

Concrete Pictorial Abstract Approach Use in Teaching Mathematics on Secondary Students' Participation in Six Selected Schools in Kicukiro District, Rwanda

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ABSTRACT

Mathematics is pivotal in science and technology, serving as a gateway discipline. Effective teaching should foster positive attitudes by engaging students with real-world examples. The specific objective of this study was to investigate how the use of Concrete-Pictorial-Abstract approach in mathematics subject affects students' participation in selected schools in Kicukiro District. This study was based on investigating the use concrete pictorial abstract approach in teaching mathematics on secondary students' participation in selected ordinary secondary schools in Kicukiro district. The population targeted in this study was all 2520 senior two students, 51 mathematics teachers and 36 directors of studies and the sample size comprised 540 students, 6 directors of studies and 6 mathematics teachers. This study applied constructivism theory. A qualitative research approach was used since the topic for research requires an in-depth study in a real situation together with descriptions which can be done better in words than in numbers. Instruments used in data collection in this research are interview and observation. To analyze the data obtained during interview and observation, this study used content analysis. Content analysis is a technique utilized for organizing spoken or written materials into predetermined categories with similar meanings. Findings reveal that when teachers use the CPA approach in teaching mathematics, students participate in lessons through manipulating concrete materials. The stage of pictorial use enables students to develop critical thinking by transitioning from concrete to abstract concepts. This final step involves students using numbers and symbols. At this step the findings show that the students try to make general rule by using numbers and symbols though students seem less active than during the steps of using concrete and pictorial. Conclusion, learners participate in learning mathematics and improve their performance through the usage of concrete pictorial abstract approach, by using tangible materials and visuals aids. This research will help the teachers who teach mathematics to involve the students in order to make the class active and stick on the learner centered approach, by engaging the students in all levels, and it will help both teachers and students to understand that mathematics concepts are tangible by using the materials, and this will attract students' interest and participation which lead to performance.

Keywords: Abstract, Concrete Materials, Students' Participation, Pictorial

I. INTRODUCTION

Mathematics is pivotal in science and technology, serving as a gateway discipline. Effective teaching should foster positive attitudes by engaging students with real-world examples (Schoenfeld, 2019). Bruner's theory of instruction from 1996 emphasizes engaging learners in mathematics education, influencing educational practices significantly (Alvares et al., 2009). In his Concrete-Representation-Abstract (CRA) sequence model, further Bruner's theory developed by (Dachi & Garrett, 2003), evolved into the Concrete-Pictorial-Abstract (CPA) model, showing effectiveness in improving student performance. The CPA approach guides students through stages of learning, integrating physical materials, pictures, and symbols to enhance mathematical understanding (Terry, 2008).

The department of Education in the Philippines has shifted to more effective teaching strategies, including the Concrete Pictorial Abstract (CPA) approach, to improve student performance, particularly in Mathematics, as indicated by better National Achievement Test (NAT) scores in recent years (Willingham, 2018). Despite initial challenges, the adoption of CPA has shown promising results in enhancing students' understanding and achievement in Mathematics at Bukidnon National High School (Fan & Williams, 2010).

In Indonesia, students' apprehension towards mathematics is addressed by the Concrete-Pictorial-Abstract (CPA) approach, aligning with Bruner's learning theory (Sarama & Clement, 2016). This method engages students actively through concrete materials, pictorial representations, and abstract symbols, fostering better understanding and confidence in mathematical learning.

African nations have chosen to prioritize increased investment in Science, Technology, and Engineering as a successful approach toward attaining sustainable development. Recent research conducted affirms that proficiency in mathematics serves as a fundamental basis for pursuing studies in Science, Engineering, and Technology. Proficiency

in mathematics is pivotal in influencing the post-school and career opportunities accessible to the younger generation. Despite this, academic achievement in mathematics among students has been reported as subpar (Fleming, 2020)..

