Effect of instruction on children's strategies for solving addition problems

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Abstract

The study is premised on the fact that instead of designing instruction with the hope that all the children in the class would understand, teachers would be more effective if they tailor instruction to the developmental level of students in their classes. In this study, eight first- and second-graders whose level of development had been determined in an earlier study were instructed according to their individual levels of development for eight weeks. The teaching sessions, which were video recorded, were aimed at helping each child to reflect on his/her actions and to improve on his/her strategies under the encouragement and pressure of the researcher. Individual interviews conducted after the teaching sessions were also video-recorded and analyzed. The study revealed that when instruction is tailored to a child's level of cognitive development his/her strategies for solving addition problems and the ability to solve relatively more "complex" problems would both be improved. Recommendations based on the findings for teachers and for further research have been presented.

Keywords developmental levels of children, adding schemes, sensorimotor scheme, preoperational scheme, operational scheme, arithmetic scheme

Background to the study

The idea that when primary-age children are presented with basic addition combinations (e.g. 1 + 1, 2 +1 up to 8 + 9) they tend to rely on informal strategies to obtain their sums has been pointed out close to a century ago (Brownell, 1928). In fact, since the time of Brownell available literature on children's strategies support this claim (see for instance, Brownell, 1941; Brownell and Chazal, 1935; Davydov & Andronov, 1981; Houlihan & Ginsburg, 1981; Resnick & Ford, 1981; Lindemann, Alipour & Fisher, 2011; Davis, 2012; Liutsko, Veraska, & Yakupova, 2017). These studies have shown that while some children count on their fingers, others solve from known combinations or give immediate answers, mostly incorrect ones, indicating that they are guessing; to mention just a few.

However, studies suggest that no matter the strategies used children are able to refine their strategies as they progress from primary one or first grade (Houlihan & Ginsburg, 1981; Adetula, 1990; Ginsburg 1975, 1976, and 1977b). For instance, Ginsburg (1975, 1976, and 1977b) contended that counting forms the core of children's practical arithmetic and that early solutions

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to addition problems involve counting strategies. Later, more efficient strategies evolve which are based either on more sophisticated counting techniques or on a core of known facts.

In spite of this, literature is replete with the fact that barriers exist to children learning using such informal strategies (see for instance, Thyne 1941; Beattie & Deichmann 1972). These studies have classified errors children commit on addition and subtraction problems as: basic fact (computation), incorrect operation (process) and unclassifiable (random). Analysis of the procedural errors suggests a variety of systematic errors. An example of a system error is one in which a child counts on from a given number to find a sum but instead of counting on the correct number of places beyond the given number, the child includes the given number in the count. For example, to add 5 + 3, the child counts 5, 6, 7 and responds that the answer is 7 (Carpenter & Moser, 1982). Apart from this, certain general solution strategies will allow children to solve some problems but not others. For instance, children who are limited to counting strategies involving the use of their fingers may face problems with addition task with addends bigger than can be modeled on the fingers.

These barriers, if not checked can prevent or delay the development of appropriate addition strategies and eventually cause the growing child to have negative feelings about himself or herself, the process of addition and mathematics in general.

Steps, therefore, need to be taken as early as possible in order to prevent such difficulties from befalling children as they progress along the academic ladder. The point can be made that the answer to this is for teachers to teach mathematics for understanding and for meaningful learning to occur so that children can form the right strategies without much limitations. In line with this a number of studies have linked mathematical concepts presented in class to the local experiences of the children. A good example is that work by Davis and Sullivan (2011) who relied on the familiarity of their Ghanaian participants with contexts involving the use of money to enhance their learning of number.

Studies by Brownell and Chazal (1935) and Thealer (1981) have, however, indicated that even when children have been given common instruction there is still a significant difference in the strategies they use in solving addition problems. Ginsburg (1977) has explained this variation in children's solution strategies by arguing that children assimilate school arithmetic into existing cognitive structures. Therefore, when given addition tasks children attempt to solve them according to the way they organize their experience at that level of development, which they find themselves. It is therefore necessary to consider individual strengths and weaknesses when planning instruction for any group of children no matter how "homogeneous" they may appear in age.

This study argues that instead of designing instruction with the hope that all the children in the class would understand, it is essential as Brownell (1928) recommends that:

> Teachers must keep fully informed concerning the stages of development of the pupils by means of continuous study . . . of the procedures and processes, which the pupils employ in dealing with numbers. (p. 143).

This means that it is necessary for teachers to first find out the level of development of their pupils, their counting unit types and the strategies they use in solving addition problems before designing instruction to take care of their individual differences. However, for the best possible result, such a study must be done in a naturalistic setting; that is "studying the variable being investigated where they naturally occur, not in researcher-controlled environments under research controlled conditions" (Gay, 1987 p. 209).

Consequently, the research under discussion was designed to investigate the effect of instruction, tailored on individual children's cognitive development, on their strategies for solving addition problems. The results of the study, when used, will help not only in designing appropriate instruction on addition but also aid children to overcome some of the barriers they face in using their informal strategies in the process of acquiring more sophisticated strategies. This will, in turn, sustain their interest in mathematics as they progress to higher grade levels and make them derive the full benefits of having a good knowledge of mathematics.

