Analyzing inflation in Nigeria: a fractionally integrated ARFIMA-GARCH modelling Approach

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Abstract
The study looked into the stochastic properties of CPI-inflation rate for Nigeria from 1995Q1 to 2016Q4. The study employed an autoregressive fractionally integrated moving average and a general autoregressive conditional heteroskedasticity (ARFIMA-GARCH) methodology as well as ADF/KPSS to investigate the long-memory properties of CPI-Inflation for Nigeria. The study found that CPI-inflation in Nigeria is shock dissipating at a geometric rate (fast mean reverting ability). The ARFIMA-GARCH process showed that CPI inflation in Nigeria is a heteroskedastic fractionally integrated process with quick mean reverting ability. The study therefore concludes that shocks to CPI-inflation in Nigeria such as sudden hikes in prices of energy products will not cause a permanent change in general price level but will eventually return to its mean state, and therefore having an implication for the Inflation-Unemployment tradeoff of the Philips curve.

Keywords: Inflation, AFIMA, GARCH, Fractional Integrated and Long Memory, ADF and KPSS

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1. Introduction

Long before the developing economies of Africa became organized economies with organized economic institutions, the developed economies of Europe and North America with developed institutions had at one time or another grappled with the issue of inflation. Haberler (1960) captured the feelings about inflation at the time while writing for the Federal Reserve Bank of the United States of America remarked that "it is commonplaceto say that inflation poses one of the most serious economic problems of our time. Many people are outraged by the social injustices which it implies. Many are alarmed by the realization that, although temporarily stimulating, it causes instability and thus reduces efficiency and retards the growth of an economy in the long run." Since the pioneering works of Granger (1980) and Granger and Joyeux (1980), attention on inflation has shifted away from attempts at conceptualization, to a new preoccupation of trying to understand its econometric properties. Notwithstanding though, existing literature is replete with efforts at conceptualization of inflation, which should be the building block for further analysis of the concept. First, to get a clear picture of the concept of inflation will require one to look at it measures. The literature has provided two measures of inflation; consumer price index (CPI) and personal consumption index [PCI]. A side by side comparison of the two methods show some slight merits of the personal consumption index over the Consumer price index. However, Economists are of the view that in the long-run, the two methods converge, yielding identical indices. The Consumer Price Index enjoys popular usage among economists than the Personal Consumption Index. Lebow and Rudd (2006) conceptualized CPI as a measure of the prices of goods and services consumed by the households. One thing however is common among the two measures identified; that is both methods take into account household spending (cost of living) in constructing measures of inflation.

Inflation is defined as a persistent rise in the general level of prices of goods and services in an economy over a period of time. "The rate of inflation – the percentage change in the overall level of prices – varies greatly over time and across countries" Mankiw (2010). When the general price level in an economy such as Nigeria rises, each unit of currency will buy fewer goods and services than the pre-inflation period, eroding the purchasing power of money in the economy. Inflation is measured by inflation rates, the annualized percentage change in the general price index (usually the Consumer Price Index) over time. The effects of inflation on an economy are numerous and can be positive or negative, but mainly negative.

Sokpo, Iorember and Usar (2017) discussed four expected negative effects of inflation outlined by Mankiw (2010) which includes; first, a distortion of the inflation tax on the amount of money people hold. A higher inflation rate leads to an increase in nominal interest rate, which in turn leads to a fall in real money balances. If people were to hold lower money balances on average, they must make more frequent trips to the bank to withdraw money, leading to a form of inconvenience to the money holder. The inconvenience of reducing money holding in the words of Mankiw (2010) is metaphorically called the shoeleather cost of inflation, because walking to the bank more often causes one's shoes to wear out more quickly.

Another cost of inflation arises because high inflation induces firms to indulge in cost incurring behaving of changing their posted prices more often. Changing prices is sometimes costly. These costs are called menu costs, because the higher the rate of inflation, the more often restaurants have to print new menus.

