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# Water quality index development using fuzzy logic: A case study of the Karoon River of Iran

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Determination of the status of water quality of a river or any other water source is highly indeterminate. It is necessary to have a competent model to predict the status of water quality and to show the type of water treatment that would be used to meet different demands. By exploring the behavior and limitations of conventional methods for quality evaluation, a better overall index for water quality in Iran and its application in Karoon River is proposed. Six variables are employed for the quality assessment. Numerical scales relating to the degree of quality are established for each variable to assess variations in quality and to convey findings in a comprehensive manner. The unit operates in a fuzzy logic mode including a fuzzification engine receiving a plurality of input variables on its input and being adapted to compute membership function parameters. A processor engine connected downstream of the fuzzification unit will produce fuzzy set, based on fuzzy variable namely dissolved oxygen (DO), total dissolved solids (TDS), turbidity, nitrate, fecal coliform and pH. It has a defuzzification unit which operates to translate the inference results into a discrete crisp value of water quality index. The development of the fuzzy model with one river system is explained in this paper. Water quality index in most countries is only referring to physico-chemical parameters due to great efforts needed to quantify the biological parameters. This study ensures a better method to include special parameters into water quality index due to superior capabilities of fuzzy logic in dealing with non-linear, complex and uncertain systems.

**Key words:** Water quality index, fuzzification, monitoring, inference system.

## INTRODUCTION

Water quality index (WQI) is an attempt used to give an imperfect answer to non-technical questions about water quality. It has no unit, with number ranging from 1 to 100; a higher number is indicative of better water quality.

Indexes by design contain less information than the raw data that they summarize; many uses of water quality data cannot be met with an index. An index is most useful for comparative purposes (what stations have particularly poor water quality?) and for general questions (how is the water quality of my stream?). Indexes are less suited to specific questions. Site-specific decisions should be based on an analysis of the original water quality data. In short, an index is a useful tool for “communicating water quality information to the lay public and to legislative decision makers”; it is not a complex predictive model for technical and scientific application. Landwehr (1979) points out that an index is a performance measurement that aggregates information into a usable form, which reflects the composite influence of significant physical, chemical and biological parameters of water quality

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**Abbreviations:** WQI, Water quality index; DOE-WQI, Department of Environment water quality index; CPCB-WQI, Central Pollution Control Board water quality index; TDS, total dissolved solids; NSF, National Sanitation Foundation; IEPA, Iranian Environment Protection Agency; NSF-WQI, National Sanitation Foundation water quality index; FIS, fuzzy inference system; FWQ, fuzzy water quality; DO, dissolved oxygen.

conditions. House and Newsome (1989) states that the use of a WQI allows 'good' and 'bad' water quality to be quantified by reducing a large quantity of data on a range of physico-chemical and biological variables to be a single number in a simple, objective and reproducible manner. Since 1965, when Horton (1965) proposed the first WQI, a great deal of consideration has been given to the development of index methods.

Water quality indices are computed for classification of water, wherein the integration of parametric information on water quality data and the expert's knowledge based on their importance and weights are considered. Considerable uncertainties are involved in the process of defining water quality for designated uses. One of the most effective ways to communicate information on environmental trends and river water quality in particular is with indices. The aggregation of indices that represents the integrated effect of individual concentration of water quality parameters was proposed (Brown, 1970) using Delphi technique as a tool in a formal assessment procedure. Some workers have considered different approaches, for example Prati et al. (1971) considered 13 different parameters of equal weight in their system. Values of these parameters are rated from 0 to 13 with values more than 8 denoting heavy pollution. It seems that the National Sanitation Foundations' (NSF's) work appears to be the most comprehensive that has been carried out to date and has been discussed in various papers (Brown et al., 1972; Deininger and Landwehr, 1971). Conceptually similar methods are used in other countries for defining water quality. The Catalan Water Agency (Catalonia, Spain) uses 150 chemical indicators to survey the condition of water (Agencia Catalana del Agua, 2005). There are at least two reasons why an index may fail to accurately communicate water quality information. First, most indexes are based on a pre-identified set of water quality constituents. For example, a particular station may receive a good WQI score, and yet have water quality impaired by constituents not included in the index. Second, aggregation of data may either mask (or over-emphasize) short-term water quality problems. A satisfactory WQI at a particular station does not necessarily mean that water quality was always satisfactory. A good score should, however, indicate that poor water quality (for evaluated constituents, at least) was not chronic during the period included in the index. Nikoo et al. (2010) believed that available water quality indices have some limitations such as incorporating a limited number of water quality variables and providing deterministic outputs. One of the difficult tasks facing environmental managers is how to transfer their interpretation of complex environmental data into information that is understandable and useful to technical and policy individuals as well as the general public. In modeling complex environmental problems, researchers often fail to make precise statements about input and outcomes, but fuzzy logic could help (Mckone and Ashok,

2005). Conventional water quality regulations contain quality classes which use crisp sets, and the limits between different classes have inherent imprecision (Silvert, 2000).

