

The impact of concept-based instruction on senior high school students' achievement in circle theorems

Forster Danso Ntow¹ & Yunus Hissan²

Abstract

The influence of two teaching methods namely, concept-based instruction and traditional method on senior high students' achievement in circle geometry was investigated using the non-equivalent quasi experimental design. Purposive sampling technique was used to select two schools which were designated as experimental and control groups. Pre and post-tests were conducted to assess participants' achievement before and after the intervention. Paired sample t-test and an independent sample t-test technique were used to analyse the data. The findings of the study revealed a significant difference in the mathematics achievement of students in the experimental group taught using concept-based instruction compared to those taught with the traditional method (control). The implications of the findings are discussed.

Keywords concept-based instruction; circle theorems; mathematics achievement; high school geometry

Introduction

Mathematics is very essential in many fields of human endeavour. For example, Kashefi, Ismail, Yusof, and Mirzaei, (2013) argue that mathematics is the foundation and an important component for most applied engineering courses. The importance of mathematics has led to discussions about how students learn mathematics and how it should be taught for an in-depth understanding of both concepts and procedures (e.g., Hiebert & Lefevre, 1986; Riddle-Johnson & Schneider, 2015). This is because how mathematics is taught is a contributory factor in students' learning opportunities. In what has become known in the literature by various names such as traditional or conventional instruction, the primary focus of teachers is on helping students to follow a set of rules and procedures required to solve questions correctly with very little explanation and justification of how and

why the formula works (Borji, Radmehr & Font, 2019). This conventional teaching approach has resulted in most students having difficulties in grasping mathematical concepts including geometric concepts. In the teaching of circle geometry for example, Lim (1992) pointed out that, in most cases, teachers do not introduce the concept of circle theorems properly for students to grasp the "concept before proceeding quickly to the properties themselves" (pp. 36).

Evidence from the results of Ghanaian basic school students' participation in the Trends in International Mathematics and Science Study (TIMSS) 2011 report shows an abysmal achievement of Ghanaian students in mathematics (Mullis, Martin, Foy & Aron, 2011). According to the TIMSS report, most Ghanaian students who participated in the assessment had conceptual difficulties in the area of geometrical concepts and relations. At

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the senior high school level, students at that level are reported to have difficulties in Circle Theorems.

Although circle theorems is one of the fundamentals and beautiful concepts in geometry since it ties together several important Euclidean geometry concepts, the various theorems students have to learn have proven a challenge for them (e.g. Hissan & Ntow, 2021; Dogwi, 2014; Lim, 1992). A major reason accounting for the reported difficulty is that students see circle geometry topic as abstract and mechanical hence, the most difficult concept in the Senior High School (SHS) core mathematics curriculum in Ghana (Bosson-Amedenu, 2017). As a result, it has become necessary to look for appropriate instructional approaches that could help effectively improve students' geometric thinking and achievement in geometry. One possible way to address this problem could be the use of concept-based instruction method with a number of researchers emphasising that this method is an effective instructional approach to teaching of mathematical concepts.

Statement of the problem

In Ghana, the Chief Examiner for Core Mathematics' reports (West African Examination Council (WAEC), 2013, 2016) indicate that an appreciable number of

circle theorems. In most cases, many students avoid questions involving the application of circle theorems. For example, during the May/June 2013 and 2016 examinations, the Chief Examiner's reports for Core Mathematics revealed that the majority of candidates did not even make an attempt to solve questions 10(a) and 11(b) respectively which involved the application of geometrical concepts. The reports further indicated that the few candidates who attempted these questions had difficulties and thus showed inadequate content knowledge and the application of geometric theorems (WAEC, 2017).

Additionally, the Chief Examiner's report for Core Mathematics in 2017 revealed that the majority of the candidates could not apply the appropriate geometrical theorems to determine the value of some given angles in Question 9 as shown in Figure 1. Candidates were requested to draw circle *RST* and also a

WASSCE May/June 2017, Question 9
PQ is a tangent to the curve SRT at point S. PRT is a straight line. $\angle TPS = 34^\circ$ and $\angle TSQ = 65^\circ$.

- i. Illustrate the information on a diagram
- ii. Find the value of:
 - a) $\angle RTS$
 - b) $\angle SRP$.

Figure 1 WASSCE May/June 2017 Question 9

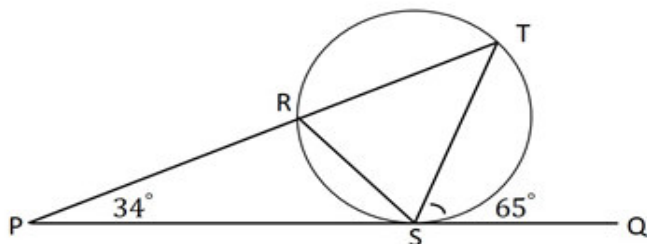


Figure 2 WASSCE May/June 2017 Question 9

students have demonstrable weaknesses in solving questions involving the application of

tangent PQ to the circle and then a straight line *PRT* as in the diagram above. According to the report, this question was very unpopular among the majority of candidates and the few candidates that attempted this question could not draw the diagram to determine the required angles and thus, displayed poor knowledge of geometry. The report further indicated that most candidates had conceptual difficulties in sketching the required diagram; hence the performance of candidates to this question was reported to be generally poor (required diagram reported in Figure 2).

The Chief Examiner for Core Mathematics Reports contend that the weaknesses observed are due to the inability of mathematics teachers to employ appropriate teaching methods in teaching the topic (WAEC, 2013, 2016). Considering that a deep appreciation of circle theorems is important to the study of other geometric concepts, it is important to identify more efficacious teaching approaches to support students' learning. This study, therefore, was conducted to compare two different teaching methods namely concept-based instruction and traditional method on students' achievement in circle theorems.