A third cost of inflation arises because firms facing menu costs change prices infrequently; therefore, the higher the rate of inflation, the greater the variability in relative prices. For example, suppose a firm issues a new catalog every January. If there is no inflation, then the firm's prices relative to the overall price level are constant over the year. Yet if inflation is 1 percent per month, then from the beginning to the end of the year the firm's relative prices fall by 12 percent. Sales from this catalog will tend to be low early in the year (when its prices are relatively high) and high later in the year (when its prices are relatively low). Hence, when inflation induces variability in relative prices, it leads to microeconomic inefficiencies in the allocation of resources.

A fourth cost of inflation results from the tax laws. Many provisions of the tax code do not take into account the effects of inflation. Inflation can alter individuals' tax liability, often in ways that lawmakers did not intend. Positive effects of inflation includes central banks' adjustment of nominal interest rates (carried out to reduce adverse effects of recessions), and encouragement of investment in non-monetary capital projects like real estates and other social infrastructure.

Beyond conceptualization of inflation, economists have devoted some considerable resources on the investigation of the econometric (stochastic) properties, that is, long-memory/persistence of inflation rate employing different econometric tools of analysis to investigate long-memory properties of inflation. The foundation for this form of investigation lies in the works of Granger (1980), Granger and Joyeux (1980) and Hosking (1981) and popularized in the works of Lai (1997) and Tsay (2000) who both investigated the long memory properties of interest rate using a fractional integration approach.

Previous analysis of inflation in Nigeria as a macroeconomic variable have focused at its impact on economic growth – Chimobi (2010), Osuala, Osuala and Onyeika (2013) and Bakare, Kareem and Onyelekan (2015); some studies have dwelled on its short-term forecasting – Ekpenyong and Udoudo (2016), Doguwa and Alade (2013), and Okafor and Shaibu (2013), while some other studies such as Sokpo, Iorember and Usar (2017), Akani and Uzibor (2015), and Taofic and Omosola (2013) looked at its effect on stock market returns in Nigeria. This paper however seeks to investigate the stochastic properties of inflation rate using an autoregressive fractionally integrated moving average (ARFIMA)-Generalized Autoregressive Conditional Heteroskedasticity (GARCH) modelling approach on monthly Nigerian Consumer Price Index (CPI) data from 1996 to 2016. This approach is an extension to the widely used ARFIMA modeling technique of Granger (1980), Granger and Joyeux (1980) and Hosking (1981), who rather than see/limit the variable as an I(0) or I(1) process, saw it as an I(d) process. The intuition behind the I(d) process lies in the fact that possibility exist that a series has an order of integration different from I(0) or I(1) which is more helpful in explaining the degree of persistence/long-memory of a variable under investigation. This paper in line with the work of Bailie, Chung and Tieslau (1996) who not just applied the Autoregressive Fractionally Integrated Moving Average ARFIMA technique but also found a ground for the modelling the generalized autoregressive conditional heteroskedasticity to account for the presence of conditional heteroskedasticity 'ARCH effect' in the model. The motivation for the study therefore rest in the application of the ARFIMA-GARCH procedure to Nigerian consumer price index (CPI) inflation data, which is novel at least by accessible existing literature on inflation rate in Nigeria.

The rest of the paper is divided into six sections. Section 2 deals with time path of inflation in Nigeria with the aid of a line graph drawn on quarterly data from 1995 to 2016. Section 3 presents a compact review of previous studies. Section 4 discusses the methodology and data issues. Section 5 dwells on results and discussions and section 6 focuses on conclusions and policy implications.

2. Time Path of Inflation in Nigeria

Inflation in Nigeria has remained highly unstable over the years. Figure 1 provides insight into the trend and behavior of the inflation series over the study period. The figure shows the monthly Consumer Price Index (CPI) inflation rate in Nigeria from 1995Q1-2016Q4. The trend shows a wild monthly fluctuation in the Consumer Price Index (CPI) inflation rate data over the study period from the year 1995 up to the second quarter of the year 2012. Slightly mild fluctuations occurred from the third quarter of the year 2012 up to 2016. From the beginning of the study period in the year 1995 recorded an aggregate Consumer Price Index (CPI) inflation rate value of 37.83% boosted by a monthly high in the period of 8.63% in April of 1995. The year 1996 witnessed a drastic decline in Consumer Price Index (CPI) inflation rate over the 12 month period, returning an aggregate of 13.38%.