African nations, including Rwanda, prioritize STEM investment for sustainable development, yet face subpar mathematics achievement due to traditional, teacher-centered methods. Rwanda and other countries are transitioning to competence-based curricula with innovative approaches like Concrete-Pictorial-Abstract (CPA), inquiry-based learning, and the 5Es model to improve student engagement and comprehension in mathematics (Kurniawan et al., 2020). Effective teaching methods, such as interactive approaches, are crucial in enhancing mathematics learning outcomes in African schools.

1.1. Problem statement

Mathematics is foundational in education for its role in mental development and practical application, yet African students consistently perform below international standards in mathematics and science (UNESCO, 2015). Research identifies challenges such as ineffective teaching methods, lack of motivation, and the abstract nature of mathematics, which hinder student engagement and performance (Ukobizaba et al., 2021). Recommendations emphasize the adoption of active learning strategies and modern teaching techniques to enhance understanding and participation in mathematics. The previous studies acknowledged the need for more active approach to teaching mathematics, though they didn't delve into specific methodologies. Based on my teaching experience, engaging students through tasks and fostering discussion enhances their understanding and participation. I found the Concrete Pictorial Abstract approach effective as it involves students in three stages: hands-on manipulation of materials, visual representation through pictures, and formulation of abstract concepts using symbols, ensuring continuous engagement throughout the lesson. So, I decided to conduct this research on investigating the impact of use of concrete pictorial abstract approach in teaching mathematics on secondary students' participation toward performance in six selected schools in Kicukiro district.

1.2 Objective

The overall objective of this study was to investigate how the use of concrete-pictorial -abstract approaches in mathematics subject affects students' participation in selected schools in Kicukiro District.

II. LITERATURE REVIEW

2.1. Theoretical review

2.1.1. Mathematics teaching and learning

Proficiency in mathematics is a crucial aspect of literacy that significantly impacts children's achievements both in academics and in broader societal contexts. (Black et al., 2012) highlight the enduring focus on mathematics education and skill enhancement, a theme spanning over a century in educational and psychological investigations. Starting in the early twentieth century, psychologists embarked on experimental inquiries into children's grasp of numerical concepts, arithmetic, and the acquisition of specific mathematical proficiencies (Fleming, 2019). These inquiries have contributed substantially to our comprehension of how children learn mathematics from a psychological perspective, an insight pivotal in fostering their cognitive development.

Since mathematics is abstract, it's best to teach it by using all possible ways to represent abstract ideas and involve the students. When students can accurately get involved to show mathematics concepts, they understand the mathematics better. Creative teaching methods, like using objects to explain concepts, can help students feel less confused or anxious about math. So, it's important to focus on making sure students understand math concepts and can demonstrate their understanding through hands-on activities.

Proficiency in mathematics plays a vital role in children's academic success and broader societal impact. Sung et al. (2017) emphasize the long-standing emphasis on mathematics education and skill development, a theme spanning over a century in educational and psychological studies. Beginning in the early twentieth century, psychologists conducted experimental studies on children's understanding of numerical concepts and arithmetic, contributing significantly to our understanding of how children learn mathematics from a psychological perspective. Teaching mathematics using various representations and engaging students in hands-on activities is crucial due to its abstract nature. Utilizing creative methods, such as using objects to explain concepts, can help students better comprehend and feel more comfortable with mathematics. The focus should be on ensuring students grasp mathematical concepts and can demonstrate their understanding through practical activities.