Research Questions/Hypotheses

The overarching question that underpinned the study was: Is there any significant difference in the achievement scores of senior high school students taught using the concept-based instruction compared to their counterparts taught using the traditional method? In answering the research question, the following null hypotheses were answered:

- H_{01} : Concept-based instruction as an instructional approach has no significant effect on senior high school (SHS) students' achievement in circle theorems.
- H_{02} : Traditional method has no significant effect on senior high school (SHS) students' achievement in circle theorems.
- H_{03} : There is no significant difference between the achievement of senior high school (SHS) students taught circle theorem using concept-based method and those taught with traditional method.

Literature review

In this section, we discuss the literature on circle geometry, conceptual and procedural knowledge, concept-based instruction and traditional instruction.

Circle Theorems

Lines and circles are the most frequently used geometry figures. Circle geometry is the aspect of geometry that deals with the relationships of the circle. The geometry of circles includes a wide range of concepts, theorems and properties of the circle. The geometry aspect in the Core Mathematics syllabus in the senior high schools in Ghana is divided into two main aspects; Plane Geometry I and Plane Geometry II. This curriculum of Geometry I is designed to include lines, circle, radius, sector, segment, tangent, chord, semi-circle, cyclic quadrilateral, arc and circumference. However, the curriculum of Geometry II, which is the focus of this paper, includes the aspects of the geometry of the circle called circle theorem in Ghana.

Several studies have highlighted students' difficulties in understanding circle geometry concepts (e.g. Dogwi, 2014 & Lim, 1992; Hissan & Ntow, 2021; Bosson-Amedenu, 2017). Many students have difficulties in understanding and relating the algebraic, geometric and numerical representations of the concepts of circle geometry. For example, in a recent comparative study conducted by Boson-Amedenu (2017) to determine the perceived "difficult concepts in the senior high school core mathematics curriculum", the researcher found that circle theorem was the most difficult concept perceived by SHS students in the mathematics curriculum in Ghana. After circle theorem concepts and similar triangles were plane geometry I in the ranking order of the perceived difficult concepts in the mathematic curriculum. The researcher concluded that most students cannot apply the appropriate circle theorem properties to solve problems.

Moreover, Chemuka (2017) who explored students' geometric thinking and understanding of circle theorem using Geogebra and Van Hiele's theory found out

that the use of Geogebra can only improve students' mathematics achievement at the basic levels as proposed by Van Hiele, that is, levels 1 and 2. According to him, the higher levels (abstraction, deduction and rigor) did not show any significant improvement in students' achievement. He suggested that the "use of Geogebra in teaching and learning of circle theorem" was limited at improving students' visualization and abstraction levels of Van Hiele. To him, Geogebra enables students to recognise and name specific circle theorems and properties but unable to carry out logical analysis. A replicated study was conducted by Ogbonnaya and Chemuka (2017) who also found similar results that were consistent with the former study. Once again, the authors suggested that the use of Geogebra in teaching circle theorem "might not help much in attaining correct results".

Conceptual and procedural knowledge

Many researchers in the mathematics education research community have investigated what constitutes mathematical knowledge with regards to teaching and learning. In this regard, many of such literature have classified mathematics knowledge in the process of mathematics teaching under different headings (e.g. Skemp, 1978; Hiebert & Lefevre, 1986). Hiebert and Lefevre (1986) introduced two dichotomous kinds of knowledge that students should possess: conceptual knowledge and procedural knowledge. They defined conceptual understanding as knowledge that is rich in relationships. Conversely, procedural knowledge in mathematics was espoused by Hiebert and Lefevre as knowledge of symbols, algorithms and rules. Drawing on these two definitions of mathematics knowledge, one way of distinguishing between these two types of mathematics knowledge is that procedural knowledge is the step by step, sequential knowledge of symbols, arithmetic operations and procedures for solving a mathematics

task. Conceptual knowledge on the other hand, is knowledge of meaning and understanding of mathematical concepts, procedures, rules and algorithms (Hiebert & Lefevre, 1986; Rittle-Johnson & Schneider, 2015; Bergsten, Engelbrecht & Kagesten, 2017). It is argued that the acquisition of both knowledge types is fundamental to mathematics teaching and learning (Bergsten, Engelbrecht & Kagesten, 2017).

Recent developments in the literature on how students learn mathematics have focused mainly on an interest in conceptual or procedural distinctions and approaches to mathematics (Borji, Radmehr & Font, 2019; Joffrion, 2005; Mahir, 2009). For instance, Bergsten et al. (2017) posited that there has been a paradigm shift towards an interest in exploring conceptual and procedural distinctions and approaches to mathematics teaching and learning. Others of such research studies have highlighted that both conceptual and procedural knowledge are needed in mathematics and that these two types of mathematics knowledge are dichotomous and bidirectional as both types of understandings complement each other (Borji, Radmehr & Font, 2019). However, Joffrion (2005) acknowledged that procedural understanding mean almost nothing without conceptual understanding. The term 'procedural knowledge' is also referred to as the traditional method and these terms are used interchangeably in this paper.

Evidence from well-controlled comparative studies related to conceptual and traditional (procedural) teaching and learning of mathematics have shown that the conceptual approach to mathematics promotes students' thinking and understanding compared to the traditional method in terms of students thinking and understanding of mathematics. As conceptual knowledge and procedural knowledge have become increasingly important in mathematics education, it is

essential to investigate into an applicable instructional approach to develop students' conceptual understanding of mathematics without compromising procedural skills (Hissan & Ntow, 2021). We now turn to discuss what is meant by traditional method of teaching in the next section.