In Iran, the classification of rivers by the Iranian Environmental Protection Agency (IEPA) is generally based on the National Sanitation Foundation water quality index (NSFWQI). This procedure always has limitations, the greatest being that, it may be too readily accepted by all in a position to make use of it. The most critical deficiency of this index is the lack of dealing with uncertainty and subjectivity present in this complex environmental issue. In this regard, some alternative methodologies have emerged from heuristic approach. Fuzzy logic has been tested with actual environmental issues (Chen and Chang, 2001). So there is a need for developing a uniform method for measuring water pollution involving biological parameters and is recognized by the scientific community and by general public long time ago. Fuzzy logic can be considered as a language that allows one to translate sophisticated statements from natural language into a mathematical formalism. Fuzzy logic can deal with highly variable, linguistic, vague and uncertain data or knowledge and therefore has the ability to allow a logical, reliable and transparent information stream from data collection to data usage in environmental application system. A suitable environmental application of inference system based on fuzzy reasoning to integrate water quality determinants has been shown (Ocampo-Duque et al., 2006).

Fuzzy logic provides a framework to model uncertainty, the human way of thinking, reasoning and perception process. Fuzzy systems were first introduced by Zadeh (1965). Raman et al. (2009) believed that fuzzy logic concepts, if used logically, could be an effective tool for some of the environmental policy matters. Jinturkar et al. (2010) used the fuzzy logic for deciding the water quality index on the basis of which, water quality rankings are given to determine the quality of water. The fuzzy index has been shown to be effective in avoiding the loss or non-detection of information crucial for classification of water quality (Roveda et al., 2010).

## MATERIALS AND METHODS

A water quality index was developed in order to integrate the composite influence of various physical, chemical and biological parameters that were measured by the Iranian Environmental Protection Agency (IEPA) at the Karoon in November and December (1999), January and February (2000).

The NSF adopted opinions from more than 140 selected water quality experts. On the basis of questionnaires, the NSF was able to draw up a list of valid parameters which had been rated on a scale of importance. They also established the relation of water quality to values in the form of rating curves. In developing rating curves, the experts were asked to attribute values for variation in the level of water quality caused by different levels of each of the selected parameters.

In this research, we assembled a panel of 180 persons in Iran

with known expertise in water quality management. Two questionnaires were mailed or fetched to each panelist to solicit expert opinion regarding the WQI and the procedure incorporated many aspects of the Delphi method. In the first questionnaire, the panelists were asked to consider 50 analytes for possible inclusion in a WQI and to add any other analytes they felt should be included. The panelists were also asked to rate the analytes that they would include on a scale from 1 (lowest significance) to 5 (highest significance). The results from the first survey were included with the second questionnaire and the panelists were asked to review their original response. The purpose of the second questionnaire was to obtain a closer consensus on the significance of each analyte. For the second questionnaire, the panelists were asked to list not more than 6 most important analytes for inclusion from the new total of 35. From these first two responses, six analytes were derived for inclusion in the WQI. Obviously, giving 35 parameters involved in the formulation of water quality index is conflicting with the main goal of this project because the aim of this project is selected to limit the number of variables entering the water quality index formula. For this purpose, a table including 35 variables in the six groups varied physical, chemical, biological, inorganic compounds, trace elements and organic elements to get expert's opinions which were designed. At this stage, experts were requested to choose only one of the most important variables from each group. Based on 60 expert opinions received, the number of five variables, including turbidity, total dissolved solids (TDS), pH, dissolved oxygen (DO) and total coliform was chosen as main parameters.

Nitrate and suspended solid variables were considered as parameters that could be used in the calibration process of index formulation. Finally, the nitrate component was used as variable involved in the formulation, and suspended solid was removed.

In this study, the fuzzy logic formalism has been used to assess river water quality. The fuzzy inference process involves three crucial steps: membership functions, fuzzy set operations and inference rules. Comparison has been done over conventional methods such as NSF, Oregon, Malaysian Department of Environment water quality index (MDOE-WQI) and Central Pollution Control Board water quality index (CPCB-WQI). River water quality data from Karoon River are used to evaluate the fuzzy model. Membership functions of the determinants and fuzzy rule bases were defined. The model was evaluated with data from 1999 of Karoon River basin based on Mamdani fuzzy inference system. Fuzzy model was developed with physico-chemical determinants of significance ratings curve to evaluate the Karoon River water quality.

