

## Stochastic Mortality Models with Birth Cohort Effects in Older People: A Systematic Review

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### ABSTRACT

*As populations age, comprehending the factors that influence mortality rates becomes ever more critical. Age, period and birth cohort are now acknowledged as essential determinants in analyzing (and projecting) mortality trends. This systematic review investigates the impact of birth cohort effects on mortality rates among individuals aged 60 and older, with a particular emphasis on stochastic mortality models. A thorough literature search was performed to identify studies published over the past three decades that examine stochastic mortality models incorporating birth cohort effects in older adults. Five primary mortality models were assessed and data were extracted concerning study characteristics, participant demographics and mortality outcomes. The findings underscore the significance of cohort effects in mortality modeling, particularly for senior demographics. These effects encompass social and historical elements that shape generational health; thus, they enhance the precision of mortality projections and guide effective policy formulation. Stochastic models that integrate cohort effects were shown to more accurately capture distinct mortality trends. This indicates that exposure to advancements in healthcare and environmental influences, significantly affects mortality results among older cohorts. However, this review identifies several challenges in model estimation and proposes avenues for future inquiry. It advocates for adapting models to more accurately mirror the unique characteristics of aging populations, improving validation techniques for mortality models and applying the results to real-world contexts to bolster decision-making in public health and social policy.*

**Keywords:** Birth Cohort Effect, Mortality Rate, Stochastic Mortality Model

### I. INTRODUCTION

As global populations continue to age, understanding the variations in mortality among older adults has become increasingly important for social security, actuarial science and public health planning. Accurate modeling and forecasting of mortality rates within this demographic are essential because of the unique and complex patterns of risks they face. These mortality risks are often shaped by historical, social and environmental factors (Dowd et al., 2011). Mortality rates are influenced by a myriad of factors, including socioeconomic conditions, advances in healthcare and generational trends. Accurate forecasts of mortality trends are critical for informing public health strategies, policymaking and actuarial calculations (Booth & Tickle, 2008). Projections based on these trends help to estimate future healthcare needs, life expectancy and the sustainability of pension systems (Lee & Miller, 2001). However, forecasting mortality among the elderly presents unique challenges due to the interplay of aging processes, emerging health conditions and the effects of birth cohorts (Hunt & Blake, 2015). Although this complexity adds difficulty, it is necessary for effective planning and response.

Stochastic mortality models have emerged as a prominent method for forecasting mortality, primarily because they embrace randomness and uncertainty, thus capturing the intrinsic variability in mortality rates over time (Hurvich & Tsai, 1989; Brouhns et al., 2002). By accommodating this variability, stochastic models yield more realistic and adaptable forecasts that can respond to unforeseen shifts in economic, demographic and environmental conditions (Cairns et al., 2010a). Over the last three decades, numerous stochastic mortality models have been crafted, each building upon prior methodologies while integrating new insights. These models are generally grounded in time-series approaches, with parameters estimated from historical death rates (Pollard, 1987). Scholars have further refined these models, however, by incorporating previously overlooked phenomena, such as birth cohort effects (Renshaw & Haberman, 2006; Cairns et al., 2007).

The notion of birth cohort effects stands as a pivotal component in the realm of mortality forecasting. A birth cohort (which denotes individuals born within a particular timeframe) may exhibit unique mortality patterns, primarily due to shared experiences, including wars, pandemics, economic climates, or advancements in medical technology. This is evidenced in various studies (Gustafsson, 2011; Hunt & Villegas, 2015). These effects, however, account for

generational variances and are vital for enhancing the precision of mortality projections. This review methodically assesses the existing literature concerning stochastic mortality models that integrate birth cohort effects, particularly focusing on older populations. The global elderly demographic is expanding swiftly, accompanied by considerable demographic transformations. Factors like chronic illnesses, lifestyle decisions, access to healthcare and societal evolutions significantly affect mortality rates within this age bracket. Accurate mortality forecasts for aging populations are crucial (because they help) alleviate the impending burden on social services, healthcare systems and pension schemes (Cairns et al., 2010b; Cairns et al., 2019).