Traditional method

Langton (1991) conceives of traditional instruction is the instruction that begins by presenting definitions, algorithms, notations and procedures without first establishing the meaning of concepts involved. For Chambell and Kilpatrick (2003), this instructional approach is characterised by procedural teaching where the development of "procedures, skills and algorithms preceded concepts development" (pp. 22). Joffrion (2007) conducted a study entitled conceptual and procedural understanding of algebra concepts in the middle grades. Additionally, Joffrion described some indicators for procedural teaching (also referred to in this paper as traditional teaching) method that were adapted from Hiebert and Lefevre' descriptions of procedural knowledge and understanding as follows:

Rules and algorithms are presented as a series of steps; solutions are presented as step-by-step and sequential; solutions are presented by the instructor as hierarchical and very structured; students are required to operate on objects or symbols; input/output model of student processes is evident or implied; steps being presented can be learned by rote (pp. 65).

That is, traditional teaching method, also known as procedural teaching, is more concerned with the teacher acting as an instructor and director of the learning environment with little or no contribution from the students (Mereku, 2010; Tay & Mensah-Wonkyi, 2018). Meanwhile, a

growing body of literature attributed Ghanaian students' low achievement pattern in mathematics to the nature of mathematics teaching and learning (Asiedu-Addo & Yidana, 2000; Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2008). Some articles (e.g. Tay & Mensah-Wonkyi, 2018; Mullis, Martin, Foy and Aron, 2011) explain that students' difficulties in mathematics are as a result of traditional method of instruction. Particularly, in the analysis of the abysmal performance of Ghanaian students in mathematical geometry by Mullis, Martin, Foy and Aron (2011), the authors noted that the teaching of mathematics in most mathematics classrooms across the length and breadth of the country are characterised by the traditional teaching methods. Such teaching method has been identified as a contributory factor in the consistently poor performance of Ghanaian students in mathematics (Tay & Mensah-Wonkyi, 2018; UNESCO, 2006).

Concept-based instruction

The term conceptual teaching used in teaching context means the teaching for meaning and understanding of concepts. The term has synonymously been used by many researchers as; conceptual instruction, concept-based instruction, concept-based learning environment conceptual teaching (e.g. Chappell & Kilpatrick, 2003; Valmoria & Tan, 2019; Langton, 1991). Among these studies, Chappell and Kilpatrick (2003) defined concept-based instruction as the instruction where the development of concept preceded the development of skills, algorithms or procedure. In this instructional approach, students work collaboratively to construct their own knowledge through problem-solving and guided discovery. Joffrion (2005) described concept-based instruction method as: "relationships between numbers, topics, or representations explicitly pointed out; concepts are connected to students' current knowledge and future learning; explanations of the reasons for

executing elements of the procedure are emphasized” (pp. 65).

In a comparative study conducted by Chappell and Kilpatrick (2003) to investigate the effect of two instructional environments (concept-based verses procedural-based) on learners' performance in calculus, an achievement test was used to measure the extent to which students in two different learning environments will respond to conceptual and procedural questions. The authors reported that students in the concept-based instruction environment performed significantly better than students who received procedural instruction in both procedural and conceptual oriented task. To further investigate on differences between conceptual and procedural instructions, Chappell and Kilpatrick (2006) also conducted a replicate study. Once again, the authors found out that the students who received instruction on calculus with conceptual approach scored significantly higher than those taught with a procedural approach. As such the results of the study were consistent with the former study.

Also, in an investigative study, Khoule, Bonsu and El Houari (2017) used conceptual and procedural teaching framework to determine the relationship between conceptual teaching, procedural teaching and students' mathematics anxiety in a school college. The study evidenced that conceptual teaching has a significant positive impact on mathematics anxiety as compared to traditional teaching. The study results affirmed that while concept-based instruction could lower the mathematics anxiety levels of students; procedural teaching method, on the other hand, could lead to mathematics anxiety. In effect, the use of concept-based instruction is reported to be more effective in improving students' learning outcomes such as achievement and mathematical anxieties. Previous studies have demonstrated that the use of concept-based instruction can develop students in developing

critical thinking skills and improve their problem-solving abilities (e.g., Langton, 1991; Borji et al., 2019). It also helps students reach conceptual understanding of a topic and create sustainability in learning (Borji et al., 2019). Additionally, Borji, Radmehr and Font (2019) suggested that students taught with conceptual teaching create more sustainable understanding and increase students' procedural abilities and efficiency.

Despite the reported benefits of teaching using concept-based instruction, there seems to be very little research on how the use of concept-based instruction in the teaching of circle geometry topic would support Ghanaian students' learning. This study, therefore, explored the impact of concept-based instruction approach on the achievement of SHS students in circle theorem beyond the identification of students' geometric thinking levels or learning difficulties (e.g. Bosson-Amedenu, 2017). This study therefore, seeks to compare two teaching methods namely, concept-based instruction and traditional (procedural) methods on senior high school students' achievement in circle geometry, specifically, circle theorems. We do so by using the two types of instructional approaches, traditional and concept-based methods, as our conceptual framework.

Methodology

Research design

A quasi-experimental non-equivalent (pre-test post-test experimental and control group)

Research Instruments

The research instrument consisted of geometry achievement test (GAT) items. The pre-test (T1 and T2) and post-test (R1 and R2) were used to compare students' entry-level

Group	Pre-test	Treatment	Post-test
Treatment group	T1	X	R1
Control group	T2	C	R2

design was adopted. Pallant (2001) encourages the use of this design when dealing with intact classes.

Concept-based method of teaching was applied to the experimental group whereas the control group received traditional method instruction. Consequently, the study used two categorical independent variables and one dependent variable. The independent variables were the two instructional learning environments used in teaching circle geometry specifically, the circle theorems, thus, concept-based and traditional methods. The dependent variable was students' performance in circle theorem achievement test.