2.2. Empirical Review

2.2.1. The concept of Concrete Pictorial Abstract Approach

The Concrete Pictorial Abstract (CPA) approach is a teaching approach that follows a structured approach with clear and explicit instructions in teaching mathematics. It consists of three phases: the concrete stage, where tangible materials are used, the pictorial stage, where pictures and visual representations are employed, and the abstract stage, which involves working with numbers and symbols. This evidence-based practice has gained significant interest among educators in various fields of education. Existing literature demonstrates that CPA is a successful approach for teaching students in both general and special education settings (Watt & George, 2021). Implementing CPA has proven to be an effective approach in enhancing the performance of students by engaging them and actively participate in mathematics learning within the classroom (Jennifer, 2021)

The uniqueness of CPA approach in class is that the topics are presented in three ways. Firstly, concepts are introduced concretely through the utilization of manipulative materials and models. Secondly, visual representations such as figures, pictures, tables, or other two-dimensional objects are employed. Lastly, symbols in mathematics, including letters and numbers, are utilized to represent mathematical ideas. Every topic is presented using these three representations. In every representation the teacher presents the topic to the students and gives a short assessment before going on to the next step. After the three representations, the teacher will give an assessment and let the students freely use any of the representations. The lesson will always end up with a reflection about the representation that they want to use in a certain topic. CPA is habitually used in primary and secondary level of education in teaching mathematics.

2.3. Theoretical framework

This research is grounded in the concept of constructivism, initially formulated by Jean Piaget. Constructivism posits that learners actively construct knowledge rather than merely receiving information in a passive manner. Through personal encounters with the world and thoughtful contemplation of these experiences, individuals develop their own mental frameworks and integrate new information into their existing knowledge structures, known as schemas. This study implies this theory because the students need to be the first participants by first using their hands when they are on the first step which is concrete where they manipulate, second, they use their eyes on pictorial and of course they say what they see and discuss after that they come up with how they can write that down by using numbers sentences here its where they go in abstract (Delaney, 2020).

Teaching mathematics through constructivist methods allows students to deepen their knowledge beyond rote memorization, develop meaningful context to comprehend the content, and take command of the learning process as an active participant rather than a sit-and-get observer. Constructivism bases on the following principles, knowledge is constructed using students' prior knowledge, they build new knowledge by drawing comparisons and making connections. In mathematics this means they use foundational concepts to build the next one, Active learning: students must be part of the lesson in order to understand new content; students do not need to receive new information passively (Arends, 2021).

III. METHODOLOGY

3.1. Research Method and Design

In this research, a case study design was used because the researcher wanted to understand how the use of concrete pictorial abstract approach in senior two class in specific schools of Kicukiro District affects students' participation to ward performance. Case studies are often selective, concentrating on one or more aspects that are critical to comprehend the system under consideration (Park et., 2021). In this case, the researcher's main focus was to investigate the use of concrete pictorial abstract approach in teaching and learning mathematics on students' participation. The case in this study is the mathematics teachers who teach senior two in selected schools and director of studies.

3.2. Target Population and Sampling

According to Kothari (2004) the target population is the large group of people, which has one or more characteristics in common on which the research study will be focused.

The population targeted in this study was all 2520 senior two students, 51 mathematics teachers and 36 directors of studies in Kicukiro district. Kicukiro district was chosen as one district because this research is not a survey for the whole country it again chosen because it is one of the districts which composed by the rural and urban area, this insured the researcher that the data presented the both areas, Kicukiro was again the accessible District to the researcher depending to the time availability and financial facilities.

A sample is a subset of the target population for generalization (Creswell, 2012). This study used stratified random, purposive, and convenience sampling methods. Stratified random sampling divides a population into strata while

purposive sampling selects based on specific research objectives (Fraenkel et al., 2011). Convenience sampling recruits easily accessible participants, commonly used in qualitative research (Creswell, 2012). This research used convenience sampling to gather data from accessible and willing participants to address research issues like geographical proximity and availability. The study focused on the concrete pictorial abstract approach in senior two mathematics classes. Teachers and directors from six schools and 540 students participated because they were actively learning during observations.

3.3. Research Data Collection Instruments/Tools

To collect data, various tools were used, including observation checklists, interview guides, and document analysis guides to answer the research questions.

The interview had two parts: demographic and open questions, conducted after each lesson observation. Teachers from six selected public schools participated in one-on-one semi-structured interviews about using the concrete pictorial abstract approach. An interview guide categorized questions on the use of concrete materials, pictorial, and abstract methods in teaching mathematics.

