components of a given argument. Toulmin's model is use in this article to analyse the arguments within the students' argumentation and proof. Toulmin model is a powerful tool for comparing argumentation and mathematical proof in mathematics. There are many researchers who use Toulmin model to analyse argumentation and proof (Pedemonte, 2002; Tsujiyama, 2012).

According to Toulmin model, an argument has tree component. The standpoint (an assertion, an opinion) which is called a claim. The data are produced supporting the claim. A warrant provides the justification for using the data in support of the data-claim relationships. This is the ternary base structure of an argument. Some auxiliary elements may be necessary to describe it: the qualifier, the rebuttal and the backing. For our study, we limit ourselves only to the first three elements described above.

Toulmin gives this example of argument: "*Harry is a British subject because he was born in Bermuda*." The argument can be analysing as follows.

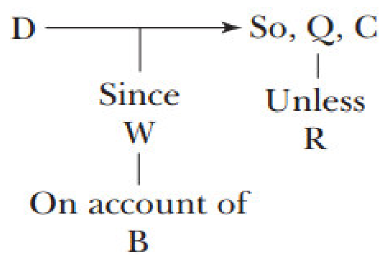
Data (D): *Harry was born in Bermuda.*

Conclusion (C): *Harry is a British subject.*

Warrant (W): since a *man born in Bermuda will generally be a British subject.*

Backing (B): on account of *the following statutes and other legal provisions.*

Rebuttal (R): unless *both his parents were aliens/he has become a naturalized American.*



This model suggests a way to categorize data and warrants. A categorization of data of an argument used in geometry activity can be obtained by questioning their origins. The question that can be asked is the following: where did the data for this argument come from? To get warrant of the argument, we answer the following question: What makes it possible to move from data to conclusion? We assume that taking this question into account is useful in describing students' comprehension of the relationship between drawing and figure trout their argumentation and proof.

Toulmin model has proven its usefulness in several research (Fukawa-Connelly, 2014; Moore-Russo, Conner, & Rugg, 2011; Pedemonte, 2002; Stephan & Rasmussen, 2002) works on argumentation. However; it is criticized by some researchers (Pedemonte, 2007) in mathematics education. For example: the structure of the arguments sometimes does not take into account the participants' knowledge bases, and explains why the warrant is or is not strong to assert the claim. The integration of the concept image and the concept definition model and the Toulmin model is the solution we have found to more accurately reflect the complexity of the argument by using these "implicit" elements. In particular, the concept image and concept definition model by expressing the external representation of the cognitive structure that mobilizes others to select the data and deduce the conclusion.

The Concept Image and Concept Definition

The concept image and concept definition model were developed by Vinner (1983) as a theoretical framework that guides the researcher in understanding the student's mental process. According to Vinner, in the cognitive structure of a student, they existed two different cells. One of the two cells is for the definition(s) of the concept and the other one is for the concept image. One cells or even both of them might be void. There might be an interaction between the two cells, although they can be formed independently. Vinner argues that there is a conflict between the structure of written mathematical definitions or statements or concepts and the cognitive process of acquiring the concepts. We use concept image and concept definition in these articles to analyse the effect of the relationship between drawing and figure on the student's argumentation and proof.

The concept image is a concept that is used to describe the total cognitive structure of an individual, associated with a given concept, it includes all mental images and properties, impression as well as the processes that are associated with it. This may not be consistent and have aspects that are very different from the formal definition of the concept (the definition accepted by the mathematical community). When a concept is mentioned or when we solve a task in relation to a concept, our memory is stimulated and something is mentioned. However, what is mentioned is rarely only the formal definition of the concept, but rather, a set of visual representations, images, properties associated with the concept, theorems related to the concept or experiments. This set constitutes the concept image (Tall & Vinner, 1981).

Various studies report that individual concept image differs from formal theory and contain factors that cause cognitive conflicts and include conception which is in contradiction with the formal axiomatic system of mathematics. For example, when a student says that the parallel lines have the same length. This is his concept image evoked on parallelism. It can be assumed that he acquired it through experience on the drawings he had to encounter. This concept image is in contradiction to the formal axiomatic theory of parallelism because the length of the straight line didn't exist. We think that incoherence of the concept image may have repercussions on pertinence and strength of students' argumentation and proof (Duval, 1991). The identification of concepts images mobilize by the students in their production should inform us about the effect of their mental representation about the figure on their argumentation and proof.