### 1.1 Statement of the Problem

As life expectancies increase globally, precise mortality rate forecasts among older people have become vital for making policies in social security, healthcare, and pensions (Lee & Carter, 1992; Cairns et al., 2006). Stochastic mortality models have become the main tools for actuaries and demographers to tackle the inherent uncertainties in mortality forecasting (Booth & Tickle, 2008).

Most traditional mortality models do not account for birth cohort effects. The birth cohort effects reflect the individual health, socioeconomic exposures, and environmental factors observed by groups born at different times, which can impact mortality trends significantly, especially in older groups (Currie, 2016).

The omission of cohort-specific variations in stochastic mortality models may lead to biased mortality projections and poor policies for managing pensions (Renshaw & Haberman, 2006). However, there is a lack of systematic evaluation on how well stochastic models incorporate birth cohort effects and the precise challenges they encounter in applying the models to older populations. A detailed review will give a better understanding of current methods and propose areas in the mortality forecast that are relevant for practitioners and policymakers concerned with aging demographics (Plat, 2009).

### 1.2 Research Objective

This review aims to provide an overview of stochastic mortality models with birth cohort effect, particularly in the elderly population, and serve as the foundation for further studies and practical applications in mortality forecasting for allocation of resources, policy-making, and actuarial planning in aging societies. This systematic review set out to:

1. Analyze the current literature on stochastic mortality models incorporating birth cohort effects in the elderly population;
2. Pinpoint the primary methodologies used to account for cohort effects in stochastic models;
3. Identify gaps in the literature and provide suggestions for future research.

### 1.3 Research Questions

The questions focus on looking into the current literature and other areas to improve the models. Below are the questions:

1. What is the extent to which current stochastic mortality models integrate birth cohorts into the older population? This question assesses how current models account for cohort effects.
2. What standard methods are used in stochastic mortality models to capture cohort effects? This question aids in understanding the methodological trends and how they relate across models and research.
3. How does the incorporation of the birth cohort effect influence the reliability and accuracy of mortality predictions in the older group? This question evaluates the effectiveness of adding birth cohort effects to the models.
4. What are the gaps in the literature with regard to the incorporation of birth cohort effects in stochastic mortality modeling? The question seeks to find areas that have not been explored.
5. How do the results of studies that incorporate birth cohort effects vary by data quality, region, and population characteristics? This question explores the difference in results across populations and how the models perform in diverse contexts and datasets.

## II. METHOD

### 2.1 Information Sources and Search Strategy

This review follows the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) guidelines (Moher et al., 2009; Page et al., 2021). A comprehensive literature search was conducted in MathSciNet, Scopus, JSTOR, Elsevier, and Google Scholar databases, using a combination of the following keywords: "stochastic mortality models," "birth cohort effects," "elderly mortality," "mortality forecasting," and "cohort mortality trends." Terms relating to the keywords were used to identify relevant studies: stochastic mortality models AND birth cohort

effects AND (elderly OR aging population OR older adults). The search was limited to peer-reviewed publications in English, focusing on studies published from 1990 to 2023 to capture the various developments in the field.

## 2.2 Eligibility Criteria

The eligibility criteria were based on the following PICOS framework: Population (P): older people (60 years and above); Intervention (I): any intervention or exposure; Comparison (C): any control; Outcome (O): changes in mortality rate and longevity; Study Design (S): quantitative observational design (cohort studies). Studies that develop or apply stochastic mortality models, including birth cohort effects as a critical factor influencing mortality rate, focused on the elderly population (aged 60 and above), and empirical studies based on actual mortality data or theoretical studies with precise model validation techniques were included in this review.

Studies were excluded if they focused on deterministic models without any stochastic elements, did not include birth cohort effects, or failed to discuss cohort-specific mortality trends; non-elderly populations or general demographic groups without a specific elderly focus, and non-peer-reviewed literature, such as reports, conference papers, and grey literature.

## 2.3 Results for Search

The database inquiry produced a total of 359 works; however, these were subsequently assessed for their relevance to stochastic mortality models that incorporate cohort effects in older populations. After the removal of 98 duplicate records, 261 unique studies remained. These studies underwent a screening of titles and abstracts, which resulted in the exclusion of 236 studies for a variety of reasons: 140 studies were considered irrelevant because they did not concentrate on mortality modeling or cohort effects in older individuals, 45 studies utilized deterministic models without stochastic processes, 16 did not adequately consider birth cohort effects, 15 investigated mortality in non-elderly populations, 10 were purely theoretical or conceptual (lacking empirical validation) and 10 were dismissed due to language barriers, as no translations were available. Following a thorough analysis of the full texts, 25 studies were ultimately included in the final review. These studies met the inclusion criteria by focusing on stochastic mortality models, incorporating cohort effects, applying these models to elderly populations and presenting substantial empirical evidence or case studies.