A data collection technique known as observation is one that "serves a formulated research purpose, is planned deliberately, is recorded systematically, and is subjected to checks and controls on validity and reliability" (Dietrich et al., 2016). As it gives the researcher an in-depth understanding of the group dynamics and interactions in various mathematics sessions, lesson observation is a crucial tool for data gathering (Maniraho, 2017).

I conducted six classroom observations in senior two and my focus was on teachers' use of concrete pictorial abstract approach on students' participation in the use of the approach. This was done using an approved class observation guide as it was represented to the teachers before being welcomed in the classroom during mathematics lesson,

3.4. Analyzing the data

To analyze the data obtained during interview and observation, this research used content analysis. Content analysis is a technique utilized for organizing spoken or written materials into predetermined categories with similar meanings (Fraenkel et al., 2011).

3.5. Trustworthiness/Validity and reliability of the study

In research, it is crucial for researchers to ensure the validity of the data they obtain. This aspect of the research focuses on establishing the reliability and trustworthiness of the data. To assess the data's trustworthiness, the researcher employed theoretical triangulation, credibility, and dependability as methods in this study. (Fraenkel et al., 2011) states that Triangulation is a technique that uses data validity that feats something else. Outside the data for checking purposes or as a comparison against the data in different sectors as mentioned. Credibility in qualitative research concerns the honesty of the inquiry's findings. Credibility or truth value involves how well the researcher recognized confidence in the findings based on the research design, participants, and context according to the questions represented in both observation, questionnaire and interview (Creswell, 2012).

IV. FINDINGS & DISCUSSION

4.1 The Use of Concrete Materials in Teaching and Learning Mathematics and Students' Participation

In this research it was found that using concrete materials in the teaching and learning mathematics boosts students' participation to ward performance shown by the marks by enhancing engagement levels. The hands-on nature of these materials captivates students' interest, encouraging them to actively participate in activities and discussions related to mathematical concepts. The use of concrete materials encourages active learning experiences. Students become more involved and participative in exploring mathematical concepts through manipulation, observation, and experimentation which helps them to learn the new concepts.

During the observation the research got the chance to visit a school where the teacher was introducing the new topic, the topic was rations, the teachers brought beads, colored papers and blocks, he shared the materials among the students by using groups, they arranged the materials without being what the topic was all about, they finally shared the ideas as entire class where they said what they did in their groups , and then teacher intervned and named the topic as it was the ratios.



Plate 1
Teaching Aids Brought by the Teacher

The teacher asked the learners how they can use the materials given, students started giving the ideas on how they can arrange the materials, some suggested that they can be arranged according to their colors, all of the ideas were welcomed, some groups arranged the materials according to the colors, others according to the shapes.



Plate 2
Materials Sorted by the Students Related to the Concept

After the organization, the teacher asked one group of the students to talk about what they have done, they have talked about the green bottle tops to yellow tops, they said that they are three green bottle tops to seven yellow bottle tops.

The teacher asked the students if they can make their own material that can be classified in different ways, the students worked in their groups and made different materials, here are some materials made by one of the groups in classroom, the students explained how they can arrange them, the first way they mentioned was by colors, where they said that they have objects of three different colors, they arranged them accordingly, 5 black, 4 red and 3 blues objects.