Conceptual framework

The work of Steffe, Fifth and Cobb (1981), Steffe, Thompson and Richards (1982), Steffe, Von Glasersfeld, Richards and Cobb (1983), and Eshun (1985) puts forward four levels in the development of children's adding schemes. The four schemes, in order of sophistication are the sensorimotor, preoperational, operational and arithmetic schemes. According to Steffe and her colleagues, a child at a higher or more sophisticated adding scheme does not lose the ability to use lower level schemes to solve different tasks. This developmental progression of the adding schemes at the four levels are characterized as follows:

Sensorimotor Scheme

According to Eshun (1985) the child with a sensorimotor adding scheme solves additive tasks by globally joining counted collections of perceptual unit items. Such a child would use physical objects like marbles or other counters, when they are available to form a pattern or collection of the two addends before joining the two collections and counting the resultant collection to find his/her sum. However, in the absence of any such marbles the child can use finger patterns as substitutes to solve the task. This was what Eshun (1985) meant when he said,

The child's interpreted task leads to the counting of the fingers until the number word for the first addend (selected by the child) is uttered, and then the child continues to utter number words until a finger pattern is recognized as representing the number for the second addend. Thus the child with the sensorimotor scheme uses counting-all models.

Preoperational Scheme

The child with the preoperational adding scheme solves additive task by joining collections of motor or verbal unit items. Eshun (1985) identified the following schemes for this developmental level. The first is the simple adding scheme involving counting collections of perceptual unit items to represent the number words or numerals and then counting all the items to find how many there are altogether. Another scheme likely to be used by a preoperational child is the simple extension scheme in which the child interprets the task as establishing a collection of perceptual, motor or verbal unit items for one addend and then continues after a pause to count only perceptual unit items beyond that addend until a collection for the second addend is established. There is also the intuitive extension scheme, similar to the simple extension scheme, likely to be used at this level. The only difference is that in this scheme the child establishes a collection of the second addend by counting motor or verbal unit items. Both involve counting from 1 but the intuitive extension scheme involves counting from 1 using motor verbal unit items and not perceptual items.

Operational Scheme

The child with an operational adding scheme solves additive tasks by using counting processes that involve counting abstract unit items to construct composite units (numerical structures). S(he) uses what is referred to as numerical extension scheme involving counting-on using either perceptual unit items, motor units, verbal units items or abstract unit items. S(he) "performs what is referred to as sequential tacit integrations for the first addend and the counting acts for the second addend when uttered beyond the first addend" (Eshun, 1985, p. 363). In this way, the last number word uttered represents a composite unit for the implied counting acts from 1 to that number word. Such a child is able to solve missing addend tasks because of their ability to use two composite units to form a third (composite) unit.

Another scheme that the child with the operational scheme is capable of using is the derived fact scheme where the child recalls number facts from memory to solve additive tasks. However, Eshun (1985) argues that the number facts that can be recalled by an operational scheme child are limited to addition fact involving numbers less than 10 or sums obtained by adding single digit number to ten (e.g. 10 + 3).

Arithmetic Scheme

The child with an arithmetical adding scheme solves addition tasks by using transformed (decomposed) composite unit to provide known and simpler additive combinations. These decomposition strategies involve either adding to a decade where the child interprets an addition task as adding a number to the larger addend to yield the next decade and then increasing the new decade by the difference between the number added and the other addend or the recalling sums using place value. In the latter scheme the addends are partitioned into tens and ones so that the corresponding place values can be first be added before adding the results.

Procedure

As earlier mentioned, the purpose of this study was to investigate the effect of instruction, tailored to individual children's cognitive development, on their strategies for solving addition problems. However, due to the fact that such an investigation could not be undertaken without first assessing the levels of cognitive development of participants, time was taken initially to assess these in the light of the framework that guided the study. This, the researcher felt, was in line with long asserted recommendation by Brownell (1928) that "teachers must keep fully informed concerning the stages of development of the pupils" (p.143).

The study therefore commenced with determining the levels of cognitive development of 30 primary one and two children who were the initial participants. To do this, each child's counting processes and problem solving behaviour was probed using interview tasks selected from those designed by Steffe et al (1981) to ascertain the most sophisticated processes and unit items they counted while solving the tasks given to them. These pre-interviews were conducted in a separate room, distinct from their classrooms, and lasted for between twenty and forty minutes each. Both direct addition and missing-addend tasks were presented with the function machine to the children and the sessions were video recorded (see Wilmot, 2008). The tapes were later analyzed to identify each child's level of cognitive development and to select the right children to use for the main study.

From these, eight children were then selected to reflect the diverse schemes in the theoretical framework so that the full range of the schemes could be re-examined in the teaching episodes of the main part of the study. Apart from selecting the eight children to reflect the diverse schemes, care was also taken to select four each from primary classes one and two and also to ensure gender balance as well.