Viholainen (2008) used the term "coherence of a concept image" to refer to the level of organization of the concept image. He lists some properties of a highly coherent concept image as follows:

1. An individual whose concept image is considered has a clear personal conception of the concept.
2. Conceptions, cognitive representations and mental images concerning the concept are well connected to each other.
3. The concept image does not include internal contradictions.
4. The concept image does not include conceptions which are in contradiction with the formal axiomatic system of mathematics.

The student can memorize the definition of a figure, which he produces when it is requested. This verbal definition, that can be memorized and repeated by the student is called by Vinner concept definition, it is a set of words used to specify this figure, it is related to the figure as a whole. It can also be the student's personal reconstruction of a definition. In this particular case, these are words that the student uses to explain his or her own concept image (evoked). Research reports that student personnel concept definition seems to be in contradiction with formal definition (Vinner, 1983). For example, the student can define a straight line as a distance between two points. This personal concept definition of the student is ambiguous and contradictory, because the straight line is not a distance. We can imagine that he constructs it by visual perception and experience on the drawings he had to encounter. It is not a reinterpretation of the teacher's definition.

Research in mathematics education reports some characteristics that a good definition in mathematics should have. This characterization can be a relevant tool for the analysis of the students' personal concept definition of the figure. According to Orit Zaslavsky and Karni Shir (2005), a mathematical definition must be:

1. Non contradicting: all conditions of a definition should coexist;
2. Unambiguous: its meaning should be uniquely interpreted;
3. Invariant under change of representation;
4. Hierarchical, that is, it should be based on basic or previously defined concepts, in a noncircular manner.

We believe that the Vinner model will help us to describe and interpret the effect of the relationship between drawing and figure on student argumentation and proof during the problem solving. We can assume that, when the drawing is part of the statement of a geometry problem, the one active in the student's cognitive structure, some elements of his concept image about the figure he is supposed to represent. When it is a proof task, for example to prove the nature of a figure, students will externally express their personal interpretation of the figure. They will develop a heuristic argumentation to find a proof strategy and after this they will construct the proof. We believe that in a student's argumentation, the arguments are developed from their concept image evoked of figures. This element of the concept image may have been activated by the figure to which it is attached or by the view of the drawing.

The diagram below shows the interactions in the relationship between drawing and figure.

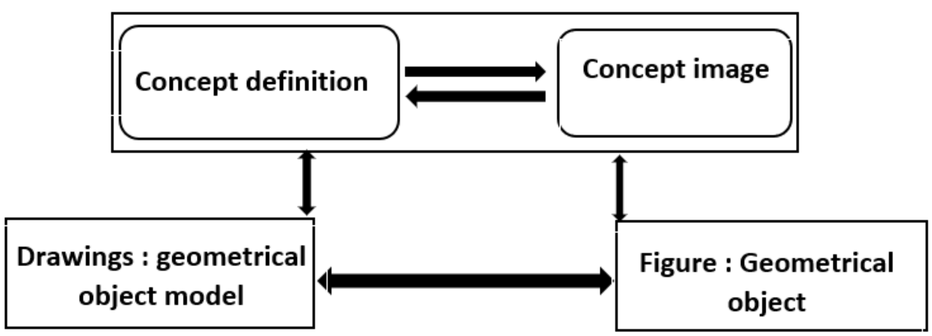


Figure 1 Relationship between drawings and figure

Relation between Argumentation and Proof

Durand-Guerrier and al. describe argumentation as a written or oral speech conducted according to common rules, and aimed at a mutually acceptable conclusion of a proposal whose content or truth is the subject of debate (Hanna & de Villiers, 2008).

Mariotti (2001) believes that the practice of argumentation can lead to the learning of the mathematical proof. Thus, there is a continuity between argumentation and mathematical proof, known as cognitive unity. Cognitive unity is a process analysis tool that allows: to highlight the potential of certain problematic situations. This is particularly true when problems are used to introduce learners to mathematical proof. According to cognitive unity hypothesis, the conjecture is usually produced by the learner at the end of the argumentation process. The arguments resulting from this phase are organized to build a mathematical proof of the statement which thus becomes a theorem (Rossella Garuti, Boero, & Lemut, 1998). From our point of view, cognitive unity can also be observed when students solve problems that do not necessarily lead to the production of a conjecture. We believe that during the resolution of the open problem that leads to the proof of an assertion, the student is involved in an exploratory activity during which argumentation is produced and the arguments used in this argumentation can be reused, restructured and reorganized to produce proof.