Participants

The two schools were purposively selected to ensure that the schools belonged to the same rank using the Ghana Education Service (Ghana Education Service) rankings. This provided a base line for comparison by ensuring that all other factors that could affect the results of the study are held constant except the teaching approaches. From these two schools, two intact classes were randomly selected. These classes were SHS 2 classes from two different schools (School A and School B). Hence, the sample consisted of 78 students of which 41 students (24 males and 17 females) who were in the control group and 37 students (21 males and 16 females) in the experimental group.

and treatment effect respectively between intervention and control groups. The essence of the pre-intervention test was to find out whether the two groups were similar in geometry abilities before the treatment, and this provided a base line for the learning of the topic (circle theorem). Conversely, the post-test was aimed at finding out the performance of students after the treatment. Both the pre- and post-intervention test items were similar to ensure an accurate comparison.

The geometry achievement test items were constructed mainly on circle theorems and the items were based on the learning objectives in the SHS core mathematics teaching syllabus. Some of the items were constructed by the second researcher to ensure that the items were within the Ghanaian core mathematics syllabus and the appropriately fit the Ghanaian context. Other items were adapted from previous research studies that related to the Ghanaian syllabus. The test contained fifteen essay-type questions. In selecting the questions, each item selected had to pass through: expert critique, item difficulty and item discrimination analysis. The reliability of the test instrument was established using Pearson Product Moment Correlation. To check the inter-rater reliability of the test since the test was open-ended, the students' answers were rated by two different scorers who have several years of experience marking national examinations and following moderation. The result of reliability was 0.996 between scorer 1 and scorer 2. The results revealed significant

relationship between the scores of the two raters, hence the test was reliable.

In addition, a semi-structured interview guide was administered to some students in the experimental group to explore their experiences the concept-based instruction learning environment. This was done to provide a deeper understanding of data collected through the intervention test. Ten (10) students who took part in the conceptual lesson (experimental group) were randomly selected and interviewed individually by the second researcher after the post-intervention test.

Treatment

Throughout the study, the experimental group was taught using concept-based instruction whereas the control group received traditional method of instruction. In the control group, definitions and theorems, rules and algorithms were presented as a series of steps consistent with our conceptual framework. The main focus of teaching in the control group was on solving many problems as possible to reach a correct answer using mainly teacher demonstrations of solved questions and practice questions for students. Nevertheless, in the traditional approach, methods such as demonstration and illustration using examples, questions and answers method,

lecture approach were used. For example, in introducing the concept of circle theorem, instruction begins by presenting definitions, theorems and algorithms on the various angle properties of circle theorem, and working through examples on how to calculate the angles properties of the circle.

In the experimental group, the students were engaged in hand-on activities by constructing, measuring, and investigating angles formed and possible relationships using mathematical set instruments. This was done to help students discover the various properties and theorems of circle concepts by themselves such as alternative segment theorem, angles in a cyclic quadrilateral and tangent and radius theorem etc. The idea was to conceptualise the concept of cyclic quadrilateral by realizing that angles in opposite segments are supplementary. The concept-based instruction lesson in the experimental group focused on verifying and justifying each step of procedure, hand-on activities investigations in an experiential way to help them explore geometric concepts about the circle and form their own knowledge (see Appendix A). See an example of an activity used in the concept-based instruction related to cyclic quadrilateral (see Figure 3).

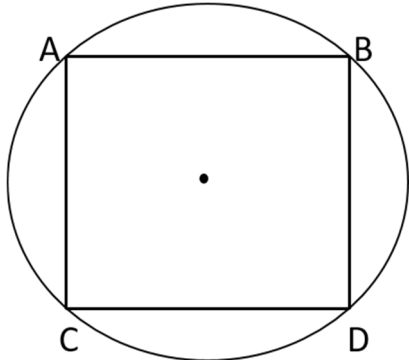
<p>Learners were instructed to:</p> <ol style="list-style-type: none">Draw a circle of any radius.Draw any quadrilateral with all the vertices touching the circumference of the circle.Measure the four angles of the quadrilateral.What is the sum of two pairs of opposite sides; $\angle A + \angle D$ and $\angle B + \angle C$.What conclusion can you draw from your result in d) about the cyclic quadrilateral?	
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Figure 3 Cyclic quadrilateral (9)

The two groups, both the control and the experimental groups were facilitated by the same instructor. However, the lessons in the two groups were taught strictly in accordance to the lesson plans designed for each group thereby minimizing any possible researcher interest. Additionally, the two instructional lesson plans were reviewed by two experienced mathematics instructors to ensure that the two instructional methods were compatible with the learning environments defined. Furthermore, each lesson segment was observed and evaluated by two independent observers using Hiebert and Lefevre (1986) conceptual and procedural teaching indicators (adapted from Joffrion, 2005). The lessons segments were examined and coded as conceptual (C), procedural (P) or neither (N) depending on the type of knowledge emphasized. The independent observers compared their evaluations and discussed for consistency and there was an agreement between them for each lesson segment and thus reliability was assured. These strategies were implemented to ensure the reliability and validity of the findings by either minimising or completely eliminating researchers' bias in the implementation of treatments.

Data Collection

At the beginning of the study, the Geometry Achievement Test (GAT) was administered to the students to both experimental and control groups as the pre-and-post-tests. Additionally, data were collected using a semi-structured interview guide. The researchers gained approval from the Institutional Review Board

(IRB) of the University of Cape Coast and all the participating students voluntarily agreed and signed the informed consent form.