The main study involved the conduct of clinical interviews and teaching episodes with the selected eight children. During the teaching episodes children who were identified to be using identical schemes were paired and taught together after which they were separated and interviewed individually. The researcher worked with each pair (consisting of children for each of the adding schemes) for four sessions in eight weeks and the individual interviews were conducted immediately after each teaching session. Each session (both the teaching and the individual interview that followed) lasted for between thirty and forty minutes and was video recorded. This enabled the researcher as a teacher to formulate hypotheses about each child's counting unit type, adding scheme, strategies that have been learnt and use it to plan appropriate activities and tasks for the next session.

The teaching episodes were directed toward encouraging the children to individually develop their strategies while noting the progress they made under the encouragement and pressure of the experimenter. Essentially, these episodes were planned to enable each child reflect on his/her own actions and to make possible abstractions and adaptations. In this way, each child was give direct assistance to adapt existing schemes to solve novel tasks and to possibly construct more sophisticated adding schemes.

Analyses of the behavior of participants during the teaching episodes

In this section the behavior of the different categories of the participating children during teaching episodes are presented. For easy reading, sessions with participants who were found to be operating at the same level of mathematical development as discussed in the conceptual framework have been focused on together, beginning with the least level of development. In addition, for anonymity, pseudo names have been used for the participants.

Children observed using Sensorimotor Schemes

During the first part of the study (i.e., the pre-interview session), these children were observed during the pre-interviews to be mostly using counting all strategies to solve direct addition tasks. They, therefore, faced problems when addends were more than ten. An effort was therefore made, during the teaching sessions to encourage them to, at least, count-on from one (without using perceptual units) to solve additive tasks. Two of these children, pseudo-named Joshua and Kofi, who were in primary class one and class two respectively at the time of the study, were selected for the teaching sessions. Because of their use of counting all strategies the teaching episodes with them were aimed at encouraging them to advance to the use of the counting-on strategy. The behavior of two of these children, during the teaching sessions is presented in this section.

The first task of the first teaching session was used to confirm the hypothesis that Joshua and Kofi could not use any sophisticated strategy than counting-all. The protocol below illustrates their behaviour on adding numbers with sums less than 10.

J: P1

R: (With ten bottle tops visible to Joshua) Joshua, what is "5 plus 3"?

J: (Gathers five bottle tops one after another while uttering in synchrony) 1, 2, 3, 4, 5 (pauses and then continues gathering another collection while uttering in synchrony) 1, 2, 3. (He then puts the two collections together and touches each bottle top in turn while uttering) 1, 2, 3 . . . , 8. Eight.

Kofi also behaved in the same manner to solve "5 + 4", making two collections with the bottle tops before joining them together to count. The children used the same strategy of counting-all to solve problems with sums greater than 10 using perceptual inputs. The protocol below shows how Kofi solved "7+8".

K: P1

R: Kofi can find "7 + 8"?

K: (Groups 7 bottle tops while uttering in synchrony) 1, 2, 3 . . . , 7 (then continues grouping another set while uttering in synchrony) 1, 2, 3 (pauses and then continues) 4, 5, 6, 7, 8 (while clenching five fingers). (He then puts the two collections together and counted the items by pointing and uttering) 1, 2, 3 . . . , 10 (Next, he counted the fingers by extending the clenched fingers in turn while uttering) 11, 12, 13, 14, 15. It is fifteen.

Kofi realized he needed more items than the bottle tops. Since his aim was to count objects to find the sum he resorted to his fingers. He used the five fingers as substitutes for the bottle tops and to create the collection for the second addend, 8. It is believed that because he was limited to the use of the counting-all strategy he was compelled to use the fingers as perceptual substitutes. This behaviour confirms the assertion by Steffe et al. 1983 that, "a counter of perceptual unit items can use counting to solve problems. If no marbles are presented the child may still solve the problem by using a perceptual substitute; for example, fingers may be extended simultaneously and counted" (p.47).

The shortage of the bottle tops in the course of Kofi's counting can be likened to the non-availability of marbles discussed by Steffe et al. (1983) and hence his use of the fingers as substitutes. However, in the course of the teaching episodes Kofi and Joshua were able to use more sophisticated strategies. For instance, during the interview that followed the first session of the teaching episode, both of them were able to utter number words as countable verbal unit items for the first addend (starting from one) and use some or all of the bottle tops available to count-on the second addend. Below is a protocol of how this was done by Joshua when he was asked to find the sum "7" and "5".

J: P2

R: Joshua, can you solve "7 plus 5"?

J: (Gathers five bottle-tops at one place and utters number words only starting from one without putting up fingers) 1, 2, 3, 4, 5, 6, 7, (pauses for about 5 seconds and then continues uttering number words) 8, 9, 10, 11, 12 (while sequentially touching each of the already assembled five bottle-tops). Twelve.

Joshua was able to use the above strategy during the second session of the teaching episodes, two weeks later, to add 5 and 3. He uttered number words for 5 starting one and counted on 3 bottle tops to complete the task.

When it was the turn of Kofi to be interviewed after the first teaching session he also used the same strategy for uttering number words as countable items to find the sum 7 and 5. He was able to adapt this strategy to solve tasks in which both addends were 10 or more. This was a significant behaviour because he had only been exposed to solving task where both addends were less than 10 in the teaching episodes. His behaviour is illustrated in the following protocol.