A membership function is a curve that defines how each point in the input space is mapped to a membership value between 0 and 1. The input space is called the universe of discourse. The output space is called the membership value  $\mu$ .  $\mu_A(x)$  is the membership function of  $x$  in  $A$ . A membership function is an arbitrary curve whose shape is defined by convenience. The standard fuzzy set operations are: union (OR), intersection (AND) and additive complement (NOT).

Fuzzy logic is the flexible tool to develop classification model with a simple framework and constructed with natural language. In this study, water quality index value was obtained to express the classification of river in order to make water quality assessment more understandable, especially in public consideration. It has been demonstrated that computing with linguistic terms within fuzzy inference system (FIS) improves the tolerance for imprecise data. A fuzzy model for river water quality assessment has been developed. Different shapes of membership functions can be used, depending upon the type of application (Pedrycz and Card, 1992). The right prediction of the fuzzy model depends on the number of fuzzy sets used in the mapping process, since it facilitates to give more continuity to the universe of discourse. However, in this research, each of the six input quality determinants has been

divided into five categories, and the Gaussian curve membership function. The membership functions were assigned as shown in Figure 1. Gaussian curve membership functions were used and the parameters are given in Table 1, and linguistic classification of water quality index is shown in Table 2. Five fuzzy sets have been considered to be suitable for this study. The amount of overlap, the width and the shape of fuzzy sets should be considered by an expert for each input variable. Ranges for fuzzy sets were based on interim national quality standards for Iran. Iranian rivers are classified in the interim national water quality standards based on water quality criteria and standards for several beneficial uses.

Ranges of fuzzy sets used are shown in Table 1. Six quality determinants have been selected to evaluate water quality by means of an aggregated index called fuzzy water quality (FWQ) index. Defuzzification of output is achieved by centroidal method as it is the most prevalent and physically appealing to all available methods. For the selected set of six water quality determinants, the most prominent 58 rules have been used. For example, fuzzy rules are chosen, "if the levels of organic matter in a river are very low, and the levels of dissolved oxygen are very high, then the expected water quality is very good. In fuzzy description, it could be pronounced as follows:

Rule 1: If TDS is good, and DO is good then WQI is good. In the same way, other rules can be enunciated. Robustness of the system depends on the number and quality of the rules.

### Study area

The largest river by discharge in Iran, the Karoon River's watershed, covers 65,230 km<sup>2</sup>. The river is about 950 km long and has an average discharge of 575 m<sup>3</sup>/s (20,300 ft<sup>3</sup>/s). The largest city along the course of the river is Ahvaz, with over 1.3 million inhabitants. Other important cities include Shushtar, Khorramshahr (a port), Masjed-Soleyman and Izeh. The Karoon continues towards the Persian Gulf, forking into two primary branches on its delta: the Bahmanshir and the Haffar that joins the Shatt al-Arab (Arvand Rud in Persian), discharging into the Persian Gulf (Figure 2). It originates in the Zagros Mountains of western Iran, on the slopes of 4,548 m (14,921 ft) Zard-Kuh.

## RESULTS AND DISCUSSION

The water quality for the Karoon River at the Shatit Station has been assessed with the FWQ index. Data sets (1999 to 2000) from Iranian Environment Protection Agency of Iran which were used to assess water quality were shown in Table 3. The calculated FWQ indices according to FIS are given in Table 4. On the other hand, comparison has been done between FWQ and the existing WQ indices such as National Sanitation Foundation water quality index (NSFWQI), Oregon, Malaysian Department of Environment water quality index (DOE-WQI), CPCB-WQI, Kaurish and Younos, and Ahmed Saied. In the fuzzy model, DO, TDS and turbidity shows high acceptability and are mainly affected by total dissolved solid values and fecal coliform. The sampling station exhibit an acceptable water quality in different months.

The IEPA-WQS rating scale considers dissolved oxygen levels from 88 to 112 percent of saturation as excellent and less than 20% and more than 200% saturations as poor. The dissolved oxygen in the Karoon