Data Analysis

Research hypotheses one and two which sought to determine whether or not there were substantial improvement in the performance of students taught circle theorem with concept-based method and those taught with traditional method were answered using the pre-test and post-test scores obtained from students in both groups. The paired sample t-test was used to compare pre-test and post-test of the experimental and control groups. Research hypothesis three which sought to determine which teaching intervention method was more effective for teaching circle theorems was answered using an independent samples t-test. Moreover, students' responses from the interview items were reported qualitatively. These responses were also coded as 'yes' and 'no' responses and then expressed as percentiles. The students' views threw more light on the perceived students' perception of concept-based method of instruction.

Results

In this section, we present and discuss the results of the findings in relation to each of the research hypotheses that guided the study.

Analysis of the achievement of students in the experimental group

The first research hypothesis sought to determine whether or not there was a statistically significant difference in the scores

Table 1 Result of the paired sample t-test comparing the pre-test and post-test scores of students treated with the concept-based instruction

Test	Mean	SD	df	t-value	p-value	Eta. square
Pre-test	8.805	3.37	40	41.475	0.000	0.977
Post-test	25.05	3.74				

of students taught circle theorems with concept-based instruction by comparing their pre-test and post-test scores. To illustrate the significant improvement in the performance of students in circle theorem, a box-and-whisker plot was used as indicated in Figure 4. From the box-and-whisker plot, comparatively, the post-test box plot had a

3.75); $p < 0.0001$]. Additionally, eta squared statistic indicated a magnitude of (0.977), a large effect size further highlighting the magnitude of the effect observed.

Analysis of the achievement of students in the control group

The second research hypothesis sought to examine whether or not there was an improvement in the performance of students taught circle theorem with the traditional method using their pre-and-post test scores. To compare the distribution of scores visually, a box and whisker plot was constructed to show the increase in performance of students in the control group. This can be observed in Figure 5 by the distance travelled. Looking at the graph, the post-test box plot had a higher first quartile value, higher median value and higher last quartile value compared to the pre-test box plot values, suggesting evidence of improved performance in the post-test scores of students compared to the pre-test scores.

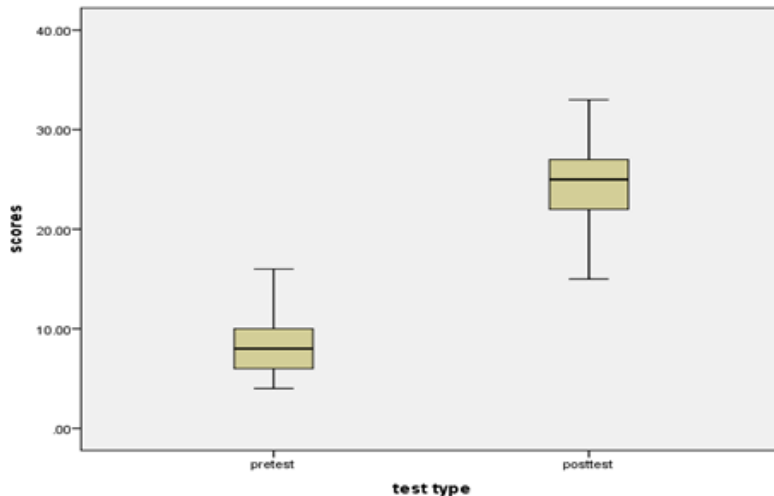


Figure 4 Box Plot Showing Differences in Achievement of the Experimental Group

higher first quartile value, higher median value and higher fourth quartile value more than the pre-test box plot values, indicating improved performance in the post-test intervention scores. The pre-test box plot is almost symmetrical and has no outstanding outliers or extreme values on either ends of the whiskers.

To check if the difference in the average scores between the pre-and post-test scores in the treatment group was statistically significant, a paired sample t-test was conducted.

The results (as shown in Table 1) of the paired sample t-test in the experimental group showed a statistically significant difference in performance between the pre-test scores ($M = 8.80, SD = 3.37$) and posttest scores [$M = 25.05, SD =$

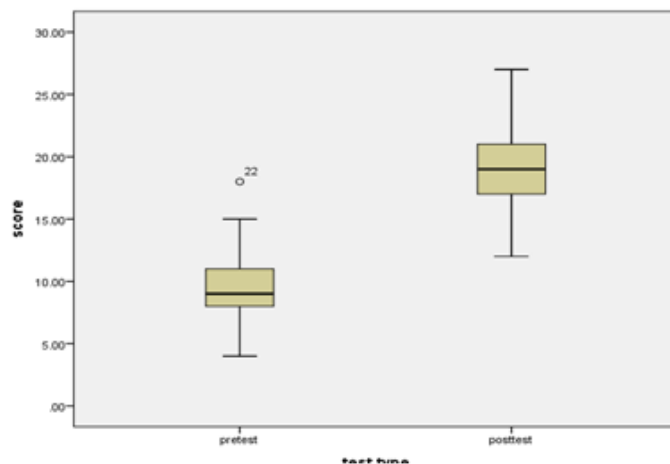


Figure 5 Box Plot Showing Difference in Performance of Students Taught with TM

To check if the differences in the average scores between the pre and post achievement test of the control group were statistically significant or students' performance has improved significantly, a paired sample t-test was conducted at 5% significance level to test the null hypothesis that: "Traditional instruction approach has no effect on the achievement of students in circle theorem". The results (as shown in Table 2) of the paired

academic abilities before the intervention. Table 3 includes the results of descriptive statistics of the pre-test for control and intervention groups. In the pre-test, the intervention group obtained a mean score of ($M = 8.804; SD = 3.37$) while the control group obtained a mean score of ($M = 9.8108; SD = 3.098$). There was a slight average point difference of (1.007) in the pre-test between the groups.