K: P 2

R: Kofi, can you do "24 plus 10"?

K: (Extends sequentially the ten fingers while uttering) 1, 2, 3 . . . , 10 (Brings the two hands together, pauses for about 5 seconds and then utters number words only to establish 24 starting from 1) 1, 2, 3 . . . , 24 (He then continues to fold his fingers while uttering in synchrony) 25, 25, 27 . . . , 34.

Thus Kofi, after modeling the second addend using his fingers apparently to facilitate his counting, began by uttering number words to establish the first addend (starting from one) before counting-on the second addend. He used the same strategy to solve the task, 13 plus 12. In this case too he uttered number words up to 13 starting from 1 without extending fingers before counting on the second addend, 12, using his ten fingers and two of the bottle tops that were available.

However, his behaviour later in the course of the teaching session indicated that he could not effectively perform the addition in the absence of extra bottle tops when the addends were each greater than 10 using his fingers independently and in the absence of counters. This limitation was observed during the interview session after the fourth teaching session when Kofi was given "45 + 16" to solve. His behaviour on this task is captured in the following protocol.

K: P 3

R: Now, Kofi, solve "45 plus 16"

K: (utters only the number word 45) forty-five. (He then continues extending fingers while uttering simultaneously) 46, 47, 48 . . . , 55 (stops).

It was Kofi's inability to model 16 in the absence of extra perceptual items that made him stop after counting-on his ten fingers. He could not reuse his fingers because he might have thought that this was not proper to do.

Kofi had earlier during the third teaching session, upon the prompting of the researcher, been able to use the counting-on strategy involving uttering only the number word for the first addend, without starting from 1. This is illustrated in the protocol below.

K: P 4

R: Kofi what is 17 plus 9?

K: (folds 9 of his fingers and utters). Seventeen. (He continues extending his nine folded fingers while uttering in synchrony) 18, 19, 20 . . . 26. It is twenty-six.

Thus Kofi was able during the teaching session to progress from the counting-all strategy he was earlier using. He could now use the more sophisticated counting-on strategy. It is worth noting that he continued to use the counting-on strategy discussed in the protocol above during the interview session that followed the fourth teaching session two weeks after he had been introduced to it. Kofi had therefore come to realized that uttering any number word say, forty-five, could be taken as representing the activity of counting 45 objects or uttering simply number words from 1 to 45.

Joshua was also to progress from the counting-all strategy to a more sophisticated counting-on strategy. He, like Kofi, had also come to realize that the number word for one of the addends could be taken as representing the activity of counting a number of objects equal to the numerosity of that addend and count on the other addend using his fingers. However, unlike Kofi who could not solve direct addition tasks with addends greater than 10 in the absence of perceptual items, Joshua was able to do so. The latter was able to model numbers greater than 10, in the absence of perceptual items, and use them to solve direct addition tasks involving them. He exhibited this behaviour during the fourth session of the teaching episodes. The following protocol illustrates this behaviour.

J: P 3

R: Joshua, can you solve 14 plus 12?

J: Yes (folds his 10 fingers while uttering) 1, 2, 3 . . . , 10 (He continues pointing to his wrist and elbow while uttering) 11, 12, (He then utters) 14 (pauses for about 5 seconds and continues to utter number words while extending his fingers simultaneously) 15, 16, 17. . . , 24 (points to the wrist and elbow in turn while uttering) 24, 26, the answer is twenty six.

Thus, though Joshua had not been taught how to use other parts of the body besides the fingers to model numbers greater than 10, he developed his ability to use it to solve not only task discussed above but also 15 + 11 and 12 + 13. This and other counting-on strategies that he could not use before the teaching episodes were now freely used by Joshua.

Children observed using Pre-operational Schemes

The children at the pre-interviews were observed to be counting on mostly with objects or their fingers to do direct addition. This behaviour was confirmed during the first session of the teaching episodes as illustrated in the protocol that follows involving Felix (a class two child) and Edna (who was in class one).

F: P1

R: Felix, what is "5 plus 8"?

F: (utters only the number word 5). Five (pauses for about 5 seconds and then continues) 6, 7, 8, 9, 10, 11, 12, 13, while extending eight fingers, initially clenched, in synchrony…). It is thirteen.

Thus, Felix had to first model the second addend before using the finger pattern, to count-on from the first addend. He only counted on with perceptual substitutes, which in this case are the fingers. Edna also used the same strategy to solve tasks with sums more than ten and whose addends are both less than ten. They were both limited in their counting by the ten fingers, in the absence of any other objects, and could therefore not solve tasks in which both addends were more than ten. This is illustrated in the protocol below in which Edna is asked to find the sum of thirteen and fourteen.

E: P1

R: Edna, can you do "13 plus 14"?

E: Yes (pauses for about 5 seconds and utter) 13 (and then continues while extending her fingers simultaneously) 14, 15, 16. . . , 21, 22, 23 (stops after exhausting the ten fingers). Twenty-three.

Felix also behaved in a similar way when asked to add 13 to 12. He uttered a number word for 13 and then counted on 10 instead of 12 using the fingers to get 23.

