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Full Length Research Paper

Effects of microwave heating on the thermal states of biological tissues

Nabil T. M. El-dabe, Mona A. A. Mohamed and Asma. F. El-Sayed*

Math. Department, Faculty of Education, Ain Shams University ,Heliopolis, Cairo, Egypt.

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A mathematical analysis of microwave heating equations in one-dimensional multi-layer model has been discussed. Maxwell's equations and transient bioheat transfer equation were numerically calculated by using finite difference method to predict the effects of thermal physical properties on the transient temperature of biological tissues. This prediction of the temperature evolution in biological bodies can be used as an effective tool for thermal diagnostics in medical practices.

Key words: Microwave heating, Maxwell's equations, bioheat, multi layer.

INTRODUCTION

The use of microwave radiation for heating is now common in many industrial situations such as smelting, sintering and drying, it also has many applications in joining mathematical and medical fields especially in clinical cancer therapy hyperthermia (Deuflhard and Seebass, 1998; Hill and Pincombe, 1992). Body surface temperature is controlled by the blood circulation under neath the skin, local metabolism and heat exchange between the skin and its environment. Changes in any of these parameters can induce variation of temperature and heat flux at the skin surface, reflecting the physiological state of human body. For example, highly vascurized skin tumor can lead to an increase of blood flow and thus to an increase in skin temperature. In burn injures low skin temperature can occur due to insufficient blood supply. Apparently the abnormal temperature or heat flux at the skin surface might indicate irregular peripheral circulation which can be used in clinical diagnosis (Liu and Xu, 2000). Effects of thermal properties and geometrical dimensions on the skin burn injures has been discussed by Jiang et al. (2002). Ng and Chua (2002) proposed a comparison of one and two-dimensional programmes for predicting the state of skin burns. Lu et al. (1998) studied the simulation of the thermal wave propagation in biological tissues by dual reciprocity boundary element method.

Preliminary survey on the mechanism of the wave-like behaviors of heat transfer in living tissues has been discussed by Liu (2000) who introduced a new concept of multi-mode energy coupling, a phenomenological thermal wave model of bioheat transfer. Marchant and Liu (2001) considered the steady-state microwave heating of a finite one-dimensional slab. The temperature dependency of the electrical conductivity and thermal absorptivity were assumed to be governed by the Arrhenius law, while both the electrical permittivity and permeability were assumed constant. The microwave heating of three-dimensional blocks with a transverse magnetic waveguide mode in a long rectangular waveguide is considered by Lui and Marchant (2002).

*Corresponding author. E-mail: sky_12@masrawy.com.

The aim of our paper is to show the effect of microwave heating equations on the thermal states of biological tissues and to predict the effects of the thermal physical properties on the transient temperature of tissues and damage function. This research can be very beneficial in widening the idea of clinical thermal technology and thermal medical practices.

Nomenclature

ρ density of tissue
c_p specific heat of tissue
t time
c_b specific heat of blood
E electric field
x space coordinate
H magnetic field
T tissue temperature
T_a artery temperature
T_b blood temperature
T_c core temperature
T_w wall temperature
L distance from skin surface to body core
m positive integer number
ρ_b density of the blood
ω_b blood perfusion rate
κ thermal conductivity of tissue
Q body heating coefficient
μ_e magnetic permeability
ε electric permittivity
σ electrical conductivity
H_0 is the magnetic field in the free space upon the tissue
E_0 is the electric field in the free space upon the tissue
P_r Prandtl number, $P_r = \frac{\mu c_p}{\kappa}$
ν kinematic viscosity, $\nu = \frac{\mu}{\rho}$
μ viscosity of the tissue

MATHEMATICAL MODELS

The diagram of one dimensional multi-layer tissue model is illustrated in Figure 1. The temperature, electric and magnetic fields for microwave heating are given respectively by

$$T = T\,(x,t), \qquad \vec{E} = (0,\ E\,(x,t),\ 0\,), \qquad \vec{H} = (0,\ 0, H(x,t)), \qquad (1)$$