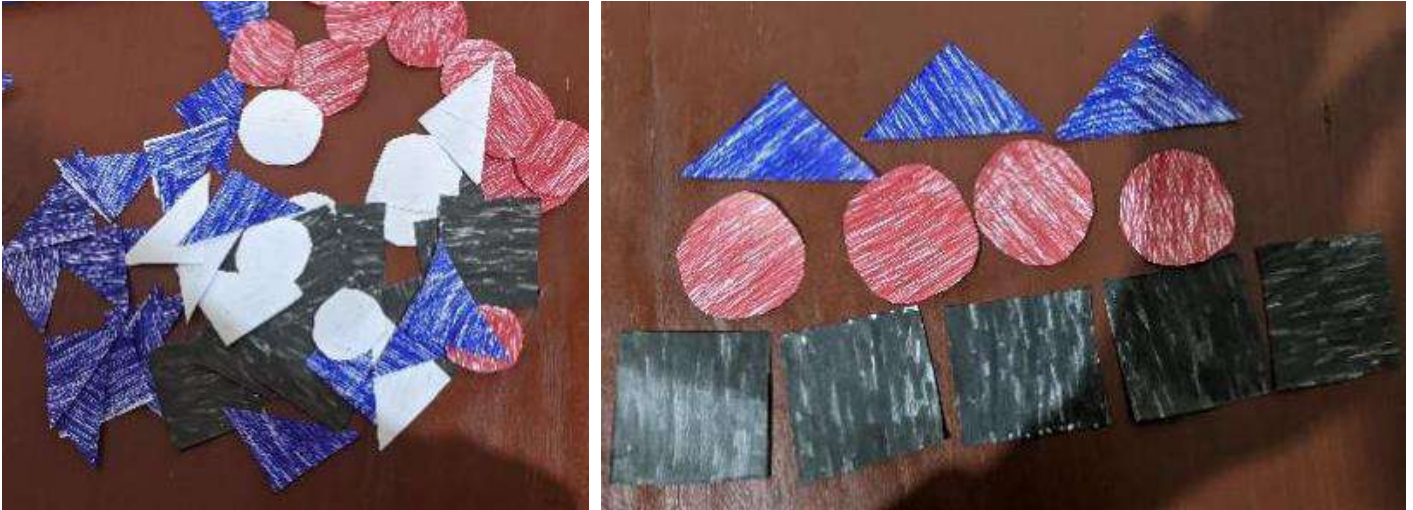


Plate 3
Concrete Materials Made by the Students

This showed the researcher that it is possible for the students to learn the new topic as they are exposed to the concrete materials and given opportunity to discuss and use them.

Hands-on activities with concrete materials often lead to increased focus and sustained attention among students. This heightened engagement promotes greater participation in mathematical tasks. According to (Iliev & D'angelo, 2014) concrete materials are important as they help mathematics to become meaningful for students by ensuring some concepts, theories and operations to be expressed intangibly, in this research the teachers showed that when the students are allowed to use the concrete materials, it helps them to participate in class.

The use of pictorial representations in teaching mathematics significantly influences students' participation, as evident from this research finding indicated that incorporating visual aids like diagrams, graphs, and visual models enhances student engagement during math lessons. Pictorial representations stimulate active involvement by capturing students' interest, encouraging them to ask questions and share interpretations this, in turn, leads to increased participation in problem-solving tasks, as students feel more equipped to express their thoughts and strategies using the visual representations provided which leads to higher level of thinking. Overall, the use of pictorial representations uplifts students' willingness to engage, communicate, and explore mathematical concepts, fostering a more dynamic and participative learning environment.

During the interview the teachers were asked if they help the students to draw the pictures related to the concrete materials which were used.

The teachers said that for using drawings as a mean for students to reinforce their understanding after working with concrete materials. This activity allows students to process their hands-on experiences and create visual representations of the mathematical concepts they've learned.

In observation, from schools where the researcher visited some students seemed to draw the pictures related to the materials used during the lesson while in two schools where the researcher observed the teachers were the ones to draw the pictures while the students follow. Again, referring to the sample lesson plan during transition to the pictorial stage the teacher asked students to represent what they have been doing the using drawings, the teacher asked them to brainstorm and complete their work in the given time, the students were given 5 minutes to discuss about it. After their discussion, the teacher took the time to go and check what they have done, they were given time to present their drawings as well. Here are some of the pictures show what they have done.