Both children ended their counting abruptly after exhausting their ten fingers. The teaching episodes were, thus, directed at encouraging them to count on with words instead of their fingers, and to assist them solve missing addend tasks. It took the persistent prompting and encouragement from the researcher for both Edna and Felix to realize that the fourteen is "ten and four" while thirteen is also "ten and three" and thus after exhausting their 10 fingers they could count on four and three more respectively. Also, the children were helped to continue the counting on their toes or recount their fingers.

Edna was able, during the interview session after the first teaching session, to count on using verbal items (that is, number words as unit) to solve direct addition tasks. She had not been taught how this was done. The following protocol illustrates this.

<u>*E: P2*</u>

R: Edna, what is "33 + 10"?

E: (pauses for about ten seconds and then utters) 34 is one, 35 is two, 36 is three . . . , 42 is nine, 43 is ten. The answer is forty-three.

Edna's behaviour above was very significant. She had suddenly realized that she count her number word uttering as countable items. This was the first time she exhibited this behaviour. It was observed that Edna used this double counting only on addends that were both ten and more. She used the counting-all or counting-on from one strategy for smaller addends, e.g. 5 + 3 (see protocol E: P3).

It was during the second session of the teaching episodes that these preoperational strategy children were taught to count on using verbal unit items or their number word utterances to solve direct addition tasks. To do this they were encouraged to utter the number word for the first addend and then count on the second addend using number words as countable items as portrayed by Edna in her protocol, E: P2, above. The prompting from the researcher only reinforced the use of the strategy by Edna. But Felix was learning for the first time to use such strategy. The protocol below shows how Felix used this strategy.

<u>*F: P2*</u>

R: Felix, can you do "8 plus 7"

F: Yes, (pauses for about 5 seconds and then continues) 8, 9, is one, 10 is two, 11 is three . . . , 15 is seven. It is fifteen

Both children were able to use this strategy of double counting to solve direct addition tasks during the second session of the teaching episodes through to the fourth session. This does not mean that they used only this strategy to solve direct addition problems in these sessions. For instance, during the third session Edna used the counting-all strategy to solve tasks with sums not greater than ten. This is illustrated in the protocol below:

<u>*E: P3*</u>

R: Edna, what is "5 plus 3"?

E: (utters number words in synchrony with clenching the fingers of her left hand), 1, 2, 3, (pauses for about 5 seconds and continues uttering) 1, 2, 3, 4, 5, (while clenching fingers on her right hand, brings the clenched fingers together and extends them one after the other while uttering them one after the other while uttering in synchrony) 1, 2, 3 . . . , 8. The answer is 8.

Also, both Edna and Felix were able to solve some missing addend tasks presented with the function machine during the interview sessions that followed each of the third and fourth sessions of the teaching episodes. They did so by counting on their fingers from the known addend. The protocol below illustrates how Edna used this strategy.

E: P 4

R: (shows a card to Edna). What is on this card?

E: Six.

R: Put it inside the first "IN" hole.

E: (Edna complies and is asked to close her eyes while being guided to put in a second card).

R: Now, Edna, open your eyes and draw out the output card (Edna complies). What is on it?

E: Eleven.

R: So what number was on the card you put in when your eyes were closed.

E: (pauses for about 10 seconds and utters) 6 (pauses and then continues while extending her fingers in synchrony) 7, 8, 9, 10, 11. The number on it was "5".

After uttering "11", Edna noticed that she had extended five fingers so the number she did not see must be five. This is the case for double counting to be used, i.e. 7 is one, 8 is two, 9 is three, 10 is four, 11 is five. But neither child used this strategy on the missing addend tasks. Both relied on their fingers to find out the missing addend. As a result they were unable to solve for missing addend greater than 10.

Children observed using Operational Schemes

Francisca and Antonia, both in class two at the time of the study, were found to be solving additive tasks by counting-on from the larger addend using their fingers during the pre-interviews. The first task during the teaching episodes was therefore aimed at encouraging them to count-on without using their fingers. They, as a result, solved the task by counting-on using verbal unit items. Below is a protocol illustrating how Francisca did it.

Fr: P1

R: Francisca, try to solve "11 plus 13" without relying on your fingers.

Fr: (pauses for about 10 seconds and then utters) 11, 12 is one, 13 is two . . . , 24 is thirteen. It is twenty-four.

Both children were able to use this strategy of double counting or of using their number word utterances as countable items to solve direct addition tasks.

Later during the same session above, Antonia was prompted to be able to count in the head without using finger patterns to solve direct addition tasks. The following protocol illustrates her behaviour.

A: P1

R: Antonia, can you solve "45 plus 16"? Don't use your fingers. Do it mentally.

A: (pauses for about 30 seconds and then utters) Sixty one.

R: Very good! How did you get the answer?

A: I counted in my head.

R: Explain your process to me.

A: (utters) 45, 46 is one, 47 is two, 48 is three . . . , 61 is sixteen. So it is sixty-one

In the protocols above Francisca and Antonia were able to create similar units and use it in counting to solve their direct addition tasks. The only difference is that Antonia did hers in the head while Francisca did hers audibly. They started by first creating composite units for their respective first addends, 11 and 45. They then performed double counting and thus used verbal units (that is number words as unit items) to solve direct addition tasks.