## 2.4 Data Extraction

Data extracted from selected studies included the type of stochastic model used, the method for incorporating birth cohort effects, the data sources, model validation approaches, and findings regarding the impact of cohort effects on elderly mortality trends.

## 2.5 Models

This section states the five (5) stochastic mortality models reviewed in this work. The models mentioned in this section all have some excellent features: the term of the Lee-Carter model makes it suitable for full age ranges, the Renshaw–Haberman model addresses the cohort effect and fits well with historical data, the Currie model has a more straightforward structure than the Renshaw–Haberman model, making it more robust the models of Cairns et al., 2006, and Cairns et al., 2007 have multiple factors, resulting in a non-trivial correlation structure, while the model structure is relatively simple.

## III. THEORETICAL FUNDAMENTALS OF STOCHASTIC MORTALITY MODELS

Due to the increasing focus on risk management and measurement for insurers and pension funds, the literature on stochastic mortality models has developed rapidly during the last decennium (Plat, 2009). This section provides an overview of the current literature on stochastic mortality models and their criteria.

### 3.1 Overview of Stochastic Mortality Models

Several methods have been proposed in the actuarial and demographic literature to capture a population's mortality trends. One of the most prominent approaches to stochastic mortality modeling is the method proposed by Lee and Carter (1992). Also, a remarkable variant of the Lee-Carter model was proposed by Cairns et al. (2006). Despite its variants and extensions, the Lee-Carter and Cairns et al. models inspired many authors to introduce more sophisticated methods by including additional parameters, for example, those of Renshaw and Haberman (2006), Currie (2006) and Cairns et al. (2009). This paper will briefly review five (5) mortality models and discuss their advantages and limitations. The subsequent models will display the following notations.

$\beta_x^i$  -reflect age-related effects

$k_t^i$  -reflect period-related effects

$\gamma_c^i$  -reflect cohort-related effects;  $c = t-x$  which denotes year of birth

$q(x, t)$  Probability that a person age  $x$ , will die at time  $t$   
 $\bar{x}$  Average of the age range under consideration

### 3.2 Lee-Carter Model

Lee and Carter developed a new method for extrapolating trends and age patterns in mortality. The Lee-Carter method is a valuable and appropriate way to extrapolate historical trends in the level and age distribution of mortality. It is predicated on a model of two variables: time and age. The logarithmic of the central death rate is said to have a linear relationship with the two parameters. Developed in 1992, this method is acknowledged as one of the most widely used approaches for estimating the mortality rate (Renshaw & Haberman, 2003). The mathematical expression for the Lee-Carter model is:

$$\log m(t, x) = \beta_x^1 + \beta_x^2 k_t^2 \tag{1}$$

The model has an identifiability problem in parameter estimation, which results in identical values for  $\log m(t, x)$ . This means that we cannot distinguish between the two parameterizations. To circumvent this, we need to impose two constraints on the parameters:  $\sum_t k_t^2 = 0$ ;  $\sum_x \beta_x^2 = 1$

### 3.3 Renshaw-Haberman Model

Renshaw and Haberman explored modeling the incorporation of cohort effects into the Lee-Carter methodology. It is the generalized Lee-Carter model to include a cohort effect. Mathematically, the model is expressed as:

$$\log m(t, x) = \beta_x^1 + \beta_x^2 k_t^2 + \beta_x^3 \gamma_{t-x}^3 \tag{2}$$

Similarly to the LC model, identifiability problems are avoided by imposing the following Constraints:  $\sum_t k_t^2 = 0$ ;  $\sum_x \beta_x^2 = 1$ ;  $\sum_x \beta_x^3 = 1$  and  $\sum_{x,t} \gamma_{t-x}^3 = 0$ .