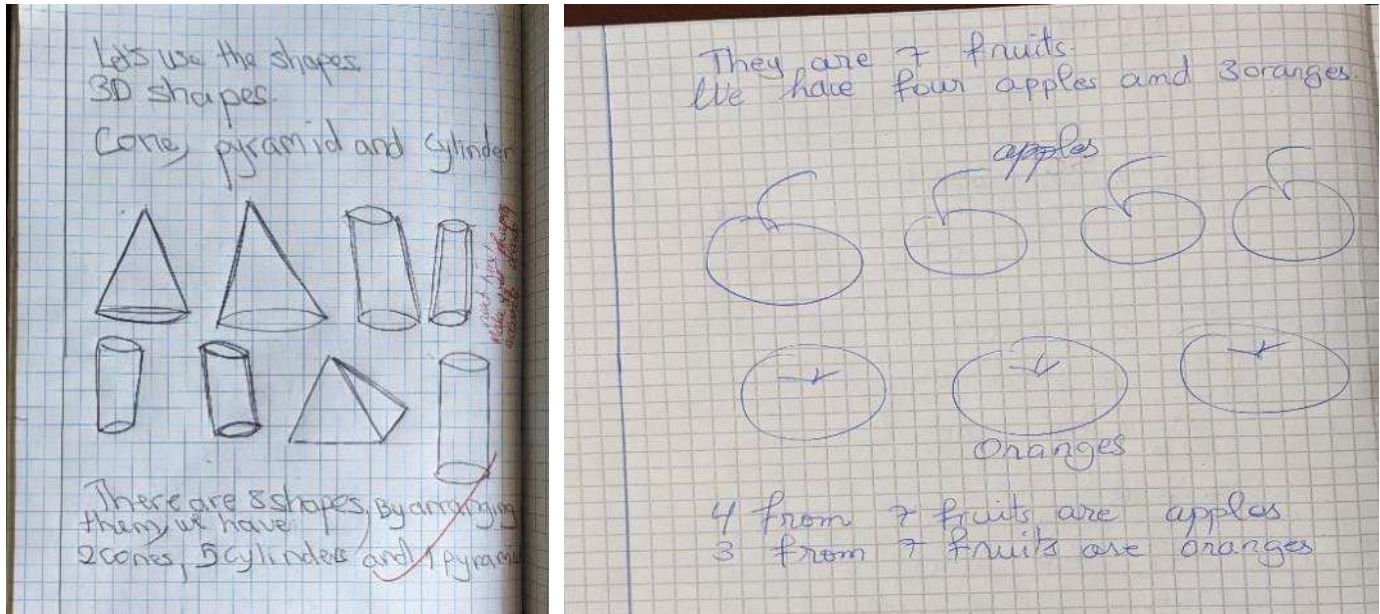


Plate 4
Students' Work by Using Pictures

According to (Hidayah et al., 2016) mathematics is abstract by its nature, can often be perplexing for children. However, by utilizing concrete and tangible resources, children are able to visually perceive and comprehend the underlying mathematical principles.

4.2 The Use of Abstract in Teaching and Learning Mathematics on Students' Participation toward Performance

Abstract representations include symbols, equations, and mathematical notation, engaging the students with abstract representations fosters the development of critical thinking, logical reasoning, and problem-solving skills. Students learn to generalize concepts, recognize patterns, and apply abstract principles across various mathematical contexts, promoting advanced cognitive abilities.

Teachers were asked if after using concrete materials and pictures, they help the students to make general rule by using symbols and numbers, they said it depends on, there is a time students discover it by themselves and there is when they get helped,

During the observation, the researchers has seen that when it comes for abstract part the students need more help than the previous stages, for instance in the schools the teachers were helping the students to generalize the rule after using concrete materials and the pictures. When it came to move to abstraction, the teacher asked students how they can present their drawings by using the numbers, they were asked to work together to make the mathematical sentence, as the first stages they worked together and discussed during the presentation the teacher asked who was ready to present the work but they seemed to be shy, the teachers got involved and explained something and then they got their flow back. One of the students explained her work in front of others, below it was the students' work.