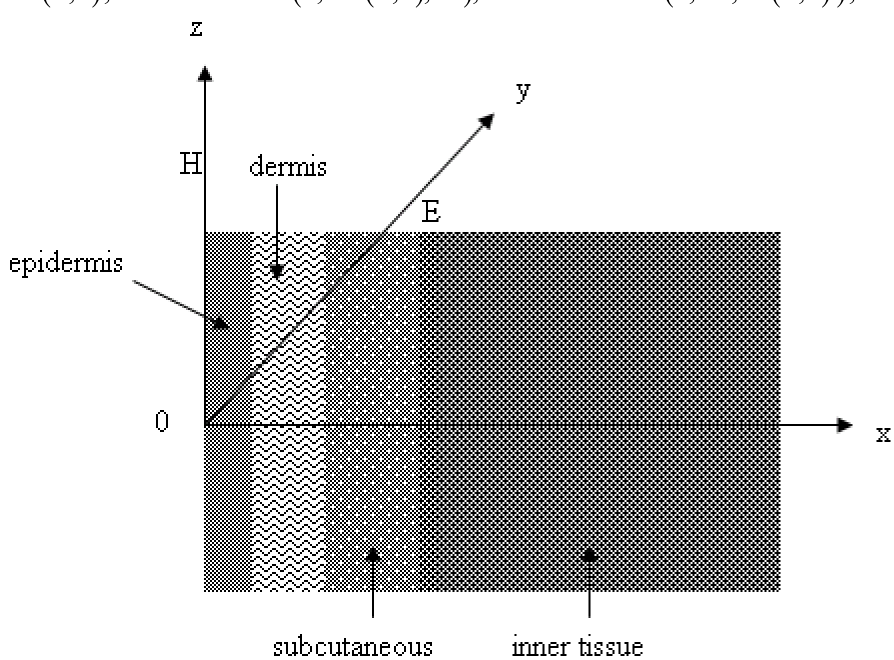


Figure 1. Schematic diagram of multi layer tissue.

The heat source arising from microwaves is proportional to the square of the modulus of the electric field intensity (Hill and Pincombe, 1992), hence the Maxwell's equations coupled with bioheat equation can be written as:

$$\frac{\partial H}{\partial x} + \varepsilon \frac{\partial E}{\partial t} + \sigma E = 0, \tag{2}$$

$$\frac{\partial E}{\partial x} + \mu_e \frac{\partial H}{\partial t} = 0, \tag{3}$$

$$\rho\, c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right) + \omega_b\, \rho_b\, c_b \left(T_b - T\right) + Q(T)\left|E\right|^2, \tag{4}$$

where the analysis assumes that no interfacial resistance exists between heating source and skin. Therefore the surface temperature remains constant during exposure. The tissue assumed to be homogeneous within each layer. ω_b is the mass flow rate of blood flow per unite volume (blood perfusion), thermal conductivity and heat capacity are assumed to be constants. The term in the left of eq. (4) is the energy storage in the tissue, the first term on the right caused by temperature gradient, the second term describes the heat transport between the tissue and microcirculatory blood perfusion and the term $\left|E\right|^2$ is the electric power density deposited in tissue. The following boundary and initial conditions were required for the solution of partial differential equations (2-4)

$$\left.\begin{array}{lll} T(x,0) = \dfrac{T_c\, x}{L}, & T(L,t) = T_c, & T(0,t) = 0, \\ E(x,0) = \dfrac{E_0\, x}{L}, & E(L,t) = E_0, & E(0,t) = 0, \\ H(x,0) = \dfrac{H_0\, x}{L}, & H(L,t) = H_0, & H(0,t) = 0, \end{array}\right\} \tag{5}$$

Marchant and Liu (2001) presents experimental evidence which indicates that the physical properties of material have power law dependence on temperature, so we can assume that the body heating coefficient has the form,

$$Q(T) = T^m, \tag{6}$$

Let us introduce the following non-dimensional variables as

$$\left.\begin{array}{l} t^* = \dfrac{t\, \nu}{L^2}, \quad x^* = \dfrac{x}{L}, \quad T^* = \dfrac{T}{T_b}, c_1 = \dfrac{c_b}{c_p}, \lambda = \dfrac{L^2\, T_b^{m-1} \left|E_0^2\right|}{\nu\, \rho\, c_p} \\ E^* = \dfrac{E}{E_0}, \quad H^* = \dfrac{H}{H_0}, \ \rho_1 = \dfrac{\rho_b}{\rho}, \omega_1 = \dfrac{\omega_b\, L^2}{\nu}, \ \lambda_1 = \dfrac{\nu\, \varepsilon E_0}{L\, H_0} \\ \lambda_2 = \dfrac{L\, \sigma\, E_0}{H_0}, \ \lambda_3 = \dfrac{\mu_e\, H_0\, \nu}{L\, E_0}, \end{array}\right\} \tag{7}$$

The dimensionless Maxwell's and energy equations, after dropping stare mark can be written as

$$\frac{\partial H}{\partial x} + \lambda_1 \frac{\partial E}{\partial t} + \lambda_2 \ E = 0, \tag{8}$$

$$\frac{\partial E}{\partial x} + \lambda_3 \frac{\partial H}{\partial t} = 0, \tag{9}$$