Research Question

Previous research has highlighted the difficulties associated with drawing and figure in high school. For example, a conflict exists between what students' seen on the drawing and what he knew about the manipulated figure. Moreover, students focus much more on the shape of the drawings than on the property that are represented. We also know that students construct their arguments from the drawing and the figure. Students who fail to produce an argument also have difficulty in producing a mathematical proof. However, there is very little work about how student use drawing and figure to produce their arguments. Therefore, it is the aim of this study to well know how the relationships between drawing and figure affects the students' arguments in problems solving situation.

Three research questions were considered in this study:

1. Which concept definition of the figure students mobilized in their argumentation?
2. How student comprehension of the relation between the drawing and figures affect the production of arguments used by students in their argumentation and proof?
3. How the relation between the drawing and figure affect the cognitive unite?

Methodology

Participants and Sample

The participants in the study are 30 students of 14–16 years old. They are in Form 4 and Form 5 attending a school in Yaoundé, Cameroon. For this article, we describe and analyse the work of four pairs of students who participate in the study during the 2018–2019 school year. These are students who allow to significantly show how the relationship between drawing and figure affect the construction of argumentation and the production of proof. The proof is supposed to be part of their culture for having practiced it and observed the teacher practicing it in their geometrical classes. Some of these students are considered to have a good level of mathematics while others have an average level. They were selected on the basis of their articulateness and willingness to devote time to the research activities. Students in this sample studied geometrical figures such as quadrilaterals and triangles in previous classes. The theorems necessary to solve the problems were taught to these students and were sufficiently reinforced in the exercises and lessons.

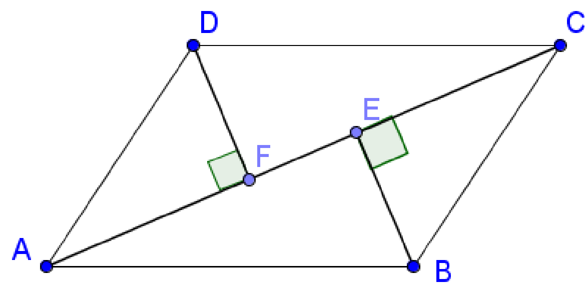
Data Collection

The participants in the study were observed in a problem-solving situation. We conducted an experiment with students during which we recorded their discussion and collected their written production (Gousseau-Coutat, 2006; Pedemonte, 2007a). We provided the students with a sheet containing a Euclidean geometry problem, and we used a tape recorder to record the students' argumentation. The problem consists of a statement composed by a text which describes a figure and a drawing that illustrates the figure describe by the text. The drawing used in this problem has two functions: representative function the drawing represents all or part of the content of the problem statement; informative function the drawing gives essential information for the resolution of the problem; the problem is based on the drawing.

The problem proposed for the experiment was written in French and can be translated into English as following:

Problem

In the ABCD parallelogram, the straight line (DF) and (BE) are perpendicular to the straight line (AC). Can we say that the quadrilateral DFBE is a parallelogram?



The participants know that the proposal that truth is the subject of debate here has to be proved as stated in the didactic contract. We chose to associate drawing with the problem statement for the following reasons: we want to observe the interactions between students and the drawing; we want students to have the same drawing; we want to avoid that students represent false drawings that may complicate the solving problems; we want students to concentrate on argumentation and proof. However, the drawing is not complete, it is up to the student to complete the quadrilateral DFBE.

The experiment took place in the evening, after school hours. In addition to that, the recordings are of acceptable quality during this time of the day because students who not participants in the study are already gone to their house. The fact that the students worked in groups led them to verbally interact. This makes easy to access their strategies and arguments. Altogether, we have analysed the activity and productions of 30 students who worked in pairs on the sheet of paper, the experiment lasted about 50 minutes. The students proceeded in two phases to solve the problem. The first phase consists of constructing argumentation and the second phase consist of producing proof. The students' argumentation was recorded. The teacher and a researcher were present in the classroom. They did not interfere in solving the problem.