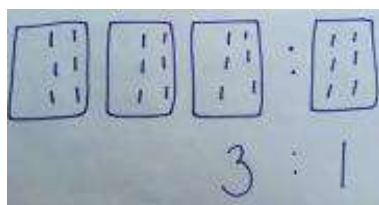


Plate 5
Abstract Presentation by the Students

The teacher appreciated their work and asked them if they can guess what the topic was about, the students gave their ideas until they said it was ratios. They discussed what a ratio is, by defining the ratio where they said that a ratio says how much of one thing there is compared to another thing.

Students were happy to be the ones to discover and participate in the lesson delivery. The teacher asked the students to reflect on how their understanding evolved throughout the lesson and how each representation helped deepen their comprehension, some appreciated being involved in using materials, others they said they enjoyed the lesson by

using the pictures, others said the last method which is summary even if it is little bit challenged but it is good because it is a summary which is easy to be taken.

During the interview, the teachers talked about the abstract stage; where they were asked if the abstract helps the students to focus on the essential and answered in these words;

Introducing symbolic representations allows students to distill the essential elements of a concept. It encourages them to see patterns and relationships more clearly, leading to a deeper understanding. For some students, working with symbolic representations can indeed help them focus on the core elements. However, for others, it might be more beneficial to continue using concrete or pictorial methods as well. It's important to adapt to individual learning styles

During the observation the researcher has seen that when it comes for the abstract part they focus more on the fundamental principles and underlying ideas, where they start noting down the notes in their books by using formulas including numbers and signs,

Abstract representations like symbols and equations help students develop critical thinking and problem-solving skills. However, at this stage, students need teacher support to make general rules. Observations showed students were less active and needed examples to start participating. The final abstract stage of the CPA approach builds on understanding and procedural fluency, but extra help is often needed for using numbers and symbols (Ban, 2021).

4.3 Challenges Faced by Teachers in Teaching and Learning Mathematics using CPA approach

The identified challenges include resources; ensuring access to varied and suitable materials for the concrete stage can be resource-intensive and is not always align with the available classroom resources. Additionally, transitioning students from concrete manipulatives and visual aids to abstract concepts demands careful guidance and some students struggle, it is not always applicable to assess the students by using all of these stages because most the time exams come in the form of questionnaire where questions are written on the papers.

V. CONCLUSIONS & RECOMMENDATIONS

5.1. Conclusions

Overall objective of this study was to determine how the use of Concrete Pictorial Abstract approach in mathematics subject affects students' participation toward performance in selected schools in Kicukiro District. After analyzing the data collected from the teachers and director of studies and through observation in this research, the researcher findings revealed that:

The teachers use concrete materials while they are teaching mathematics, students are allowed to use the materials brought by their teachers and they can even make theirs, materials are very essential for teaching and learning mathematics where the teachers said when they are using materials, it helps them to show the student that what they are talking about is real and it can be seen around them, students supported this where they said that learning by manipulating helps them to see the real meaning of mathematics.

After concrete materials pictures are also being used, as the approach is the learner centered approach, the students are always being asked to draw the pictures according to what they have touched, teachers support those who need help, this helps the learners to feel valued and participate in class.

Learners make the general mathematical rule by using numbers and symbols to present what they saw and drew, teachers intervene in this part so that they can drive to the conclusion, this part is essential because it is where they focus to the main point to be taken as the general form.

5.2. Recommendations

Recommendations basing on the research findings, some teachers claimed that the materials are not enough. According to what other teachers said it is possible to make the concrete materials by improvisation, I would like to recommend the teachers to have the culture of creativity, I would like to recommend the teachers to show all the steps that are going to be followed in order to help the students to participate in class as they are using concrete pictorial abstract approach.

The present study was conducted only in six public schools within one district; therefore, findings might not be generalized to the entire Rwandan context; rather further studies would be undertaken for example by involving more schools and/or analysis that caters for various parameters such as school types (public, private), location (urban, rural).

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