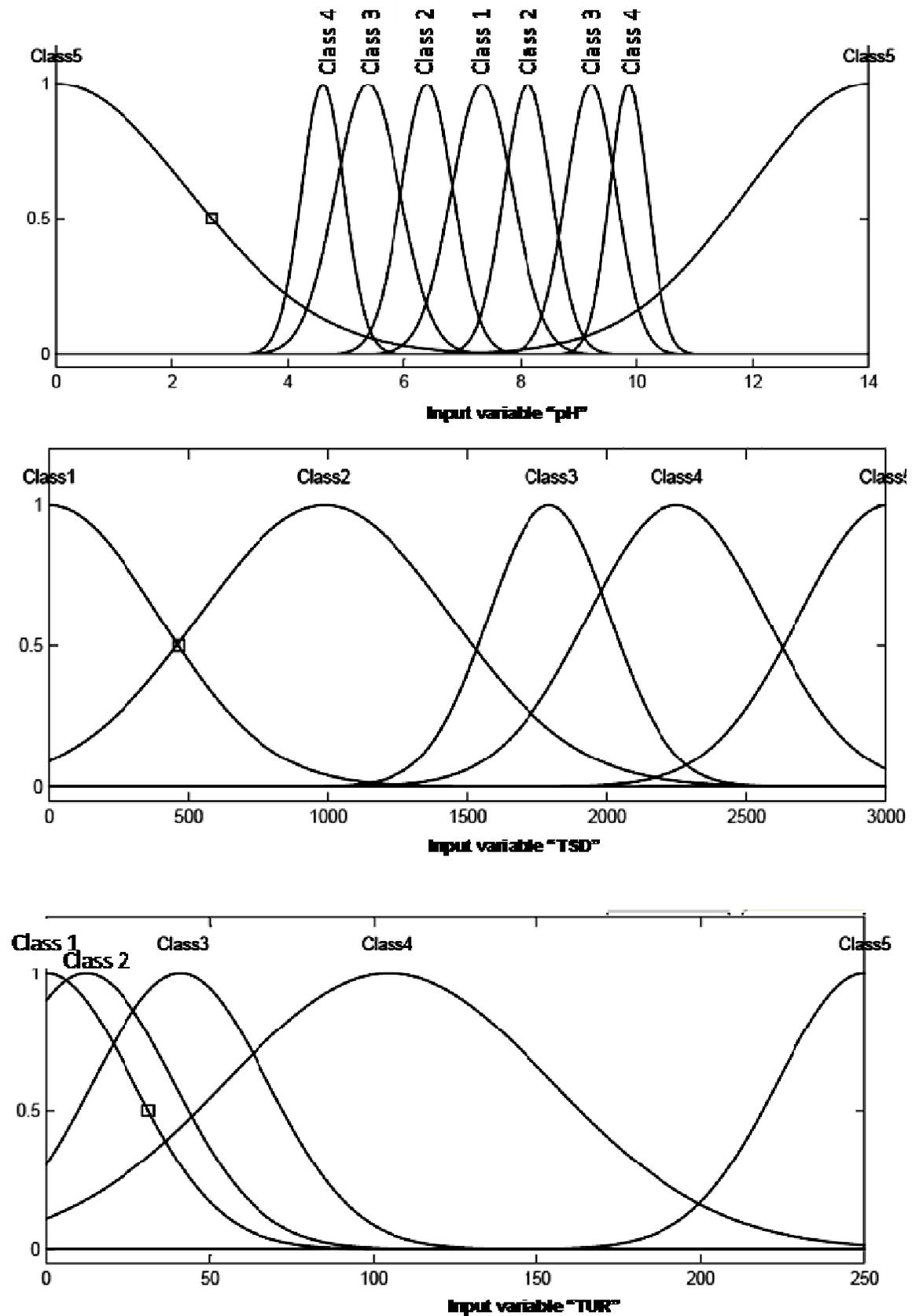


Figure 1. Membership functions defined for water quality classification.