The period from 1997 to 1999 recorded lower 12 month aggregate Consumer Price Index (CPI) inflation rate in comparison with the two preceding years, with 9.72%, 11.26% and 0.22% for 1997, 1998 and 1999 respectively. The low 12 month inflation recorded in the year 1999 may be due to the increased business confidence, testimonial to the calm and stability witnessed with the transfer of power to democratically elected government in Nigeria by the military government. However, the gains made in the government transition period in maintaining a low Consumer Price Index (CPI) inflation rate could not be sustained, with average aggregate Consumer Price Index (CPI) inflation rate over 12 month periods from the year 2000 up to the year 2012 standing at a two digit value of 12.27% over the 11 year period, with two election years 2003 and 2007 turning up the maximum and minimum 12 month aggregate Consumer Price Index (CPI) inflation rates of 21.36% and 6.36% respectively.

The period from 2010 to 2015 witnessed sustained Consumer Price Index (CPI) inflation rates with a single digit on the average over the two period 9.12% and a maximum and minimum over the period of 11.32% in 2012 and 7.63% in 2013 respectively. A global fall in prices of crude oil took its highest toll on the Nigerian economy, leading to a shortfall in foreign exchange and a rising exchange rate led to an increase in prices of local goods in the year 2016, leading inflation to jump back to two digit figures, returning a 12 month aggregate of 17.01%.

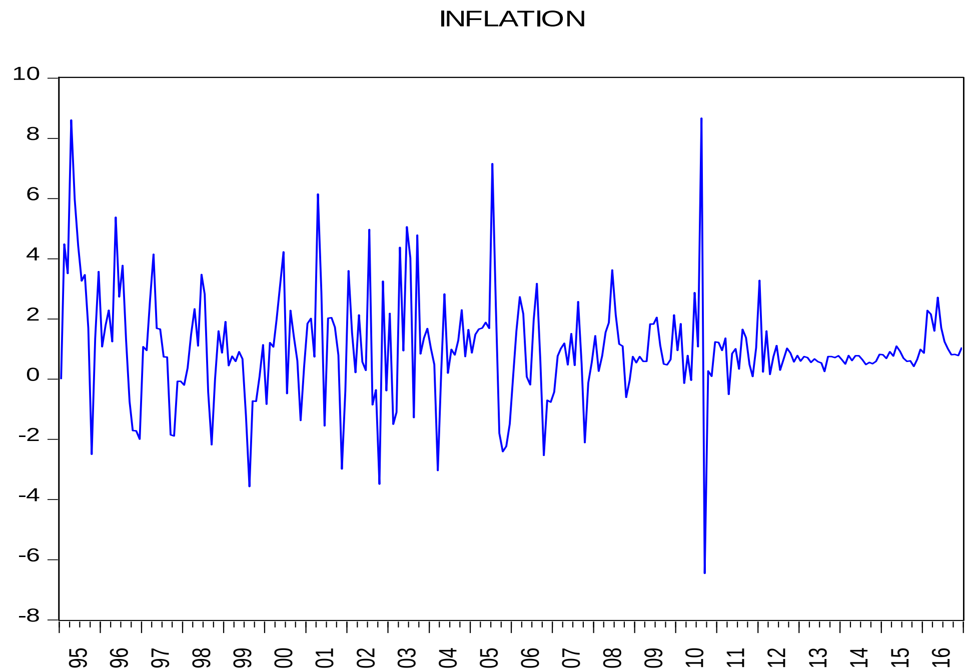

Figure 1: Trend of Inflation in Nigeria

3. Review of Previous Studies

The quest for finding the appropriate approach for analyzing inflation rate in Nigeria has led to the evolution and application of different methodologies by different researchers. Using ARIMA and SARIMA models, Doguwa and Alade (2013), Alnaa and Ferdinand (2011), Okafor and Shaibu (2013), Akdogan et al (2012) Ho and Xie (1998) and Adebyi et al (2010) showed that ARIMA models were modestly successful in explaining inflation dynamics in Nigeria.

Using GARCH models, Osarumweuse and Waziri (2013), Fasanya and Adekoya (2017), Bamanga et al., Nkoro and Uko (2016), Chowdhury (2011), Thornton (2006) and Omotosho and Doguwa (2012) have found high persistence parameters implying that the impacts of inflation shocks on their volatility dies away rather very slowly.