Data Analysis

The recordings of the students 'discussions were transcribed and translated from French into English. For this article we have retained the transcripts of the discussions of five pairs of students. Our analyses follow the same principle as those used in previous research in mathematics education (Gousseau-Coutat, 2006; Pedemonte, 2002), we proceeded by an a priori analysis where we identified possible resolution strategies and then a posteriori analysis. This is an approach generally used in didactic of mathematics (Artigue, 1990). Several strategies can be implemented to solve this problem. To prove that a quadrilateral is a parallelogram, students can mobilize one of the following definitions:

1. A parallelogram is a quadrilateral which two pairs of opposites sides are equal in length.
2. A parallelogram is a quadrilateral which two pairs of opposites sides supports are parallels. In the Cameroonian context, we are not talking about parallel segments but rather parallel lines. However, since the sides of a parallelogram are segments, we say that the straight lines containing pairs of opposite sides are parallel (or the supports of opposite sides).
3. A parallelogram is a quadrilateral which one pair of opposite sides has equal length and parallel support.
4. A parallelogram is a quadrilateral which diagonals intersect in their middle.

Our analyses focused on external representation of students' comprehension of the relationship between drawing and figure when they construct argumentation and when they produce proof. To support the presentation of the analysis, we use a two-column table with the transcription of a student's utterances in the left column and the decomposition of the argumentative steps with our comments in the right column. For our analyses, we labelled the text segments obtained after the transcription of the students' discussions to create categories and then reduced these categories to make them more precise. For this article we have selected four categories: drawing students personnel concept definition in argumentation; drawing and student conception in argumentation; symbolic representation of the figure in the students' proofs; cognitive continuity/gaps between argumentation and proof. We analyse the components of the arguments used by the students; we also analyse the attributes of the figure contained in the students' definition.

Results

Drawing and Students Personnel Concept Definition in Argumentation

Finding shows that the first activity of the students to carry out this proof task was to formulate their definition of the parallelogram. The definitions formulated are discussed in order to reach a consensual definition. The definitions proposed by two pairs of students may not be the definitions accepted as formal definitions of the figure. The attributes contained in the students' definitions do not allow to describe the parallelogram and exclude some particular cases of the parallelogram. There were ambiguities in the students' personal concept definition. For example, Nono and Kenne describe their own understanding of the parallelogram as follows.

> Nono*:* First of all, what is it?
> Kenne: it is a quadrilateral with four sides two-by-two being equal;
> Nono: no, which has four sides with equal support;
> Kenne: which has four sides with supports that are two-by-two equal and parallel;
> Nono: this means that the side here is parallel to this, and the side here is parallel to this;

Two definitions can be identified in student discussions. The first definition which comes from Nono can be formulated as follows: "a parallelogram is a quadrilateral with four sides which any pair of sides are equal." This definition is not correct, it doesn't consider all cases of parallelograms as a parallelogram. What is described in this definition is a particular case of a parallelogram, the diamond. For Kenne "a parallelogram is a quadrilateral with four sides which pairs of sides are equal length and has parallel support" she introduces the parallel relationship. This description is not correct, it is contradictory because two consecutive sides cannot be parallel. The two previous definitions are students' personal concept definitions of the parallelogram. They do not correspond to the formal definition of the parallelogram. Indeed, the students failed to specify that they are on opposite sides. We think that students have forgotten some characteristics in the definition of the parallelogram. It can be assumed that these personal concept definitions are the economic reformulations of the formal definition. However, students try to make themselves understood by indicating on the drawing relationships mentioned in their definition.

Once the definition of the parallelogram is formulated by the pair of students, they proceed from experience on drawing to verify that the DFBE quadrilateral verifies the attributes of the parallelogram contained in their personal concept definition. After this, we observe that two pairs of students modified their concept definition. We imagine that it came up when the data from the visual inspection of the drawing did not correspond to the property evoked in their personal concept definition. Here is an extract from the discussions of Amba and Djeteji who modify their personal concept definition after experience on drawing.

