered to students in the experimental group to find out their views or perceptions of the effectiveness of the Maple software used in teaching and learning of integral calculus. The data collected were analyzed quantitatively based on; views about whether or not the Maple lessons increased their participation in class activities, improved their concentration, enjoyment, self-confidence and content mastery.

Results

A descriptive error analysis under the following subtopics of integral calculus were investigated: (i) indefinite integral, (ii) integration of powers; integration of trigonometric functions, (iii) definite integral (iv) area under a curve, area bounded between two curves. The errors observed are presented next.

Errors in working with indefinite integral of functions

Students' difficulties with integral symbols and variables were identified as one of the errors in the test-item Question 1. The integral sign$\int$ and dx were omitted in the solution presented by the students. The integral sign$\int$ and dx are the symbol errors committed by some students. The errors related to symbols and variables pertaining to integral calculus might seem trivial but not. Another problem was misunderstanding on the property of integral such as $\int \frac{x-2x^2}{\sqrt{x}} dx = \int (\frac{x}{x^{\frac{1}{2}}} - \frac{2x^2}{x^{\frac{1}{2}}}) dx$. Many students had mixed up the processes involved in integration with that of differentiation, which results from forgotten of the techniques in integrating functions, or lacked practice in this area.

Another error exhibited by the students was not adding a constant $+C$, after integrating

Table 1 Students' Performance in Indefinite Integral of Function

Ability to:		N	Number of students: who didn't attempt	with correct answers	with wrong answers
i.	Split the function into separate terms	80	12(15)	32(40)	36(45)
ii.	Integrate individual terms	80	12(15)	24(30)	44(55)
iii.	Find the correct integral with the constant attached	80	12(15)	20(25)	48(60)

Percentages in parenthesis

indefinite integral. Students were not aware or had forgotten that they needed to add a constant, $+C$ after integrating indefinite integral. Table 1 describes statistically the performance of each skill under finding indefinite integral of function.

Errors with definite integral of function

The third question tested the students on how to evaluate definite integral of $\int_{-1}^{2} x(x-4x^2)dx$. From the data analysis, some students evaluated the definite integral wrongly. They were not able to expand $\int_{-1}^{2} x(x-4x^2)dx$ to get $\int_{-1}^{2}(x^2-4x^3)dx$, before integrating, but rather integrated x and $(x-4x^2)$ separately, and ended up not obtaining the correct integrand $\frac{1}{3}x^3 - x^4 + c$, first. Even those students who evaluated the definite integral correctly ended up applying the limits wrongly. In addition, errors in basic mathematical skills such as additive, arithmetic operation and reducing by common factors errors were identified as common errors from the definite integral question. The table 3 describes statistically the performance of students under definite integral of function.

Table 2 Students' Performance in Indefinite Integral of Function involving Trigonometric Function

Ability to:		N	Number of students: who **didn't attempt**	with **correct** answers	with **wrong** answers
i.	Split the function into separate terms using correct trigonometric identities	80	39(49)	5(6)	36(45)
ii.	Apply the rules of integrating simple trigonometric functions	80	39(49)	1(1)	40(50)
iii.	Find the correct integral with the constant attached	80	39(49)	0(0)	41(51.25)

Percentages in parenthesis

Error in working with indefinite integral of function involving trigonometric function

The second type of errors identified was confusion with respect to trigonometry, differentiation and integration. This errors occurred in the question 2 involving integrating trigonometric function, as students were not able to recall the trigonometric identity nor could they manipulate the trigonometric function of $\int(sin^4 x)dx = \int(sin^2 x)^2 dx$ and $\int(sin^2 x)^2 dx = \int(\frac{1}{2}-\frac{1}{2}cos2x)^2 dx$. Students' inability to identify the appropriate integration techniques was another error identified. That is, students' poor linkage between differentiation and integration as they were not able to integrate $\int(\frac{1}{2}-\frac{1}{2}cos2x)^2 = \frac{3}{8}x - \frac{1}{4}sin2x + \frac{1}{32}sin4x + c$ correctly. The Table 2 describes statistically the performance of students under the skills required in finding indefinite integral of function involving trigonometric function.

Error with area under curves of function

Question four (4) tested the students on how to evaluate definite integrals and apply integration to evaluate plane areas. The question did not provide any diagrams or sketches, required the students to integrate the

Table 3 Students' Performance under Definite Integral of Function

Ability to:		N	Number of students who **didn't attempt**	with **correct** answers	with **wrong** answers
i.	Expand and simplify the function	80	6(8)	51(64)	23(28)
ii.	Integrate simple algebraic terms	80	6(8)	46(58)	28(34)
iii.	Apply the limits	80	6(8)	43(54)	31(38)
iv.	Find the correct answer for the definite integral		6(8)	40(50)	34(42)

Percentages in parenthesis

curve $y = x(x - 4)$ from $x = 0\ to\ x = 5$. Because of this, many students failed to realize that the part of the curve $y = x(x - 4)$ from $x = 0\ to\ x = 4$ was below the x −axis whereas the part from $x = 4\ to\ x = 5$ was above the x-axis. Many students had no idea that they needed to sketch out the curve to determine how they were going to integrate the function. Many simply integrated the function from $x = 0\ to\ x = 5$ directly. They integrated the function mechanically using the given limits. However, they had difficulty explaining their answers. Table 4 describes statistically the performance of each skill under finding the area under the curve.