Several other studies have investigated the inflation dynamics in Nigeria using several methodologies such as Artificial Neural Network (ANN), Vector Auto Regressive (VAR), Bayesian Vector Auto Regressive (BVAR), Structural Vector Auto Regressive (SVAR), Simple Auto Regressive (SAR), random walk and Auto Regressive Fractionally Integrated Moving Average (ARFIMA). Expectations-augmented Philip curve, P-Star, Leading indicators and Traditional monetarist model (Uwusu, 2010).

This study employed the ARFIMA-GARCH approach proposed by Bailie, Chung and Tieslau (1996). Bailie et al (1996) consider the application of long memory process to describing inflation for ten countries. They implement a new procedure to obtain approximate maximum likelihood estimates of an ARFIMA-GARCH process; which is fractionally integrated (d) with superimposed stationary ARMA component in its conditional mean. An additional feature of their work is that the long memory process is allowed to have GARCH type conditional heteroskedasticity. Their results for the ten countries shows strong evidence of long memory with mean reverting behavior for all countries except Japan which appeared stationary. They conclude that for three high economies, there is evidence that the mean and volatility of inflation interact in a way that is consistent with the Friedman hypothesis.

4. Methodology and Data Issues

The data used in the study has been drawn from the Central Bank of Nigeria (CBN) Statistical Bulletin 2016 annual edition. Monthly data on Consumer Price Index (CPI, all items) from January 1995 to December 2016 was used to compute inflation rate using the formula below;

$$INF_t = 100 * Ln\left(\frac{CPI_t}{CPI_{t-1}}\right)$$

Where INF denotes inflation, CPI denotes Consumer Price Index t denotes time period in months.

4.1 Justification of Method

Advancements in econometrics have brought with it much needed tools to explore the stochastic properties of economic variables over time. Early efforts at investigating the empirical properties of macroeconomic variables are found in the works of Ross (1988), McDonald and Murphy (1989), Godwin and Grennes (1994), Wallace and Werner (1993), Crowd and Hoofman (1996) and Mishkin and Fisher (1995) used the Dickey-Fuller variant of unit root test developed by Dickey and Fuller (1979) to investigate the persistence of interest rate. An issue with these early works lay in the power of the Dickey-Fuller unit root test, which was considered to have weak power.

The perceived low power of the Dickey-Fuller variant unit root test led to the emergence of studies by Lai (1997) and Tsay (2000) investigating the stochastic properties of macroeconomic variables using the Autoregressive Fractionally Integrated Moving Average conceived in Granger (1980) and Granger and Joyeaux (1980). The ARFIMA models' purpose was to serve as an alternative to the unit root test and at the same time overcome the weaknesses of the unit root test technique.

Recent studies like Rapach and Weber (2004), Sofiane and Ahmed (2007), Neely and Rapach, (2008), Awomuse and Alimi (2012) Haug (2014) and Apergis, Christou, Payne, and (2015) have inculcated both the Dickey-Fuller variant of the unit root test, with modifications to overcome the problem of low power of the test. Other studies used cointegration test to investigate the stochastic properties of macroeconomic variables and some recent studies have found the ARFIMA technique an indispensable tool for the investigation of the stochastic properties of macroeconomic variables.

4.2 Test for Long Memory in Inflation

The study applied two conventional tests for a unit root in a series to determine the long memory properties of Consumer Price Index Inflation in Nigeria. The augmented Dickey Fuller (ADF) test and the Kwiatkowski, Philips, Schmidt and Shin (KPSS) test of stationarity

4.2.1 Testing For Unit Root I (0) and I (1)

Augmented Dickey Fuller (ADF)

The series of consumer price index (CPI) Inflation is tested for a unit root using the standard augmented Dickey-Fuller (ADF) test. For a time series (Y_t), the ADF test involves the following regression carried out under three conditions:

- A random walk process which is defined as;

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{m} \alpha \Delta Y_{t-i} + \varepsilon_t$$

- A random walk process with drift which is defined as;

$$\Delta Y_t = \beta_1 + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha \Delta Y_{t-i} + \varepsilon_t$$

- A random walk process with drift around a stochastic trend which is defined as;

$$\Delta Y_t = \beta_1 + \beta_2^t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha \Delta Y_{t-i} + \varepsilon_t$$

Where,

δ = the difference operator; ε_t = the random error term.