Amba: no, a parallelogram is a figure that has two sides equal two by two … so DC is equal to AB and AD is equal to BC, but CB does not have the same length as DC.

Njeteji: But…

Amba: if we try to make a small figure here, we'll see that they don't have the same length.

Njeteji: but even the square is a parallelogram…

Amba: no

Njeteji: so, the square is not a parallelogram?

Amba: a parallelogram is a geometrical figure that has two parallel sides two by two;

Amba notes from experience on the drawing that the consecutive sides of the quadrilateral do not satisfy his description of the parallelogram. The students then try to agree on what a parallelogram should be, a conflict emerges over the hierarchy of parallelograms (Is the square parallelogram?). It can be seen that the drawing leads to the modification of the students' personal concept definition. Their concept image about the parallelogram does not seem to be coherent because they cannot easily make the link between the parallelogram and the square. The modification of the students' personal concept definition of the parallelogram could come from the trust they place in the drawing proposed by the teacher.

Drawing and Students' Conception in Argumentation

The students also validated the nature of the parallelogram by exerting experience and reasoning on the drawing, the information they use as data of their argument comes from their interpretation of the drawing. We observe in trees pairs of students that their concept image evoked about the parallelogram are visual model contained in their mental. The pair of students visually see that the shape of the DFBE drawing corresponds to the visual model they have in their mental image about the parallelogram. An excerpt from the discussions of Ndondi and Kenmogne which illustrates this approach is as follows.

6. Ndondi: if I draw the drawing here [complete the parallelogram DFBE], it can be a parallelogram and it can also be a diamond. 7. Kenmogne: it can be a rectangular parallelogram and a triangular parallelogram; yes, it is a parallelogram, because DF if you connect DF [D to F] and EB [E to B] now it is a parallelogram. 8. Ndondi: it is a parallelogram;	For Ndondi, the drawing obtained looks like both a parallelogram and a diamond. Argument D: the drawing DFBE look like a parallelogram; C: DFBE is a parallelogram; W: since, if a drawing of a figure looks like visual model of a parallelogram then this figure can be a parallelogram.

	The data evoked a visual perception of drawing, it is not pertinent, and the warrant is an element of student concept image of parallelograms in this case a mental image, it's not accepted in a proof.

Ndondi and Kemogne complete the drawing to obtain a DFBE quadrilateral, they then observe that the figure obtained can be both a parallelogram or a diamond or other figure that is unknown to us. For both students, the image that the drawing allows students to see is not stable. Students make a connection between the drawing of the DFBE parallelogram and the visual model of the parallelogram contained in their mental image. The information that comes from their experience and visual perception of the drawing is based only on the shape of the drawing and not on the properties represented by codes on the sides of the DFBE quadrilateral.

It can be observed that the students implement empirical control to deduce the nature of the quadrilateral DFBE. The drawing is used here as a support for reasoning. It is assumed that to proof that a quadrilateral is a parallelogram, the visual perception of the drawing should correspond to the mental image that students associate to the parallelogram in his concept image. We can see that the drawing evoked two figures in the students mine, the diamond and the parallelogram. Although a diamond is a particular parallelogram, there is no information on the drawing to specify the type of parallelogram represented. We assume that this is a superficial interpretation of the drawing, and that the hierarchical relationship between the parallelograms is not coherent in student concept image.

Finding reported that not all students were able to activate their concept image on the parallelogram during their discussion. Two pairs of students implemented the visualization of the ABCD parallelogram. They visually controlled the relationships between the sides and then observed the visual similarity with the relationships between the sides of the DFBE quadrilateral. Then, they concluded that the DFBE quadrilateral is a parallelogram. To illustrate this approach, we present an excerpt from the discussions of Ngono and Keneka who have implemented this strategy.

25. Ngono: In the parallelogram ABCD we see that (AB) is parallel to (DC) and (AD) parallel to (BC); 26 Keneka: so, we can only pose like that, they are parallel (DE) is parallel to (FB) and (EB) is parallel to (DF), so it is a parallelogram.	D: In the parallelogram ABCD, we see that (AB)∥(DC) and (AD)∥(BC), we also see that in the DFBE quadrilateral (EB)∥(DF) and (DE)∥(BF); C: DFBE is a parallelogram; W: since if in a quadrilateral, the opposite sides have parallel supports then this quadrilateral is a parallelogram. The data of this argument consist of both the information given and the information resulting from a visual inspection of the drawing. The warrant of the argument is an element of the concept image about

	parallelogram built by visualization. It is coherent.