Table 2 Result of the paired sample t-test comparing the pre-test and post-test scores of students instructed with traditional method

Test	Mean	SD	df	t-value	p-value	Eta. square
Pre-test	9.81	3.098	36	27.031	0.000	0.953
Post-test	18.97	3.345				

Table 3 Results of the independent sample t-test comparing the pre-test scores of the control and experimental groups

Group	N	Mean	SD	df	t-value	p-value
Control	37	9.8	3.10	76	1.367	0.176
Experimental	41	8.8	3.37			

sample t-test of the control group showed a statistically significant difference in performance between the pre-test scores ($M = 9.81, SD = 3.10$) and post-test scores [$(M = 18.97, SD = 3.435); p = 0.000 < \alpha = 0.05$]. This is evidence that the performance of students taught circle theorem with the traditional method of the control group had improved significantly. The eta squared statistic also gave a large effect size of (0.953) difference, indicating a significant effect on student's performance in circle theorem.

Comparing the achievement of the treatment and control groups

Prior to the experiment, both groups were given a pre-intervention test to examine if the two groups were homogeneous in terms of

An independent sample t-test was conducted to determine whether or not the mean difference observed was statistically significant. This is reported in Table 3. The p-value of (0.176) was greater than α -value (0.05) hence no significant difference between the two groups [$t(76) = 1.367; p = 0.176 > \alpha = 0.05$]. This result shows that the learners in the control and treatment groups were similar in geometric abilities before the intervention hence, any difference in post-test scores could be attributed to the treatment effect.

Research hypothesis: H_{03} : "There is no significant difference in the achievement of students taught circle theorems with the concept-based method and those taught with the traditional method".

The data presented in Table 4 is the post-test score results of both groups; the treatment

learners studying circle theorem compared to the traditional method.

Graphically, this can further be illustrated using the box-and-whisker plot as depicted in Figure 6. Looking at the graph, the experimental group post-test box plot had a higher lower quartile value, higher median

Table 4 Results of the independent sample t-test comparing the post-test scores of the control and experimental groups

Group	N	Mean	SD	df	t-value	p-value
Control	37	18.97	3.435	76	-7.43	0.000
Experimental	41	25.04	3.748			

group obtained an average score of ($M = 25.05$; $SD = 3.47$) while the control group obtained a mean score of ($M = 18.87$; $SD = 3.435$). The results indicate that both the intervention and control groups differ in post-test mean scores, with a (6.18) mean point difference. This means that the intervention group scored better, on average, than the control group in the post-test. This is evidence that the intervention activity did improve students' achievement in circle theorem and can, therefore, be suggested that concept-based instruction may have had more effect on

value and higher upper quartile value compared with the control post-test box plot values, suggesting evidence of more improved performance in the post-intervention scores of students in the treatment group. Both the post-test box plots are almost symmetrical and had no outstanding outliers or extreme values on ends of the whiskers. Moreover, by inspection, all the pre-test and post-test box plots are fairly normal suggesting normality of distribution of scores. This permitted the use of an independent sample t-test.

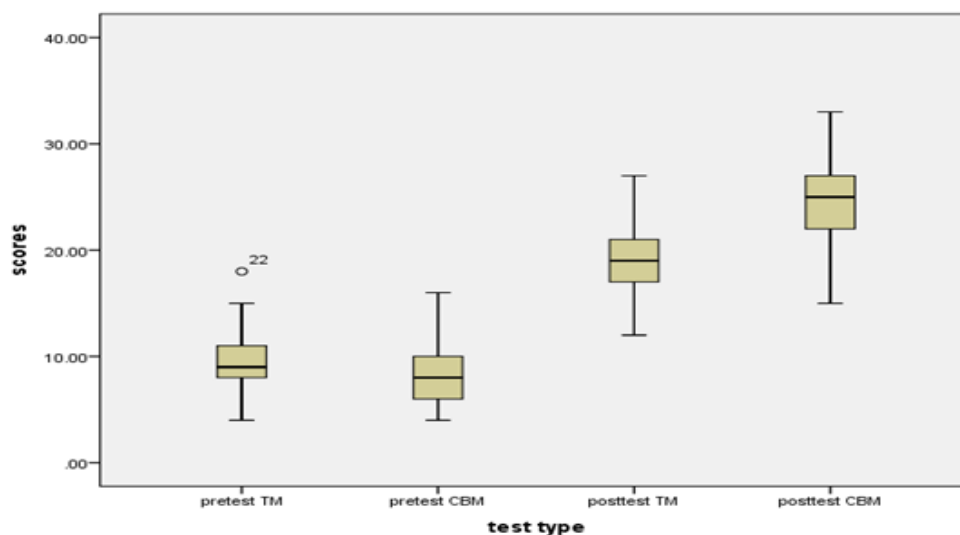


Figure 6 Box plot showing performance of students in instructed with concept-based instruction and traditional instruction

To determine which instructional intervention method tested is most effective for teaching and learning of circle geometry, the post-test achievement scores were further analysed using a sample t-test. The analysis of the post achievement scores of the control and intervention groups using an independent sample t-test is detailed in Table 4. The results showed a substantial difference between the two groups [$t(76) = -7.43; p = 0.000 < \alpha = 0.05$] in favour of the treatment group. This implies that concept-based teaching as an instructional method had significantly improved students' performance in circle theorem compared to the traditional method.

In order to gain further insight into the nature of the learning environment in the treatment group, 10 students were randomly selected and interviewed. The data were coded and analysed thematically as presented in Table 5. Five themes were obtained (see Table 5). We also provide illustrative quotes.

Most students in the intervention group confirmed that they understood the conceptual lesson with some explaining that it was easier for them to understand the properties and theorems of the circle. Some of the interviewed students (about 80%) also did indicate that they were able to discover the properties of circle theorem by themselves

through the group work and hands-on activity. Moreover, approximately 90% of the students said that they enjoyed the lesson because the lesson was interactive and all of them were engaged in the lesson. However, some students also indicated that the lesson was time-consuming as some of them could not finish some of the activities.