The ADF test considers a null hypothesis of an I(1) process against the alternative of an I(0) process.

Kwiatkowski, Philip, Schmidt and Shin (KPSS)

The KPSS stationary test developed by Kwiatkowski, Philip, Schmidt and Shin (1992) is an alternative test to the ADF test. It tests for stationarity in the series against an alternative hypothesis of no stationarity in the series. The KPSS test considers that a time series can be written as the sum of a random walk and a stationary error, and that the variance of the error in the random walk component equals zero under the null hypothesis. To carry out the test, the residual seriesε_t, is first obtained from a regression of Y_t on a constant and possibly a trend. The KPSS statistic, denoted by $\eta_\mu$is constructed as

$$\eta_\mu = T^{-2} \sum_{t=1}^{T} \frac{St^2}{S^2(l)}$$

Where,

S_t = the partial sum process of regression residuals defined by;

$$S_t = \sum_{i=1}^{t} e_i, \quad t = 1, 2, \ldots, T$$

And $S^2(l)$ is a heteroscedasticity and autocorrelation consistent variance estimator given by

$$S^2(l) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{j=1}^{1}\sum_{t=j+1}^{T} w(j,l)e_t e_{t-j}$$

With l being a lag truncation parameter and $w(j,l)$=1- $\frac{j}{(l+1)}$ a weighting function corresponding to the choice of the Barlett window.

4.3 ARFIMA-GARCH Specification

Bailie, Chung and Tieslau (1996) developed a new method of evaluating the long memory properties of macroeconomic data that takes account of conditional heteroskedasticity in the series.

$$\phi(L)(1-L)^d (INF_t - \mu - \delta\sigma_t) = \theta(L)\varepsilon_t \tag{1}$$

$$\varepsilon_t \big| \Omega_{t-1} D(0, \sigma_t^2) \tag{2}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

Where d is the fractional integration term and can assume non-integer values, INF_t is CPI inflation μ is the mean of the process, $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$, $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$, and all the roots of $\phi(L)$, and $\theta(L)$, lie outside the circle. If in equation (1) δ =0 then equations (1) and (2) describes the ARFIMA process by Granger (1980, 1981) and Granger and Joyeux (1980). With $\delta \neq 0$, the model is extended to allow volatility to influence mean inflation rate, and therefore extend to an ARFIMA-GARCH model represented by equations (1), (2) and (3).

5.0 Results and Discussions

Test for Long Memory in Inflation Series

Table 1 ADF and KPSS Tests for a Unit Root and Stationarity in the Inflation Series

Variable	INFLATION		
	Level		
Test type	None	Intercept	Intercept and trend
ADF	-6.978560 (***) [1]	-12.76831 (***) [0]	-12.87814 (***) [0]
KPSS		0.239883 (***) [0]	0.082034 (***) [0]

Note: Critical values for KPSS obtained from Kwiatkowski et al (1992) and (***) denotes 1 percent level of significance and squares brackets denotes optimal lag lengths.

Table 2 Summary of ADF and KPSS Test for a Unit Root and Stationarity in the Inflation Series

Variable	ADF (t-statistic)	KPSS (LM-Statistics)	Test Property	Level of Integration	Decision
INFLATION	-12.87814 (***) [0]	0.082034 (***) [0]	Intercept and trend	I(0)	No Unit Root/ Stationary

Note: (***) denotes 1 percent level of significance and squares brackets denotes optimal lag lengths.

Table 1 and 2 display the results for test for a long memory property in the Nigerian inflation series using an Augmented Dickey Fuller test specification at level as well as the Kwiatkowski, Philips, Schmidt and Shin (KPSS) test for stationarity as an additional validating tool for the unit root test. Both ADF and KPSS test show that Nigerian inflation rate series is level stationary. This implies that a shock to CPI inflation series will eventually die out in the long run. In other words, Nigerian CPI inflation is a shock dissipating process. ADF and KPSS tests give a pictureof the finite behavior of a long memory process but in the case of a shock dissipating process such as CPI inflation used in the study, the ADF and KPSS tests fail to point out if the series absorbs the shock at hyperbolic or geometric rate. This gives rise to the need for a fractionally integrated process, which gives non-integer values of an order of integration of a series which makes it feasible to ascertain the speed at which shocks to a series that is shock dissipating die out.