Research Question on impact of the Maple software on students' understanding of integral calculus. The mean scores of both groups in the pretest and posttest were compared using the paired sample t-test. The results of the analysis for the control and experimental groups were summarized in Tables 5 and 6 respectively.

A paired sample t-test carried out which compared the mean difference of posttest and pretest scores of the Experimental group as the data met all the assumptions of paired sample

Table 4 Students' Performance under Areas of Curves of Function

Ability to:		N	Number of students who **didn't attempt**	with **correct** answers	with **wrong** answers
i.	Sketch the curves	80	32(40)	31(39)	17(21)
ii.	Locate the required areas and find the respective differences	80	32(40)	12(15)	36(45)
iii.	Apply integration to find the area between the curves in the specific intervals	80	32(40)	3(4)	45(56)
iv.	Find the total area between the curve and the x-axis in the given interval	80	32(40)	0(0)	48(60)

Percentages in parenthesis

Table 5 Paired samples t-test results of posttest and pretest scores of experimental group.

Group	Test	N	Mean	SD	df	t-value	p-value
Experimental	Post-test	40	25.75	3.46	39	-6.94	0.00
Experimental	Pre-test	40	4.18	5.42			

Table 6 Paired samples t-test results of posttest and pretest scores of Control Group.

Group	Test	N	Mean	SD	df	t-value	p-value
Experimental	Post-test	40	18.98	2.64	39	-4.01	0.00
Experimental	Pre-test	40	16.71	4.34			

t-test. The result verify mean difference between the posttest and pretest scores indicated that there was a significant improvement in the achievement of posttest scores (M = 25.75, SD = 3.46) over pretest scores (M = 4.18, SD = 5.42) at $\alpha < 0.05$ level of significance, with conditions [t (39) = -6.94, P = 0.00]. It was therefore concluded that, there was a statistically significant difference between the posttest and pretest scores of students when taken through integral calculus with Maple software. Table 6 showed paired sample t-test of posttest and pretest scores of the control group.

The results on Table 6 showed that there was no statistically significant difference of the pretest scores (M = 16.71, S.D = 4.34) against posttest scores (M = 18.98, S.D = 2.64) at [t (39) = -4.01, P = 0.00], the traditional method seems not to be significant difference in teaching as compared to Maple software approach of teaching integral calculus. This was because the mean difference of 2.27 showed that the control group exposed to the traditional method was not very good in the understanding of integral calculus concepts and its application to real life situations.

The descriptive statistics of the students' rating on their perceptions of the use of Maple to learn integral calculus were presented on a multiple bar graph showing the percentage of students who disagreed or agreed to each of the questionnaire items (see Figure 1).

With one voice, 99.25% of the students strongly agreed/agreed on the following: (1) All SHS students should be exposed to the use of Maple software to learn integral calculus, (2) Mathematics teachers have to use Maple software to teach integral calculus, (3) Maple software helps mathematics teachers to explain integral calculus concepts better and (4) Maple software helps mathematics teachers to make their lessons enjoyable. Moreover, the rest of the students, constituting 0.75% disagreed to these four perceptions about the use of Maple software by mathematics teachers. By implication, mathematics teachers when able to use Maple software well to teach integral calculus concepts, it will eventually reduce the many errors of students in learning integral calculus.