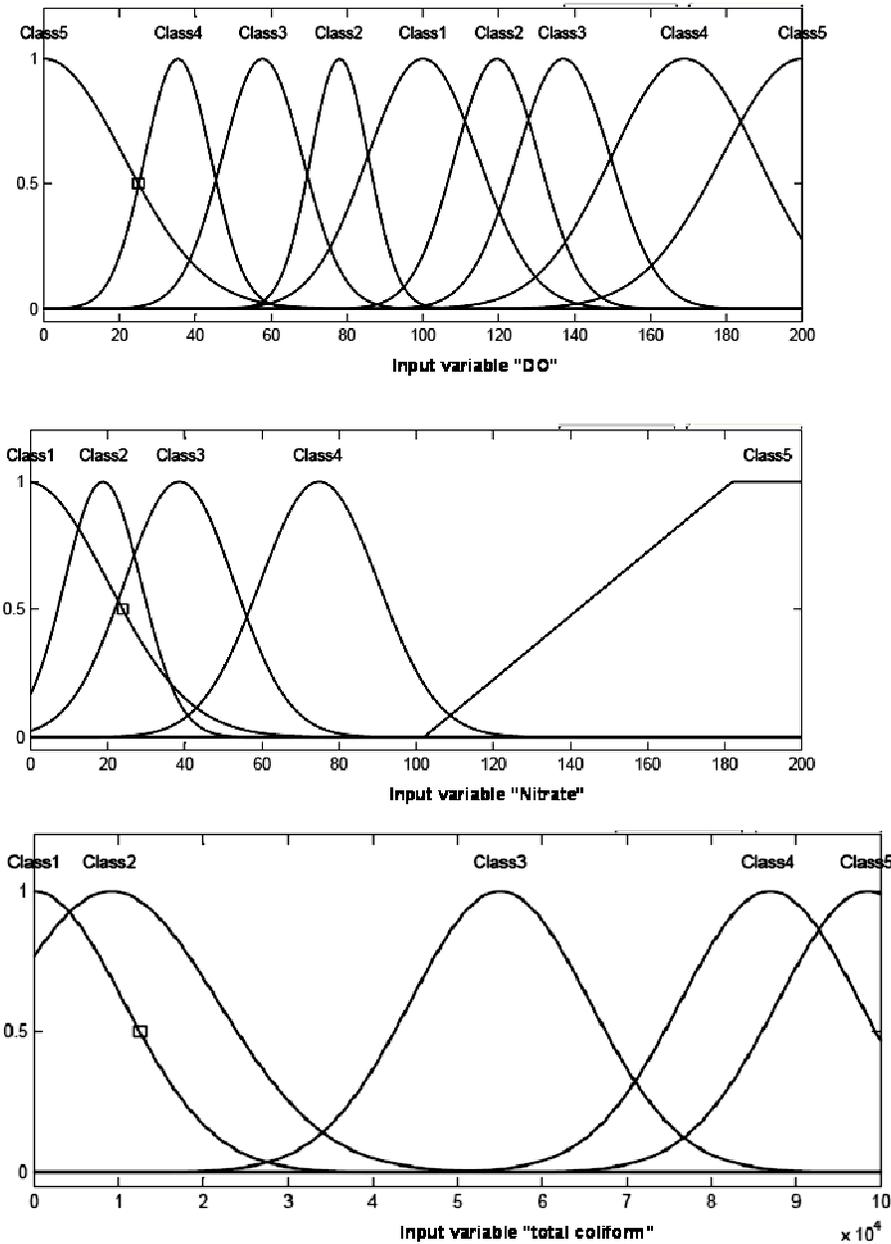


Figure 1. Continued.

River is very good based on fuzzy model results. Overall DO in the Karoon River is acceptable on the bases of fuzzy results.

Total dissolved solids are a measure of materials that are dissolved in the water. Karoon River monitoring indicates that TDS is in a good range in November and December but is marginal to poor range in January and February. Most TDS in the Karoon River originates from sediment runoff and bank erosion and makes the water look muddy. The IEPA-WQS rating scale designates 0 to 500 mg/L TDS as low (rated as excellent), 500 to 1500 as medium (rated as good) and 1500 to 2100 as slightly high

(rated as fair), and 2100 to 3000 as high (rated as marginal), and more than 3000 as very high (rated as poor). Based on adequate fuzzy model results, the TDS levels in Karoon River are in the fair range. Fishes live the best in waters with a pH between 6.5 and 8.4 (Raman et al., 2009). Fish are harmed if pH becomes too acidic (falls below 4.8) or too alkaline (goes above 9.2). The IEPA-WQS rating scale designates a pH between 6.5 and 7.5 as excellent, between 6.0 and 6.5 and between 7.5 and 8 as good, between 5.0 and 6.0 and between 8.0 and 9.0 as fair, between 4.5 and 5.0 and between 9.0 and 9.5 as marginal, and below 4.5 and above 9.5 as