### 3.4 Age-Period-Cohort Model

The APC model was initially proposed in medicine and demography (e.g., Clayton & Schifflers, 1987) and later introduced in actuarial literature by Currie (2006). Referring to the latter, the APC model involves an age, period, and cohort effect. This model introduces the simpler Age-Period-Cohort (APC) model. The structure of the model is as follows:

$$\log m(t, x) = \beta_x^1 + k_t^2 + \gamma_{t-x}^3 \tag{3}$$

Such a model requires parameters constraints defined as:  $\sum_t k_t^2 = 0$  and  $\sum_{x,t} \gamma_{t-x}^3 = 0$ . Currie (2006) uses Penalized-splines to fit  $\beta_x^1$ ,  $k_t^2$  and  $\gamma_{t-x}^3$  to ensure smoothness.

### 3.5 Cairns-Blake-Dowd Model

The model was constructed based on the supposition that cohort effects, age, and period have distinct characteristics and exhibit unpredictability within each year. The purpose of this model's creation was to account for increased age mortality. On the other hand, the Cairns, Blake, and Dowd model allows for the period impact while also assuming smoothness in age. This model is not affected by identifiability issues, hence mathematically expressed as follows:

$$\text{logit } q(t, x) = \beta_x^1 k_t^1 + \beta_x^2 k_t^2 \tag{4}$$

However, simple parametric forms were assumed for  $\beta_x^1$  and  $\beta_x^2$ :  $\beta_x^1 = 1$  and  $\beta_x^2 = (x - \bar{x})$ . Equation (4) is then structured as:

$$\text{logit } q(t, x) = k_t^1 + k_t^2 (x - \bar{x}) \tag{5}$$

### 3.6 Cairns-Blake-Dowd Model with Cohort Effect

This model is the first generalization of the CBD model to include a cohort effect. The set of cohort years of birth included in the analysis.

$$\text{logit } q(t, x) = \beta_x^1 k_t^1 + \beta_x^2 k_t^2 + \beta_x^3 \gamma_{t-x}^3 \tag{6}$$

Similar to the CBD model, simple parametric forms were assumed as follows:  $\beta_x^1 = 1$ ;  $\beta_x^2 = (x - \bar{x})$  and  $\beta_x^3 = 1$ . The model is then expressed as:

$$\text{logit } q(t, x) = k_t^1 + k_t^2 (x - \bar{x}) + \gamma_{t-x}^3 \tag{7}$$

Contrary to the CBD model, constraints have to be introduced:  $\sum_{c \in C} \gamma_c^3 = 0$  and  $\sum_{c \in C} c \gamma_c^3 = 0$



#### IV. DATA APPLICATION & FINDINGS

The Lee and Carter (1992) model introduced a stochastic framework for mortality forecasting. The model was applied to U.S. mortality data from the Human Mortality Database (HMD) and the National Center for Health Statistics (NCHS) from 1933 to 1987 using Singular Value Decomposition (SVD) to estimate the parameters (Lee & Carter, 1992). It provided accurate forecasts for mortality trends in the U.S. over time and formed the basis for many subsequent developments in the field of mortality modeling. Despite some limitations, particularly in forecasting mortality at ancient ages and ignoring cohort effects, its simplicity, interpretability, and probabilistic nature have made it a cornerstone of modern mortality forecasting.

The Renshaw and Haberman (2006) study applied the model to the Human Mortality Database (HMD), which covers various countries and national statistics offices. The primary dataset used in their original study was focused on England and Wales mortality data for males from 1961 to 2002 for ages 20 to 100. The model was fitted using a Generalized Linear Model (GLM) framework. The Renshaw and Haberman (2006) version introduced significant advancements in mortality modeling by incorporating cohort effects. Including these effects improved the accuracy of mortality forecasts, especially for elderly populations where cohort influences are most pronounced. The model's application in actuarial practice has provided more reliable estimates of life expectancy and longevity risk, particularly for pension funds and life insurers (Renshaw & Haberman 2006).

Currie (2006) used several mortality datasets from different countries, including England, Wales, and Sweden, to demonstrate the model's flexibility and applicability across various populations. The APC model is formulated using the Generalized Additive Model (GAM). The Age-Period-Cohort model introduced an essential innovation in mortality modeling through penalized regression splines. By smoothing the age, period, and cohort effects, the model provided more reliable and interpretable estimates of mortality trends and improved long-term mortality forecasts. The model's ability to capture cohort effects was precious for studying generational differences in mortality rates. The APC model remains vital in mortality forecasting, especially in actuarial and demographic applications.