It is worth noting that both children were able to use the double counting strategy to perform missing addend tasks as well. The protocol below illustrates how Antonia went about her solution on a task presented to her using the function machine.

A: P2

R: (Gives a card to Antonia) what number is on this card?

A: Seven.

R: Put it in the first "in-hole" (Antonia complies and is asked to close her eyes as she is helped to put in a second card. She complies and the second card is put in). Now draw the output card. (Antonia complies). What is on it?

A: Fifteen.

R: What number do you think was on the second card you put in while your eyes were closed?

A: (pauses for about 5 seconds and utters) 7, 8 is one, 9 is two . . . , 15 is eight. It was eight.

Fr: P2

R: Francisca, what must be added to 16 to give 28?

Fr: (pauses for about 5 seconds and then utters) 16, 17 is one, 18 is two . . . , 28 is twelve. The answer is twelve.

Thus both Antonia and Francisca knew that in order to solve missing addend tasks like "7 + [ ] = 15" and "16 + [ ] = 28" they had to create verbal units from "8 to 15" and "17 to 28" respectively. This is in contrast with the behaviour of Edna and Felix (see Edna's protocol E: P10) who only used a counting-on strategy using their fingers to solve missing addend tasks. Hence whereas Edna and Felix could not solve the task when the missing addend was greater than ten, Antonia and Francisca who did not rely on their finger patterns but on the number words they created could solve them.

Also several tasks were used to confirm the hypothesis that Antonia and Francisca could be directed to use derived fact strategies to solve additive tasks. Below is how Francisca used the strategy.

Fr: P3

R: Francisca, can you do "8 plus 7" using a different strategy?

Fr: Yes (pauses for about four seconds utters) Fifteen.

R: Good. How did you do it?

Fr: "7 plus 7" is 14 so "8 plus 7" is one more, Fifteen.

Another strategy Francisca and Antonia used, as a result of prompting from the researcher, was the strategy of adding to base. Below is how Antonia used that strategy to solve direct addition tasks.

A: P3

R: Antonia, what is "8 plus 6"?

A: (pauses for about five seconds and utters) Fourteen.

R: How did you figure that?

A: I broke the 8 into 4 and 4 and added one of the fours to the six to get 10, and 4 more is fourteen.

Thus whereas Francisca used the fact that because "7 + 7 = 14", "8 + 7" should be one more than fourteen, Antonia used the strategy of counting or adding to the base while at the same time demonstrating knowledge of additions to ten (Her statement "and 4 more is fourteen" suggested that she knew what to get when 10 is added to 4). Antonia used the same strategy to solve tasks like 5 + 7 and 7 + 6. Francisca too later solved 5 + 7 is the same way by breaking 5 into 2 and adding the 3 to the 7 to get 10 and then the 2 to finally get 12. Their behaviour later showed that they knew how to add multiplies of ten.

With encouragement of the researcher these children could later use the method of decomposition to add any two-digit numbers. This is illustrated in the following protocol of Antonia.

A: P4

R: Antonia, can you do "25 plus 24"?

A: (pauses for about 10 seconds) Forty-nine.

R: Good but how did you get it?

A: I broke the 25 into 20 and 5 and 24 into 20 and 4. Then I added two twenties to get 40, the 5 to the 4 to get 9 before adding the 40 to the 9 to get 49.

To be sure that Antonia was not just quoting the "25 + 24" as an addition fact (since the two addends were both in the twenties and differed by just one) but was actually doing the decomposition she described several other tasks were given her. The results confirmed, after two weeks, that Antonia could actually use the method of decomposition very well. An example of her behaviour is captured in the protocol below.

A: P5

R: What is "55 plus 39", Antonia"?

A: (pauses for about 10 seconds) Ninety-four.

R: Can you demonstrate how you did it to me?

A: Yes. I broke the 55 into 50 and 5 and the 39 into 30 and 9. I then added the 50 and 30 to get 80 and 5 to 9 to get 14. This 14 I broke into 10 and 4, added the 10 to the 80 to get 90 and 4 to finally get 94.

Francisca also used the same method to solve tasks like "26 + 5, 53 +28, 35 + 27, 47 + 23, and 45 + 19". In each case, she like Antonia added the tens together separately from the units before adding the results of the two (breaking the sum of the units down into ten and the left over first).

It is however worthy to note that in spite of the refinement through which the addition strategies of these children had gone Francisca was, during the interview session after the last teaching episode, counting with her fingers to solve missing addend tasks presented with the function machine. This is illustrated in the protocol below.

Fr: P4

R: (Gives a card with a number on it to Francisca) Francisca what is on that card?

Fr: Thirteen.

R: Put it inside the machine (Francisca complies and is guided to put in a second card while her eyes were closed). Now get us the output card (she complies). What is on it?

Fr: Twenty-five.

R: What do you think was on the second card?

Fr: (pauses for about 5 seconds and utters) 12 (pauses and then continues) 13, 14, 15 . . . , 25 (while extending her fingers in synchrony) Twelve.