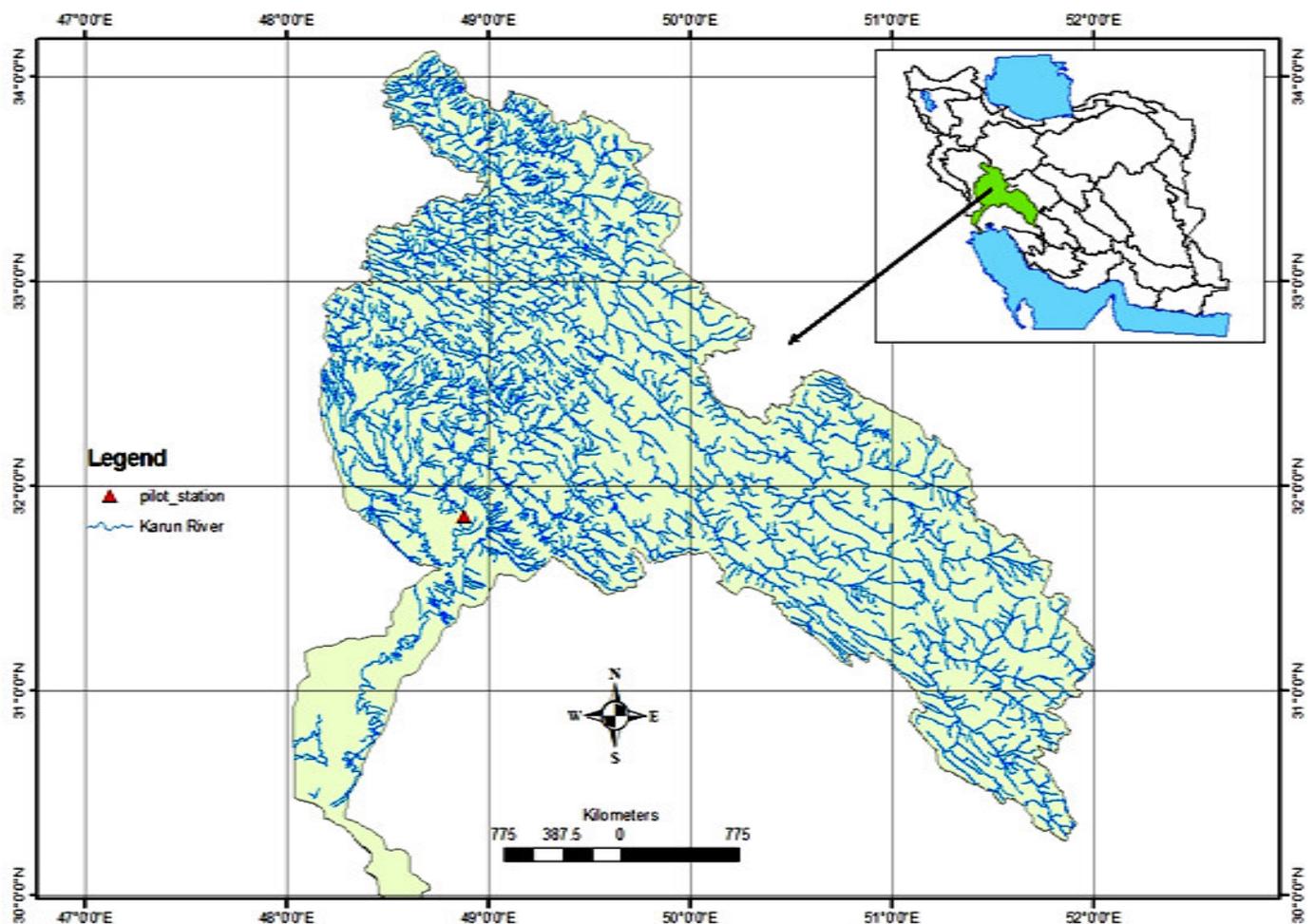
**Table 1.** Water quality classification based on IEPA.

Determinant	Class I	Class II	Class III	Class IV	Class V
DO (%)	88–112	75–125	50–150	20–200	<20 and >200
Turbidity(NTU)	5	10	20	250	>250
Nitrate (ppm)	11	22	45	90	>200
Total coli (MPN/100 ml)	100	5000	50000	100000	>100000
Ph	6.5–7.5	6.0–6.5 and 7.5–8.0	5.0–6.0 and 8.0–9.0	4.5–5.0 and 9.0–9.5	<4.5 and >9.5
TDS (ppm)	500	1500	2100	3000	>3000

DO, Dissolved oxygen; TDS, total dissolved solids.

**Table 2.** Water quality index and linguistic classification.

Index number	Class	Linguistic classification
0-25	Class I	Excellent
25-49	Class II	Good
50-74	Class III	Fair
75-94	Class IV	Marginal
95-100	Class V	Poor



**Figure 2.** Map of Karoon Basin.

**Table 3.** Water quality data set.

Parameter	Unit	NO1	NO2	NO3	NO4
COD	ppm	18	21	12	0
hardness CaCO <sub>3</sub>	ppm	397.5	353.5	340.5	316.5
SiO <sub>2</sub>	ppm	2.88	3.84	3.95	4.13
Organic nitrogen	ppm	3.27	0.54	0.77	
Total nitrogen	ppm	4.96	1.17	2.05	1.12
Fe	ppm	0.062	0.017	0.034	0.03
SO <sub>4</sub>	ppm	248.2	176.2	266.4	112.8
NH <sub>3</sub>	ppm	0.4	2.01	1.01	0.34
Cl	ppm	251	340.8	266.6	377.01
Nitrate	ppm	6	2.98	1.73	3.14
Organic Phosphate	ppm	0.031	0.021	0.029	0.14
Total Phosphorus	ppm	0.058	0.025	0.03	1.67
TSS	ppm	1426	88	91	66
Turbidity	NTU	18.52	43	20	6
pH	-	8.5	8.2	8.3	8.1
EC	μS/cm	1426	1620	1698	1730
Temperature	°C	14	13	18	10
BOD5	ppm	3.2	3.5	3.6	4.4
DO	ppm	8	9.8	8.4	8.6
DO%	%	75	91	86	91
FECAL	MPN/100 ml	20000	11800	21200	22000
TDS	ppm	912.64	1036.8	1086.72	1107.2
Date	-	Nov-99	Dec-99	Jan-00	Feb-00