Ngono and Keneka visually observed the parallelism of the pair opposite side supports of the ABCD parallelogram proposed by the teacher. Then they carried out a visual control to verify the parallelism between the opposite sides of the DFBE quadrilateral, which allowed them to conclude that this quadrilateral is a parallelogram. It can be assumed that the visual inspection made it possible to construct a concept image that allowed them to proof. The relationships between the sides of the DFBE quadrilateral are the result of an abusive interpretation of the drawing because no information represented on the drawing makes it possible to establish a direct relationship between the sides of this quadrilateral. We can imagine that the students have established the link between the drawing and the visual image of the parallelism contained in its mental image.

From our point of view, the students' concept image of the parallelogram seems to have remained inactive at the beginning of the problem solving. We can assume that the students' concept image and concept definition of a parallelogram can be empty because they forget all the knowledge that they learn. However, the modes of thinking used to acquire information on parallelism of sides were based on drawing, it mobilizes students' mental image associated with parallel lines. They are not coherent because no information on the drawing allows to conclude directly that pair of opposite sides of DFBE parallelogram are parallels.

Students discussions and proof, they write highlight the inconsistency of their concept image on the relation between figures. The students' speech brings out misuse of equality relationship as well as parallelism relation between figures. This can be illustrated by the excerpt from the discussion of the pair Amba and Djeteji.

> Amba: The line (DF) is equal to the line (BE) and the line (BF) is equal to the line (DE), it is possible to say that it's a parallelogram, but the line (DE) is not equal to the line (DF).

We think it is a mistake, Amba means parallel lines and not equal straight lines. This student's speech contains a contradiction, because the straight lines have no length. They use equality between two angles instead of equality between angle measurements. We assume that the students' concept image evoked about angels and straight lines are not coherent and has not reached maturity, it may be a matter of forgetfulness or negligence.

The observations we have made on the productions of two pairs of students show that they have mobilized unteach properties of parallelograms as a Warrant in their argument. We can imagine that they built them during their experience with the parallelogram. Some of these Warrants are consistent with the formal axiomatic system on the parallelogram. The following excerpt corresponds to a point where Nono and Kenne are trying to conclude that two triangles are equal in the parallelogram.

Nono: we didn't say anything about the triangles here, look, we can see that [DF] and [EB] are the heights of the two triangles;	D: ADC and ABC are triangles from the division of the ABCD parallelogram by the diagonal AC;
Kenne: And the two triangles are equal because they are divided by the diagonal;	C: ADC and ABC are equal;
Nono: the two triangles are equal because they form a parallelogram first;	W: since the diagonal in a parallelogram is the divides into two congruent triangles.
Kenne: if the diagonal passes through the middle, it means to say that it divides the quadrilateral into two parts equal;	The data for this argument comes from an interpretation of the drawing. The conclusion is correct. The warrant is consistent with the axiomatic theory of the parallelogram.
Nono: who tells you that it's the rectangular triangle ADC?	
Kenne: it is not a rectangular triangle, since, in a parallelogram the diagonal divides the quadrilateral in two equal parts means ADC and ABC are equal	

Nono and Kenne used as a warrant of argument, the following property of the parallelogram: the diagonal in a parallelogram divides it into two equal triangles. These triangles are those calls congruent triangles. This property is not part of the theory taught on a parallelogram, either in textbooks or in syllabus recommendations. However, it is a student's concept image evoked, it is consistent with the theory of parallelograms. We can imagine that students built this element of their concept image through experience on the different drawings that they encountered as they progressed through their schooling.

Symbolic Representation of Figure in Students Proof

Analyses of the proof texts of three pairs of students show that they have difficulty in representing symbolically their concept image evoked about the figure. Confusion is observed in the use of symbols associated with the manipulated figure. The following excerpt corresponds to a point where Ndondi and Kenmogne try to prove that the DFBE quadrilateral is a parallelogram.