The original CBD model was applied to mortality data for England and Wales male populations aged 60 to 89 from 1961 to 2002. The CBD model is typically fitted using Maximum Likelihood Estimation (MLE). The data was sourced from The Office for National Statistics (ONS). The CBD model provided an excellent fit to the observed mortality data for older ages. The model balanced simplicity and accuracy by focusing on the two most important factors affecting mortality at higher ages—overall mortality levels and the age slope. The CBD model has become particularly popular in pension funds and life insurance due to its ability to capture mortality dynamics at older ages, where longevity risk is most significant (Cairns et al., 2006).

In Cairns et al. (2007), the CBD model with a cohort effect was tested on mortality data from England and Wales for ages 60 to 89 from 1961 to 2004. Similar to the original CBD model, Maximum Likelihood Estimation (MLE) is used to estimate the parameters in the extended CBD model with the cohort effect. The Cairns-Blake-Dowd model with cohort effect enhances the original CBD model by incorporating a cohort effect, significantly improving the model's ability to capture mortality trends influenced by generational differences. The model better fit the data and provided more accurate long-term mortality forecasts by accounting for the distinct mortality experiences of different birth cohorts. This development has been precious for actuaries working with pension schemes and life insurance, as it allows for better management of longevity risk and more precise forecasting of future liabilities.

#### V. EMPIRICAL EVIDENCE FOR COHORT EFFECT MORTALITY MODELS

This section supports the idea that people born during specific periods experience different mortality patterns than other cohorts. This empirical evidence helps validate cohort effect mortality models, incorporating these differences into their predictions of mortality trends over time. This section answers the whys and the reasons for the birth cohort effect in stochastic mortality models.

##### 5.1 Incorporation of Birth Cohort Effects

A significant finding from the review is that most stochastic mortality models now incorporate birth cohort effects to account for variations in mortality rates across different generations. Cohort effects are significant for elderly populations, as individuals born during periods of hardship (e.g., World War I or the 1918 influenza pandemic) tend to exhibit higher mortality rates at older ages than those born in more prosperous periods (Li & Lee, 2005; Bamia et al., 2008).

Studies in the review generally agree that including birth cohort effects leads to more accurate and robust mortality forecasts for the elderly (Currie, 2016). For example:

Díaz and Ouburg (2019) used a cohort-extended stochastic model to analyze elderly mortality in Spain. They found that mortality improvements in post-war birth cohorts were much higher than earlier, reflecting advancements in healthcare and nutrition.

Hyndman and Ullah (2007) highlighted that neglecting cohort effects can lead to significant biases in mortality forecasts for older populations, particularly in countries like Japan and the UK, where post-war improvements in living conditions profoundly impacted mortality.

## 5.2 Importance of Birth Cohort

Empirical studies have consistently shown that birth cohort effects play a critical role in shaping mortality trends in the elderly population (Tuljapurkar et al., 2000; Cairns et al., 2008). According to Finch and Crimmins (2004), generations that experienced specific historical events, such as wars, economic crises, or medical breakthroughs, tend to exhibit distinct mortality patterns compared to other cohorts. For example, individuals that were born during the Great Depression or World War I often experience higher mortality rates in old age due to poor early-life conditions.

In actuarial applications, stochastic mortality models with cohort effects provide improved estimates of future longevity risk (Bloemen et al., 2013). For pension funds and life insurers, accurate mortality projections are essential for pricing annuities, assessing solvency, and managing longevity risk (Stoeldraijer et al., 2013; Lam & Wang, 2021). Actuaries increasingly rely on cohort-extended models to account for the variability in mortality patterns across different generations (Hyndman et al., 2017).