5.1 ARFIMA-GARCH Model Results

The ARFIMA-GARCH methodology is made valid if there exist conditional heteroskedasticity in the variable under consideration, an absence of which the ARFIMA-GARCH model will reduce to a strictly ARFIMA model. A test for 'ARCH effect' in the Nigerian CPI Inflation data is presented below.

Table 3 Pre-estimation Test for 'ARCH Effect' in the Nigerian CPI Inflation Data

Heteroskedasticity Test: ARCH			
F-statistic	23.09622	Prob. F(1,260)	0.0000
Obs*R-squared	21.37510	Prob. Chi-Square(1)	0.0000

Source: extracts from Eviews 10 outputs

Table 3 shows the results of the test for 'ARCH effect' in the study's variable of interest. A significantly large value of F-statistic leads to the rejection of the null hypothesis of the absence of ARCH effect and the acceptance of the alternative hypothesis of the existence of 'ARCH effect' in the CPI inflation variable.

Table 4 ARFIMA (2, d, 2)-GARCH (1, 1) Results

Variable	Coefficient	Std. error	t-stat	p-value
d	0.331277	0.091848	3.606814	0.0004
ω	0.015137	0.020088	0.753525	0.4511
α	0.833022	0.860602	0.967952	0.3331
β	0.722088	0.048427	14.91093	0.0000

Source: Compiled by author from Eviews 10 output

Table 4 presents the results of the ARFIMA-GARCH estimates for Nigerian CPI-inflation data. The fractional integration parameter *d* in the ARFIMA model is statistically significant which implies CPI-inflation rate in Nigeria is an I(*d*) process. The fractional integration coefficient is closer to zero and by implication aligns with the results obtained by the ADF and KPSS that CPI-inflation is a shock dissipating process but although, different from zero. The fractional integration parameter provides more information about the pace at which CPI-inflation would absorb shocks. The fractional integration coefficient is 0.33 reveal that shocks to CPI-inflation die out at a geometric rate, although more slowly than an I(0) process.

The general autoregressive conditional heteroskedasticity GARCH component of the result provide information about how long it will take to halve the impact of shocks on the CPI-inflation series in Nigeria. The half-life computed from the GARCH component of the ARFIMA-GARCH model shows that it will take approximately 2 months to halve the impact of shocks on CPI-inflation series in Nigeria. This agrees with the results from the ARFIMA component of the ARFIMA-GARCH model that shocks to CPI inflation in Nigeria will die out quickly at a geometric rate, which imply CPI-inflation has a fast mean reverting ability in the event of an external shock to the series.

6.0 Conclusion and Policy Implication

Clear conclusions can be drawn out for the CPI-inflation series in Nigeria as regards it long-memory properties. This properties also are of interest in policy making as the government will be interested in knowing how deliberate changes in prices of very important items such house hold and firm energy (electricity, cooking gas, diesel, kerosene, PMS and aviation fuel) will affect the general price level in the economy and to what extent the changes in the general price level will persist over time. Aside deliberate government policy to alter prices of important selected items directly, monetary policy instruments which are inflation targeting may also cause shocks to the general price level in Nigeria.

This study provides quiet useful insights into the behavior of the general price level over time in Nigeria in situations of deliberate government action in changing the price which takes the consuming public by surprise or in a situation where inflation targeting monetary policy causes changes to the price level. The study's finding avail that despite sudden increases in prices of very essential items either by government fiat or monetary policy instruments, these changes do not persist in perpetuity as the general price level eventually readjust over time to a mean level.

These findings have an implication for the inflation-unemployment tradeoff in the Philips curve. Given the CPI-inflation is a mean reverting process, policy makers can afford a trade-off by targeting unemployment while allowing some deliberate price rises in the hope that prices will revert to their mean levels over short period of time.

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