Thus even though Francisca had earlier demonstrated the ability to use more sophisticated strategies like the counting-on with words, decomposition and derive fact strategies she still decided to count but extend her fingers to keep track of what was counted (a less sophisticated strategy). This confirms the assertion of Steffe et al (1983) that "Children will continue to use primitive counting types and less sophisticated strategies even when more sophisticated ones are available" (p. 70).

Children observed using Arithmetic Schemes

As a result of their behaviour during the pre-interviews it was hypothesized that Timmy (a class one child) and Maame (a class two child) knew some number facts. This was confirmed during the first session of the teaching episodes as they called out sums of doubles from "1 + 1" to "9 + 9" and those addends that differ by one with ease and insisted they knew the latter from the former. This is illustrated in the protocol below.

T: P1

R: Timmy, what is "9 plus 8"?

R: How did you figure it out?

T: I know that "8 plus 8 is 16" so I just added one to the sixteen to get seventeen.

During the second session Timmy and Maame also demonstrated knowledge of sums to the ten (as in 9 + 1, 8 + 2, 7 + 3, 6 + 4 . . . , 1 + 9) as they called out these with ease and at times derived some from the others. The following protocol illustrates how Timmy derived what "3 plus 7" was.

T: P2

R: Now Timmy, can you do "3 plus 7" for me?

T: (Pauses for about 3 seconds and utters) Ten.

R: How did you get it?

T: I know that "4 plus 6" is ten so "3 plus 7" will also be ten.

Maame also used a similar strategy to figure out what "8 plus 2" was. The protocol below shows this.

M: P1

R: Maame, what is "8 plus 2"?

M: (pauses for about 2 seconds and then utters) Ten.

R: Tell me how you figured that out.

M: I know that "9 + 1" is ten so "8 +2" is also ten.

Thus according to their explanations Timmy and Maame could apart from using number facts to derive novel sums, as in the first protocol in this section (see T: P7) also figure equivalent sums as in the last two protocols (T: P8 & M: P5) above.

The first attempts at finding out whether these children could use decomposition methods effectively on double digit additions still showed some weaknesses on this strategy. The protocol below shows an instance of these weaknesses.

<u>*M: P2*</u>

R: Maame, now try to do "19 plus 28"?

M: (pauses for about 10 seconds and utters) Thirty-seven.

R: How did you get your answer?

M: I know that "9 plus 8" is "17" so I put only seven down and added the "1" to "2" to get "3". So the answer is "37".

Maame was only trying to add the units column separately and the tens column separately. But her mistake stemmed from the fact that she forgot to add the ten that was left, after "putting down only the seven" in the seventeen (to use child's own expression), to the tens column.

As for Timmy he just failed to keep track of what he received after adding the units column first. Below is an illustration of such a difficulty.

<u>*T: P3*</u>

R: Can you do "55 plus 39", Timmy?

T: (pauses for about 10 seconds and then utters) Ninety-one.

R: How did you get your answer?

T: I know that "50 plus 30" is 80 and "9 plus 5" is 14 so I added 10 to 80 to get 90 and 1 more giving 91.

R: Timmy but you said you had 80 and 14.

T: (pauses for about 5 seconds) sorry then the "80 plus 14" is 94. So the answer is ninety-four.

Thus after adding the units column and the tens column correctly Timmy forgot that he had "14" instead of "11" and rather added the later to the "80" received by adding the tens column. However, upon the intervention of the researcher he was finally able to do it correctly.

The behaviour of these children during the second session made the researcher to hypothesize that they knew sums of multiples of ten. The first part of the third session was therefore used to test his hypothesis. They were therefore given several tasks involving multiples of "10" and their behaviour confirmed the hypothesis as the protocol below illustrates.

<u>*M: P3*</u>

R: What is "50 plus 80", Maame?

M: (pauses for about 5 seconds and utters) one hundred and thirty.

R: How did you figure that?

M: 5 plus 8 is "13" so 50 plus 80 will be "130".

Timmy also used the same strategy to figure other sums "30 plus 50", explaining that because "3 plus 5" is "8", "30 plus 50" is "80".

Now, it was realized as in the last but one and last but two protocols above that in his use of the decomposition strategies the main mistakes Timmy and Maame respectively made was when they came to add the sum of the tens column to that of the units column. They could not remember exactly what was obtained at the units column and thus failed to complete the process correctly. To help them overcome this short-coming they were first taken through additions in which one of the addends was a multiple of ten. This is illustrated in the protocol below.

M: P4

R: Maame, what is "48 plus 70"

M: (pauses for about 20 seconds and utters) one hundred and eighteen.

R: Can you explain how you got it?

M: "7 plus 4" is "11" so "70 plus 40" is "110" and 8 more give 118.

It was only after performing a number of additions of the kind above, in which one addend is a multiple of ten, that they were introduced to additions with both addends being non-multiples of ten. By the end of the third session of the teaching episodes Timmy and Maame had started and were actually applying the decomposition strategy on novel two digit additions correctly. Below is an illustration of how Timmy went about figuring one of such tasks.

T: P4

R: Can you do "45 + 16", Timmy?

T: (pauses for about 10 seconds) It is sixty-one.