## VI. DISCUSSIONS

Stochastic mortality models with birth cohort effects have provided several significant findings regarding elderly populations. These findings show that birth cohort effects have greatly enhanced the ability of stochastic mortality models to forecast mortality trends, particularly for elderly populations, leading to more precise longevity estimates and better risk management strategies. Here are some key insights:

**Improved Accuracy in Longevity Forecasts:** Cohort-specific trends help capture more accurate mortality patterns, particularly for elderly populations (Finch & Crimmins, 2004). For example, models incorporating cohort effects show that individuals born during specific periods (e.g., post-World War II) experience different mortality rates than those from earlier cohorts. These models can better account for the gradual decline in mortality rates observed in elderly populations over time, enhancing projections of future longevity.

**Impact of Early-Life Conditions on Elderly Mortality:** Cohort effects highlight the long-term influence of early-life conditions (such as nutrition, healthcare access, and disease exposure) on older-age mortality. For example, cohorts born during economic hardship or widespread disease (e.g., the Great Depression, World War I, or the 1918 influenza pandemic) tend to experience higher mortality rates in later life than those born during more prosperous times (Debón et al., 2010).

**Faster Mortality Improvements for Post-War Cohorts:** Post-war cohorts, particularly in developed countries, often exhibit faster mortality improvements due to better living standards, advances in healthcare, and reduced exposure to early-life hardships (Willets, 2004). This trend is often driven by medical advancements, such as vaccinations, antibiotics, and better treatments for chronic diseases, which disproportionately benefit the elderly (Cutler et al., 2006).

**Reduced Mortality Uncertainty for Elderly Populations:** By incorporating cohort effects, stochastic models help to reduce the uncertainty in mortality forecasts for older populations (Wong-Fupuy & Haberman, 2004; Cairns et al., 2006). This allows for better risk management and more accurate pricing for life insurance, annuities, and pension plans (Nigri et al., 2019). These models are instrumental in mitigating longevity risk resulting from underestimating how long elderly individuals will live (Plat, 2009).

**Changing Patterns of Age-Related Mortality:** Stochastic models with cohort effects capture the changing patterns of age-related mortality, showing that the rate of improvement in mortality is often not uniform across ages (Booth et al., 2002). For example, while mortality at younger ages has improved significantly in recent decades, improvements for ancient populations have been more gradual. Cohort models help capture these graduations (Li & Lee, 2005).

**Effect on Pension Funds and Life Insurance:** Incorporating birth cohort effects in stochastic mortality models has provided better estimates for longevity risk, which is critical for pension funds and life insurers (Cairns et al., 2008). Accurate mortality forecasts for the elderly allow for better assessment of liabilities, annuity pricing, and more effective solvency management (Renshaw & Haberman, 2006).

## VII. CONCLUSIONS & RECOMMENDATIONS

### 7.1 Conclusion

Stochastic mortality models with cohort effects significantly improve over traditional age-period models, particularly for forecasting mortality in elderly populations (Wang et al., 2018; Wilmoth, 1993). By incorporating generational differences in mortality patterns, these models enhance the accuracy of actuarial forecasts and provide better insights for managing longevity risk. According to Plat 2011, despite challenges related to model estimation and data limitations, cohort-extended models are increasingly being adopted in actuarial practice. Further research to refine these models and address identifiability challenges will continue to advance the field and improve mortality forecasting.

### 7.2 Recommendations

The systematic review on stochastic mortality models with birth cohort effects in older people identified areas where current research and modeling practices could build up. The recommendations seek to advance academics of mortality modeling and improve the reliability and usability of these models in demographic planning for the elderly population, policymaking, and actuarial science. The recommendations are listed as follows:

**Model adaptation for the aged population:** Stochastic mortality models should be able to adapt to capture notable mortality trends for the older population accurately. Current mortality models focus basically on age, period, and cohort. Subsequent studies could consider capturing lifestyle changes, health conditions, and socioeconomic factors since they impact mortality trends in the elderly population significantly (Booth & Tickle, 2008; Currie, 2016).

**Improve model validation techniques:** The model output can be compared to the observed historical data, especially for older people, to help promote a more thorough validation of stochastic mortality models. Cross-country and cross-cohort validation can be done to show how they effectively generalize and adapt to various demographic patterns (Cairns et al., 2006).

**Real-world applications and practices:** Mortality models should be adjusted to provide practical insights for policymakers and actuaries. This involves developing simple and interpretable versions of complex stochastic models for easy application in policy and financial planning (Cairns et al., 2010a).

### Conflict of Interest

The authors declare no conflict of interest or financial interest related to this work.

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