TSS, Total suspended solids; BOD5, biochemical oxygen demand; DO, dissolved oxygen; EC, electrical conductivity; TDS, total dissolved solid; COD, chemical oxygen demand; Nov 99, November 1999; Dec 99, December 1999; Feb 00, February 2000; Jan 00, January 2000; NO, number.

**Table 4.** Comparison of FWQI and other WQIs results.

Code	NSF	Oregon	CPCB WQI	MDOE WQI	Kaurish and Younos	Ahmed Saied	FWQI
NO1	58 (Medium)	13.98 (Very poor)	60.30 (Medium to good)	73.91 Fair (Class III)	5.25 Good (Class 5)	1.37 (Medium)	37.1 (Good)
NO2	64 (Medium)	18.99 (Very poor)	58.28 (Medium to good)	67.72 Fair (Class III)	5.0 Good (Class V)	1.96 (Medium)	33.8 (Good)
NO3	66 (Medium)	16.10 (Very poor)	61.43 (Medium to good)	64.39 Good (Class II)	5.5 Excellent (Class 6)	2.09 (Good)	34 (Good)
NO4	58 (Medium)	16.09 (Very poor)	53.56 (Medium to good)	66.89 Fair (Class III)	5.0 Good (Class V)	1.762 (Medium)	31.3 (Good)

NO, number.

poor. The pH of the Karoon River is stable based on fuzzy results (due in parts to the dissolved carbonate minerals in the water). The IEPA rating scale designates 0 to 11 mg/L nitrate as very low (rated as excellent), 11 to 22 mg/L as low (rated as good), and 22 to 45 as medium (rated as fair), 45 to 90 mg/L as high (rated as marginal), and over 90 as very high (rated as poor). Based on fuzz

results, the nitrate in Karoon River is excellent.

If sufficient DO is present, ammonia can easily be broken down by nitrifying bacteria to form nitrite and nitrate. An analysis of variance over the FWQ results has shown that there are significant differences between years assessed. It indicates that policies to diminish pollution are not giving optimistic results. Fuzzy model