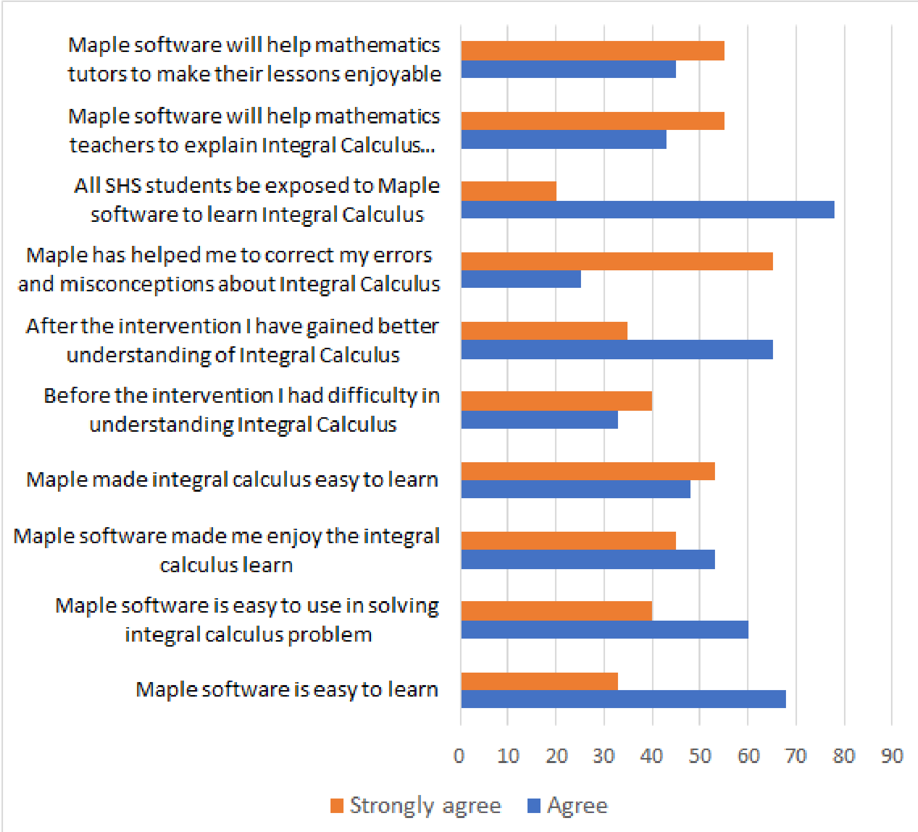

Figure 1 Bar graphs of students' rating on their perceptions of Maple software

Discussion

The pretest item analysis shows general poor performance of the students coupled with several conceptual, procedural, and technical errors committed by students which are:

- misunderstanding on the property of integral such as $\int \frac{x-2x^2}{\sqrt{x}}dx = \frac{2}{3}x^{\frac{3}{2}} - \frac{4}{5}x^{\frac{5}{2}} + c.$
- difficulty in manipulating the trigonometric function of $\int (sin^4 x)dx = \int (sin^2 x)^2 dx$
- students' inability to recall the trigonometric identity needed to manipulate the trigonometric function of $\int (sin^2 x)^2 dx = \int (\frac{1}{2} - \frac{1}{2}cos2x)^2 dx$
- students' inability to identify the appropriate integration techniques, thus confusion with respect to differentiation and integration as $\int (\frac{1}{2} - \frac{1}{2}cos2x)^2 = \frac{3}{8}x - \frac{1}{4}sin2x + \frac{1}{32}sin4x$
- error with constant $+c$ at the end of the integration.

There was an improvement in the integral calculus mean scores in pretest to the posttest of control group from (M = 16.71, S.D = 4.34) to (M = 18.98, S.D = 2.64) and experimental group from (M=4.18; S.D=5.42) to (M=25.75; S.D=3.46).

The independent samples t-test results revealed that there was significant difference in students' understanding of integral calculus between the control and experimental groups in the posttest. That is those in the control group obtained mean and standard of 18.98 and 2.64 respectively and experimental group obtained mean and standard of 25.75 and 3.46 respectively. In the analysis, the difference was significant at 5% with a p-value of 0.00 which is less than the significance level (i.e. $0.000 < 0.05$), indicating a statistically significant difference between the mean scores of the control and the experimental groups. Thus, the mean and standard deviation of the experimental group (M = 25.75; S.D = 3.46) was significantly higher than the mean and standard deviation of the Control group (M=18.98; S.D=2.64) in the posttest. Hence, the use of Maple software to teach students integral calculus produces a highly significant and tremendous improvement in students' understanding as against the performance of students taught by the traditional approach. The findings of this study agree with those of Okoro and Etukudo, (2001); Egunjobi, (2002); Karper, Robinson and Casado-Kehoe, (2013) that students taught with CAI packages in Chemistry, Mathematics and Education in general, perform better than those taught with normal classroom instruction.

The questionnaire findings revealed that using Maple software in teaching and learning, not only increases achievement in general, but also motivates students. All the attributes in the questionnaire coined to be the six motivation attributes such as increased participation, improved concentration, enjoyment, self-confidence, content mastery and recommendation affirm that Maple software enhances student motivation to learn integral calculus. This finding was in consistence with the findings of (Tsarava, Halkidis, Venardos & Stephanides, 2013).

Conclusion

Based on the findings of the study, the researcher recommends Maple assisted instruction in the teaching and learning of integral calculus and the need for teachers to employ blended teaching and learning methods, in which computer software such as Maple software are used simultaneously with traditional teaching strategy. The blended teaching and learning process is a system that combines face-to-face instruction with computer-mediated instruction. Lloyd-Smith (2010) argued that blended instruction offers more choices for content delivery and is more effective than teaching that is fully online or fully classroom based. Garnham and Kaleta (2002) reported that students learn more in blended learning environments than they do in comparable traditional classes. Blended teaching offers advantages to both the school and the students. The method of instruction is not over-reliant on the physical presence in one room of both the teacher and the student, and it offers greater flexibility for students to carry out their work independently (Lloyd-Smith, 2010).

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