It can be said that the DFBE quadrilateral is a parallelogram because the DF segment is perpendicular to AC and EB is perpendicular to AC too. And another remarks the triangle CEB is parallel to the AFD triangle.	D: $DF \perp AC, EB \perp AC, CEB \parallel AFD$ C: EB is equal DF W: since, if two triangles are parallel and the … The data seem to be information given and an abusive interpretation of the drawing. The concept image used as a Warrant is not consistent.

The following excerpt corresponds is another point where Ngono and Keneka try to prove that the DFBE quadrilateral is a parallelogram.

From our point of view, the quadrilateral DFBE is a parallelogram because it is observed that on the ABCD parallelogram the straight lines $DC \parallel AB\ and\ DA \parallel BC\ d'ou\ DF \parallel EB\ et\ DE \parallel FB$ so the quadrilateral DFBE is a parallelogram	D: ABCD is a parallelogram $DC \parallel AB\ and\ DA \parallel BC\ d'ou\ DF \parallel EB\ et\ DE \parallel FB$; C: DFBE is a parallelogram; W: since a quadrilateral which opposite side has their supports parallel, then it is a parallelogram. The data of this argument are not correct, we observe that the symbols used by students to represent lines are inappropriate because they represent distance in practice. The information which constitutes data come from visual perception of drawing, we can imagine that student makes the connection of their interpretation of drawing and the visual model contains in their mental image about parallel lines.

The proof of Ndondi and Kenmogne allows to observe several concept images evoked on the figure manipulated which is not coherent. They use the relationship of perpendicularity between the segments then the relationship of parallelism between the triangles which is an error. However, the symbol they use to represent the segment is the one that is used to represent the distance between two points. In practice, the segment which ends are A and B is written [AB], a straight line which contains two points A and B is written (AB) and the distance between two points A and B is noted AB. The confusion observed in the students' proof makes the proof ambiguous. One can imagine that the student's mental image of the symbolism used to represent the figures include internal contradictions.

Cognitive Continuity/gaps Between Argumentation and Proof

All pairs of students who participated in this study wrote proof to show that the DFBE quadrilateral is a parallelogram. We observe in written or oral speech conducted by the students that the proof summarizes some of the properties develop to construct argumentation. Proof using the same arguments that had gradually emerged, in different forms during the construction of argumentation. We would like to point out that the data and warrants that make up some of the argumentation steps used by the four pairs of students to construct their argumentation do not appear in their proof.

The analysis of the proofs constructed by the students shows that the data and warrants that are the components of the arguments that the students used to produce their proofs are the same as the components of the arguments that they previously used to build their argumentation. Students proof is based on concepts images evoked about the parallelogram that has been mutually accepted by a pair of students. Pairs of students who used the concept image on the manipulated figure as a warrant for arguments in their discussion also used it in their proof. The following production is the proof written by Amba and Djeteji where they try to show that the DFBE quadrilateral is a parallelogram by using the properties that emerged in the argument.

We know that $DC = AB$ and $AD = BC$ given that. $E \in [AC]$ and $F \in [CA]$. We have $(EB) = (DF)$ then $(EB) \parallel (DF)$ and $(DE) = (BF)$ then $(DE) \parallel (BF)$ since the opposite angles are equal: the angle $\widehat{DFB} = \widehat{DEB}$ and $\widehat{FDE} = \widehat{FBE}$ so DFBE is a parallelogram	Argument 1 D: $(EB) = (DF)$ C: $(EB) \parallel (DF)$ W: since if two straight lines are equal then they are parallel. Argument 2 D: $(DE) = (BF)$ C: $(DE) \parallel (BF)$ W: since if two straight lines are equal then they are parallel. Argument 3 D: $\widehat{DFB} = \widehat{DEB}$ and $\widehat{FDE} = \widehat{FBE}$; $(EB) \parallel (DF)$ $(DE) \parallel (BF)$ C: DFBE is a parallelogram W: since if a quadrilateral has opposite sides with parallel supports and equal opposite angles then it is a parallelogram.

We can observe in the proof of these students the presence of three steps of argumentation. The data's shows traces of abusive interpretation of the drawing. The students refer to equal straight lines as well as equal angles, these are relationships that make no sense in geometry. It can be assumed that these are the lengths of the segments that represent the manipulated straight lines as well as the angle measurements of the drawing. The concept image evoked on the straight lines is not coherent, it is in contradiction with the theory of parallel lines. The concept image evoked on the parallelogram seems coherent. It is superfluous because it is a conjunction of two elements of the parallelogram theory.