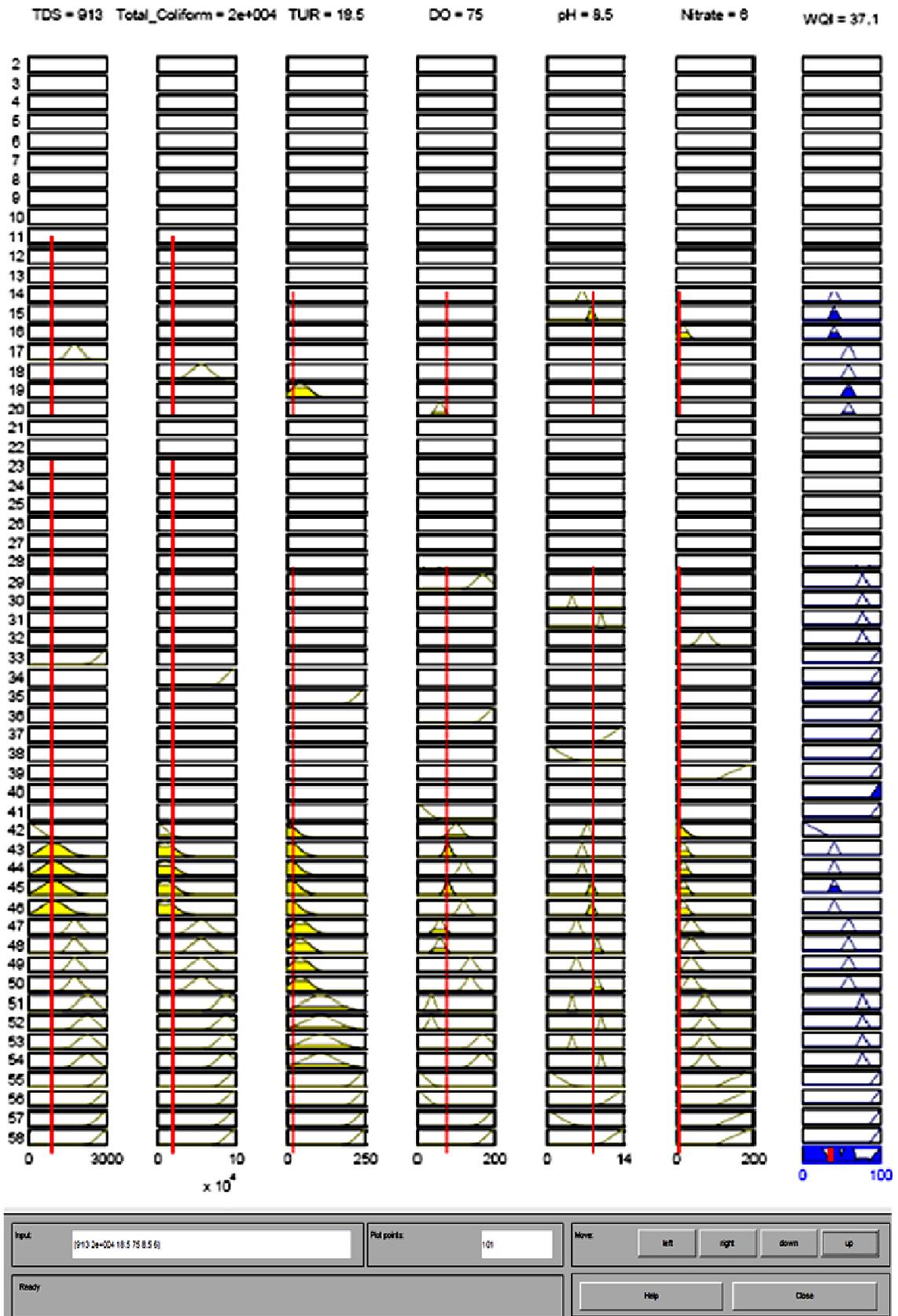


Figure 3. Fuzzy result for NO1 sample.

has been validated with four month independent sets of data.

The validation results have been shown in Table 4, the FWQ index is compared with observed WQI, which is used by IEPA for river water classification. FWQ results have shown status of water quality in Karoon River as good generally. FWQ outputs better agree with the real condition reported by IEPA-WQI. The results are in perfect agreement with observed values of 90%. Further, an expert survey was conducted by selecting most prominent rules out of derived fuzzy rules of the model.

This is accumulated from 60 respondents involved in the field of study namely, graduate researchers of higher learning institutes, IEPA and its local offices in the state. The results are also in good agreement with the expert survey for about 75%, indicating that human also make mistake sometime. They may forget and give wrong responses in expert survey. If the number of respondents is more, there is a possibility to get more accurate fit with fuzzy model. The developed fuzzy model determines the water quality index (FWQI). With FWQI, one can get guidance for the type of treatment for which the water has to be subjected to. Some of the criteria are derived through this research and are presented in Table 1.

FWQ is not objective at discussing the changes of the strength of a single pollutant or the alternation of a physical parameter. It is used as an estimator of the status of water quality generated by physico-chemical determinants. Fuzzy model has been validated with the two years independent sets of data. The validation results are shown in Table 4. FWQ results give a water condition which is good in the Karoon River. The FWQ outputs agree with the real condition as reported by IEPA-WQI. Figure 3 shows fuzzy result for NO1 sample.

## Conclusions

This study comprised the evolution of a new index called the Fuzzy water quality index. It provides a simple representation of the extensive and complex variables (physical, biological and chemical) that govern the overall quality of surface water that is intended for potable use. Based on expert opinions and national experiences, six water quality parameters including DO, turbidity, pH, TDS, nitrate, and fecal coli form were considered as the significant indicator parameters of FWQI to assess the quality of surface water sources. The application of the new index was demonstrated at a sampling station on Karoon River in Iran, based on observed water quality data.

Fuzzy model has demonstrated that water quality has high sustainability with the expected results in the Karoon River. The new index is believed to assist decision makers in reporting the condition of water quality and investigation of spatial and temporal changes in the river. The authors believe that fuzzy logic concepts, if used

logically, could be an effective tool for some of the environmental policy matters. Model based on FIS can be used for future determination of WQI for six parameters. More stringent methodologies are then required to molt the ideas of decision maker and manager to apply fuzzy model in practice. This new index is believed to assist decision makers in reporting the state of the water quality, and investigation of spatial and temporal changes. In addition, it is useful to determine the level of acceptability for the individual parameter by referring to the concentration ranges defined in the proposed classification scheme.

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