We provide some excerpts from the interview data as further illustration of the positive feedback the experimental group gave regarding the use of concept-based instruction. The following quotes and comments from audiotape best explained the experiences and perceptions of students regarding the teaching method:

Student 3: Yes, I was enthused with the lesson because we participated fully in the lesson and was able to discover all the properties and theorems of the circle by ourselves.

Student 7: Yes, I was able to understand the concepts very well because the explanations were very clear. Knowing how the theorems came about help me understand better and will not easily forget the theorems. In fact, I will not waste time to chew and pour these theorems again.

Table 5 Students' perception of the nature of the learning promoted by the concept-based instructional approach to teaching in circle theorem (n = 10)

Students' response	Yes	No
1. The teaching method was practical	(8) 80%	(2) 2%
2. Easy to understand	(9) 90%	(1) 1%
3. We participated fully in the lesson	(9) 90%	(1) 1%
4. We enjoyed the lesson	(9) 90%	(1) 2%
5. We prefer this teaching method to the others	(10) 10%	(0) 0%

Student 5: *I was working with full concentration and everybody was busy doing geometric construction.*

The sampled students' responses to the interview questions are used as an illustration of how concept-based instruction or a learning environment which affords students opportunities to make meaning of mathematical concepts can help ease some of the problems students' face in learning circle theorem. Most of the students explained that it was easy for them to understand the concepts and theorems of the circle. They indicated that the concept-based approaches to mathematics instruction gave them the opportunity to explore, interact and work actively in groups performing different activities. These observations made by the students in their responses agree with the observations made by Chappell and Kilpatrick (2003) that indicated that students can achieve outstanding gains with respect to the understanding of mathematical concepts when instructed in a concept learning environment.

Discussions

The present study reported evidence on the impact of concept-based instruction on the achievement of students in circle theorem by comparing students' achievement on circle theorems when exposed to two different teaching methods. The study found that the use of concept-based instruction in teaching circle theorems had a significant influence on students' performance in geometry as compared to the traditional method. Some of the reasons for the observed difference are that concept-based instruction helps students to understand the mathematical concepts and procedural skills in an experiential way. This helps motivates students and bring students level of reasoning to the expected level of the topic as advocated by Lim (1992). Students in the treatment group reported that they were fully involved in the teaching and learning process with the effect being that there is no

need to "waste time to chew and pour these theorems again".

Moreover, the most glaring elements in the experimental group were explicit connections between algebraic and geometric representations, between prior learning and current lesson, and between different problems. There were multiple students' inputs and students were asked to justify their reasoning. This conceptual emphasis exemplified the conceptual teaching indicators espoused by Hiebert and Lefevre (1986). The explicit connections between the algebraic, numerical, symbolic equations and geometric representations contributed to students' remarkable performance in the experimental group. On the contrast, the lessons in the control group were focused primarily on the step-by-step methods of finding answers to each problem encountered. The mathematical content in the control group was not rich in explanations and connections between topics, concepts and procedures were not emphasized (Joffrion, 2005). Definitions, theorems and procedures were presented as sequential and stand-alone skills to be mastered. The lessons of the control group did not encourage students to use their own procedures in solving questions. This approach is consistent with the traditional viewpoint of teaching and also in line with the descriptions and indicators of procedural teaching outline by Hiebert and Lefevre (1986).

Apparently, the use of concept-based instructional method made lessons in the experimental group more practical, afford students the opportunity to explore and verify mathematical concepts in an experiential way. This approach to mathematics instruction helps students understand mathematical concepts and the reasons behind using each procedure or formula and how they relate to each other. A detailed analysis of the individual questions of students' response to

the mathematical task in the pre and post-test confirmed this. For example, most students could not recognize the required circle theorem properties to answer pre-test questions that required informal arguments. However, in the post achievement test, most of the students provided meaningful arguments of theorems in answering the post-test. The analysis here seem to suggest that the hand-on activities investigations and constantly verifying and justifying elements of procedures and each concept of circle theorem property in an experiential way using various geometric and algebraic methods enables students to reach a higher geometric thinking and understanding of circle geometry concepts. This helps students to examine the relationship between geometric figures and properties as well as establishing links and connections between concepts, rules and procedures. The use of practical investigations, experientially, motivates students and helps them to appreciate the usefulness of geometry as well as its structural beauty (Lim, 1992).

Some of possible reasons that might have also accounted for these findings could be that students taught with the concept-based method in the experimental group were exposed to the concept of discussion, group work, problem-solving, hand-on-activity and guided discovery. The conceptual instruction provided an opportunity for learners in the experimental group to understand mathematical concepts and techniques involving meaningful definitions as well as helping them know the reasons behind executing every step of the procedure. Consequently, this improved students' conceptual understanding of the topic and would create sustainability in the learning (Borji, Radmehr & Font, 2019). Notwithstanding, the result of the study is consistent with other contemporary studies on conceptual learning (e.g. Khoule et al., 2017; Chappell & Killpatrick, 2003; Borji, et al.,

2019). For example, the finding of the study is affirmed by the findings made by Chappell and Killpatrick (2003) that students can achieve outstanding gains with respect to the understanding of mathematical concepts when instructed in a conceptual learning environment.

Conclusion

Based on the findings of the present study, the study, therefore, concludes that if concept-based instruction method is used in teaching and learning of circle theorem in SHS in Ghana, the abysmal performance of students in mathematics questions involving circle theorem will be minimal if not a thing of the past. The students also have positive perceptions on concept-based teaching approach in terms of their interest, understanding and practicality. The study affirmed that the use of concept-based instruction method has the potential of helping students to improve their understanding of circle geometry concepts. The study calls for the need for mathematics teachers to focus on concept development rather than emphasising the teaching of skills, procedures and algorithms. That is, when a concept is taught in class, both geometric and algebraic expressions must be given coupled with real life examples (Mahir, 2009). When topics and concepts are learned conceptually, it is more likely to be recalled and applied in new situations (Joffrion, 2005; Hiebert & Lefevre, 1986).