When we rehearse to the students' argumentation, we realize that the data as well as the concept image evoked on the figures use as warrant which are components of arguments use in Amba and Djeteji proof are taken directly from the arguments constructed in their argumentation. We can therefore speak of a cognitive unit. However, the equality between the angles found in the student's proof is not mentioned in the student's discussions; we can talk about the cognitive gap at this level.

Discussion and Conclusion

The objective of this article was to better know how the relationship between drawing and figure influence the students' argumentation and proof. To achieve this objective, we conducted an experiment in which we invited students to solve a problem that puts into debate the truth of a proposal. The students' argumentation and proofs were analysed by articulating Toulmin's model and the concept image and concept definition developed by Vinner (1983). The analyses of the student protocols show how four pairs of students use their comprehension of the relationship between drawing and figure to construct their argumentation and to produce their proof.

To prove that a quadrilateral is a parallelogram, students mobilize their personnel concept definition that allows them to describe the parallelogram. Our finding shows that this personal concept definition does not often correspond to the formal definition of the parallelogram (Nono and Kenne). Drawing contains in problem statements help students to modify their personnel concept definition about the figure. After using instruments and a visual control of the drawing, the pairs of students realize that the visual information does not correspond to the attributes of the figure contained in their personal concept definition. They prefer to give confidence to the visual information of the drawing contained in the problem statement. These results reinforce those obtained by Souvignet (1994).

Our analysis of argument produces by our participant during the construction of their argumentation show that the data which are components of argument contains information which comes from abusive interpretation of the drawing. This is information that is not represented by the code on the drawing. This kind of data is not accepted in natural axiomatic geometry. We assume that all pairs of students who used this mode of thinking connected the visual perception of the drawing with their mental image evoked about the manipulated figure. It may come from previous classes where knowledge was acquired by applying experience and reasoning on drawing. The results of this research correspond to those obtained by Fujita (2012) which reports that learners are likely to recognize quadrilaterals primarily by prototypical drawing. This has consequence to make them do abusive interpretation of drawing which is not accepted in proof. We observe that students have difficulty in understanding the inclusion relations of quadrilaterals. The example that illustrates it is that of Amba and Djeteji for whom the link between the square and the parallelogram is not obvious (Fujita, 2008, 2012).

Difficulties have emerged in the student’s argumentation and proof. They reveal the incoherence of their concept image evoked on the manipulated figure. For example, students talked about parallel triangles, equal lines. These relationships used by students between triangles and straight lines are in contradiction with the formal theory. The proof written by the students shows that they have difficulty to establish the connections between their conceptions about figures and the symbols contained in their mental image evoked about this figure. The symbols used by students to represent certain figures are not appropriate, for example, confusion in the representation of the straight line has been observed (Ndondi and Kenmogne). This misuse of figure symbols can make the students proof incomprehensible and inconsistent.

Analyses of students’ argumentation and proofs allow us to observe cognitive continuity as well as cognitive gaps between the argumentation and proofs produced by pairs of students. The data and warrants that are components of the arguments used by the pairs of students in the production of their proofs have been used in the arguments used in the construction of their argumentation. However, data and warrants that are components of the arguments used in some of the student’s argumentation are not present in their proofs. We also observe that data and warrants that are components of the arguments used in some of the student’s proofs are not present in their argumentation. We observed that students who had a poor comprehension of the relationship between drawing and figure and who used it to construct their argumentation also used it to produce their proof.

In conclusion, further research is needed to better understand problems of students’ acculturation to the relationship between drawing and figure so that they can use it appropriately to learning mathematics proofs. For the teaching of mathematical proof in all grades of secondary school, we recommend that teachers develop semiotic activities that will allow students to learn how to interpret a drawing within the natural geometry paradigm. This study has limitations insofar as the students' transcripts do not allow us to see the actions carried out by the students of the drawing, the hesitations. The number of participants is small and does not allow the results to be inferred with certainty. One perspective for this research work could be to repeat the experiment with a large sample. The experimental situation could be a didactic situation that aims to construct a definition or a theorem.

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