R: Good, how did you do it.

T: I added 40 to 10 to get 50 and 5 to 6 to get 11. I then took ten from the eleven to add to the fifty to get sixty before adding remaining one to get sixty-one.

When it was Maame's turn to answer the same question during the last session she used the same strategy. Below is a protocol illustrating her behaviour on the same task.

M: P5

R: Now Maame try "45 plus 16"

M: (Pauses for about 10 seconds and utters) sixty-one.

R: That is correct. But how did you get it?

M: I know that 40 plus 10 is 50 and 5 plus 6 is eleven. I then added ten from the eleven to the fifty to get sixty and one more is sixty-one.

Thus both pupils used the same strategy on this and other tasks given to them. It is important to note that during the pre-interview these children could not solve the direct addition task "45 plus 16" illustrated in the last two protocols above. However, these children did not always use such sophisticated strategies. At certain times, depending on the nature of the task given they used other less sophisticated counting strategies to do missing addend tasks presented using the function machine. Below is a protocol illustrating this.

M: P6

R: (Gives a card to Maame) Maame what is on this card?

M: Twelve

R: Put it in the "in-hole" (Maame complies and is asked to close her eyes as she is guided to put in a second card, which she complies also). Now Maame remove the output card (Maame brings this card out of the "out-hole") What is on it?

M: Twenty-five

R: What do you think was on the second input card placed in while your eyes were closed.

M: (utters) 12 (pauses and then starts extending her fingers while uttering in synchrony) 13, 14, 15 . . . , 25. It was thirteen.

Timmy also counted on his fingers to do the missing addend tasks "[ ] + 11 = 25" and [ ] + 15 = 31" presented using the function machine. Timmy's use of this less sophisticated strategy confirms the claim by Bobis (2007) that , "When a child is placed under some form of cognitive demand, such as an imposed time limit, mental fatigue or even boredom, they will often revert to a less sophisticated strategy that they know well and can perform with minimal effort" (p. 24). This statement is relevant because when a relatively more complex task such as "33 + [ ] = 44" was presented to Timmy, still with the machine, Timmy decided to shift away from the counting strategy to using a derived fact strategy to get his answer because he found it to be more efficient. The protocol below illustrates this.

T: P5

R: (Gives a card to Timmy) Timmy what is on it?

T: Thirty.

R: Put it in the machine (Timmy complies and is aided to put in a second card while his eyes were closed) Draw the output card (Timmy complies) What is on it?

T: Forty-four

R: So what do you think was on the second card?

T: (pauses for about 10 seconds and utters) Fourteen.

R: How did you figure that?

T: I know that 30 plus 10 is 40 so I just added 4 to it.

Conclusions and recommendations

First, this study has confirmed the fact that the child's adding scheme was restricted to the four developmental level; sensorimotor, preoperational, operational or arithmetic level and that these schemes are in developmental order from the least advanced sensorimotor through to the preoperational and then to the operational and finally to the most advanced arithmetic scheme. Thus, the developmental levels and their progression in the theoretical framework are confirmed in this study. This finding is important because the theory put forward by Steffe et al (1981) was a result of working with white children in the US while the work by Eshun (1985) involved black children in the US. However, the present study has been conducted in Ghana, an African country using Ghanaian children. Knowledge of these developmental levels of scheme is recommended to classroom teachers since it has the potential of helping teachers know how to influence and encourage primary aged children to construct more advanced adding schemes and develop problem solving skills.

Also confirmed in this study is the fact that a child at a higher scheme period does not lose the ability to use less advanced ones. For instance in this study, Maame, an arithmetic scheme child was observed to be capable of using relatively less advanced operational scheme to solve some of the additive tasks presented to her (see protocol M: P 6)

One striking observation in the study was that children from the same class and being taught by the same teacher had developed different schemes. For instance, Timmy, Felix and Joshua from the same class one classroom had developed arithmetical and preoperational sensorimotor schemes respectively while Maame and Francisca from the same class two classroom had developed arithmetical and operational schemes respectively These different developmental levels at which children from the same classroom were "located" confirm the fact children's adding schemes were their own spontaneous constructions in the context of learning mathematics under the care of their teachers (cf. Greon & Resnick, 1977). This finding has serious implications to the teacher. That is no matter how homogeneous children are in terms of age or grade level they can be operating at different developmental levels. The teacher should therefore determine the level of development of each child in order to design instruction to take care of such individual differences. In this way each child will be able to make the necessary adaptations in order to progress and construct more advanced schemes.

Most importantly, this study has shown that when instruction is tailored to the child's level of cognitive development both his/her strategies for solving addition problems and the ability to solve relatively more complex problems will be improved. This is evident from the fact that each pair of children of the same developmental level when given instruction for only four sessions of 30 minutes each they showed improvement in their additive problem solving skills. This has serious implications for the teacher. That instead of giving common instruction to a class with the hope that all children would understand, it is necessary for teachers to design instruction to take care of the children's individual differences in cognitive develop and help them refine their strategies along their levels of development. Instructional activities or path followed in this study to this effect are therefore recommended for classroom teachers and for researchers who would want to pursue such lines of research.

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