Another implication of the present study to teaching and learning is that teachers at the SHS level need to verify the various properties of circle theorems to students either by geometric followed by the algebraic verification to reduce the abstract nature in which the theorems are introduced. The verifications of theorems or sketch overviews of geometric proofs may help students to appreciate the structure of space which is what geometry is all about (Lim, 1992).

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Appendix A

Summary of Lesson Plan Activities for Experimental Group

Step	Activity Description
Step 1	<p data-bbox="342 394 841 428">Presenting (examples and non-examples)</p> <p data-bbox="342 457 1500 695">Examples and non-examples of simple geometric figures were presented and shown to students in the form of visualization. Students were able to identify the geometric figures by their shapes and properties. For example, students were able to distinguish between figures, a circle or not a circle and also draw circles of varying radii. This task was done to develop the basic geometric concepts, by recognizing of shapes and properties of figures. The concept of circle was then formed through practical experience, by drawing circles and introducing terminologies such as radius, chord, centre, tangent, diameter etc.</p> <p data-bbox="451 709 1230 793"> a) Tell why each of the following figures is or is not a circle. b) Can you give a general rule to fit the above answers? </p> <div data-bbox="380 806 1273 953" style="text-align: center;"> </div>
Step 2	<p data-bbox="342 1024 639 1058">Developing the concept</p> <p data-bbox="342 1087 1500 1352">The students were engaged in an activity by constructing, measuring, and investigating of angles using mathematical set instruments. The hand-on activities offer learners the opportunity to participate in the learning process. These tasks were done to help students discover and verify the various properties and theorems of circle concepts by themselves such as alternative segment theorem, angles in a cyclic quadrilateral and tangent and radius theorem etc. For example, the teacher draws a cyclic quadrilateral on the chalkboard with opposite angles $\angle ABC$ and $\angle ACD$. The idea was to conceptualize the concept of cyclic quadrilateral by realizing that angles in opposite segments are supplementary.</p> <p data-bbox="266 1457 591 1486">Learners were instructed to:</p> <p data-bbox="266 1507 1029 1801"> a) Draw a circle of any radius b) Draw any quadrilateral with all the vertices touching the circumference of the circle c) Measure the four angles of the quadrilateral d) What is the sum of two pair of opposite sides; $\angle A + \angle D$ and $\angle B + \angle C$ e) What conclusion can you make of the cyclic quadrilateral </p> <div data-bbox="1062 1457 1377 1738" style="text-align: center;"> </div>

Step 3 Recognizing the concept

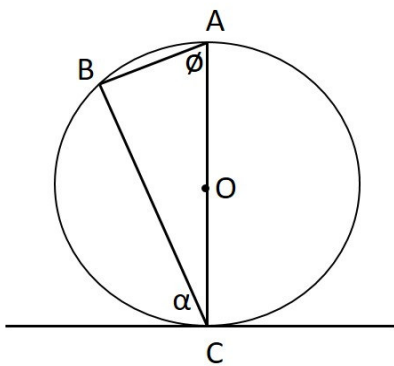
Following concept development through exploration of activities and interactively, students were then guided to recognize the concepts based on what they have observed and discovered on the activities. Students were allowed to express and share their opinions on what they have discovered. This task enabled students to become conscious of the new concept. The task also provided a means of assessing students' understanding of the concept. Interactively, students discovered all the various properties of circle theorem on their own. For example, from the activity above (in step 2), intuitively, students concluded that the opposite angles of a cyclic quadrilateral were supplementary. That is $\angle A + \angle D = 180^\circ$ and $\angle C + \angle B = 180^\circ$.

Step 4 Solve problems

At this stage, instruction began on the algorithms of the concept. Students learnt how to solve questions regarding the concept. In solving problems, students were also provided with explanations and reasons behind using a particular procedure, theorem, formula or technique. Multiple methods of solving problems were emphasized. This task was done to provide students with the skills and procedures on how to calculate the angle properties of circle theorem.

Furthermore, to build connections among mathematical ideas and achieve higher geometric thinking, students were also guided after the geometric proves of theorems to verify some of the various properties of circle theorem algebraically. For example, students were guided to prove the alternative segment theorem (i.e. the angle between a tangent and a chord is equal to the angle in the alternative segment) as shown below.

Theorem: to prove that the angle between a chord and a tangent is equal to the angle the same chord subtends in the opposite segment:



Alternative segment theorem:

We want to prove that $\phi = \alpha$

$\angle ABC = 90^\circ$ (students shared that the angle subtended by a diameter is a right angle)

$\angle BCA + \alpha = 90 \dots (1)$ (students shared that the angle between a tangent and radii is 90)

Therefore, $\angle ABC + \angle BCA + \phi = 180$ (students notice that sum of interior angles in a triangle is 180)

Thus, $90 + \angle BCA + \phi = 180$

Simplifying the equation gives $\angle BCA + \phi = 90 \dots\dots (2)$

$\angle BCA + \phi = 90 \dots\dots (1)$

$\angle BCA + \alpha = 90 \dots\dots (2)$

Solving the two equations simultaneously

$\alpha - \phi = 0$ and thus $\alpha = \phi$ as required.

Step 5 Summarize/ homework assignment

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At the last stage, students were guided to summarize what they have learnt about the concept of circle theorem and the correct geometric language was emphasized. The instructor gave some questions for students to answer as homework.