$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial x^2} + \omega_1 \rho_1 c_1 (1 - T) + \lambda |E|^2 T^m , \tag{10}$$

Subjected to the following non-dimensional initial and boundary conditions

$$\left.\begin{array}{lll} T(x,0) = \dfrac{T_c\, x}{T_b}, & T(1,0) = \dfrac{T_c}{T_b}, & T(0,t) = 0, \\ E(x,0) = x, & E(1,t) = 1, & E(0,t) = 0, \\ H(x,0) = x, & H(1,t) = 1, & H(0,t) = 0, \end{array}\right\} \tag{11}$$

Hill and Pincombe (1992) represented a similarity solution to solve the microwave heating equation (10) without studying the effect of convection due to the blood flow, he assumed that the electrical field decays exponentially with distance $|E|^2 = E_0^2\, e^{-k\,x}$ for certain constants E_0 and k. In our paper we will solve Maxwell's equations and transient bioheat transfer equation numerically and we will assume for simplicity $m = 1$.

Numerical technique

A one dimensional finite difference method is used to solve Maxwell's equations together with the bioheat transient equation which describe the skin burn process resulting from the application of a high temperature heat source to a skin surface. The space steps and time steps are small enough to ensure that the transient temperature were mesh independent. A finite difference scheme of Crank-Nicolson was applied to system of the partial differential equations $(8) - (10)$ to yield the following finite difference equations

$$\frac{H_{i+1,j} - H_{i-1,j}}{2\,\Delta x} + \lambda_1 \frac{E_{i,j+1} - E_{i,j}}{\Delta t} + \lambda_2\, E_{i,j} = 0, \tag{11}$$

$$\frac{E_{i+1,j} - E_{i-1,j}}{2\,\Delta x} + \lambda_3 \frac{H_{i,j+1} - H_{i,j}}{\Delta t} = 0, \tag{12}$$

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = \frac{1}{P_r}\left(\frac{T_{i+1,j} - 2\,T_{i,j} + T_{i-1,j}}{(\Delta x)^2}\right) + \omega_1 \rho_1 c_1 (1 - T_{i,j}) + \lambda |E_{i,j}|^2\, T_{i,j}, \tag{13}$$

where the appropriate mesh sides $\Delta x = .01$ and time steps $\Delta t = .04$ are considered for calculations $H_{i,j} = H(i\,\Delta x, j\,\Delta t)$, $E_{i,j} = E(i\,\Delta x, j\,\Delta t)$, $T_{i,j} = T(i\,\Delta x, j\,\Delta t)$, i and j denote the grid functions which approximate the exact solutions of $H(x,t)$, $E(x,t)$ and $T(x,t)$.

The thermal dose calculation

The thermal dose at $43^0 C$ defined by Sapareto and Dewey (1984) can describe the extent of thermal damage or destruction of tissue, and its expression is used as following:

$$\Omega = \int R^{(T-43)} \, dt \,, \tag{14}$$

where $R = 2$ is defined for $T \geq 43^0\ C$ and $R = 4$ for $37^0\ C < T < 43^0\ C$, numerical integration formula of eq. (14) can be written as (Chapra and Canale, 2002).

$$\int_{t_j}^{t_{j+1}} R^{(T(x,t)-43)} \, dt = \left(\frac{R^{T_{i,j}-43} + R^{T_{i+1,j+1}-43}}{2} \right) j \, \Delta t \quad , \tag{15}$$

RESULTS AND DISCUSSION

A one dimensional multi-layer transient finite difference model for predicting temperatures in living tissues such as prostate tissue undergoing microwave heating is presented. The microwave energy can be transmitted into the body from applicator on the skin tissue. For example in the case of prostate tissue, microwave energy is emitted from the transurethral catheter. Pennes perfusion term is assumed to predict the effect of perfusion on heat transfer.

The thickness of the tissue differs from different people and different location.

The transient temperature for different thickness is shown in Figure 2. The temperature distribution decreases with the increasing of tissue thickness L. In Figure 3 the temperature distribution of tissue decreases as time t increases. The effect of variation of blood perfusion ω_b on the tissue is presented in Figure 4. It is clear the transient temperature increases with increasing of blood perfusion ω_b.

Figure 5 illustrates the effect of three different values of thermal conductivity of tissue κ on the transient temperature. It is shown that the temperature distribution decreases with increasing of thermal conductivity κ of epidermis and dermis and it is increases with the increasing of thermal conductivity of inner tissue. Figure 6 shows the relation between the electric field in the free space E_0 and the temperature distribution. It is seen that the temperature distribution increases as E_0 increases. Figure 7 shows the relation between the temperature distribution and time for different thermal conductivity κ of tissue. The temperature decreases with the increasing of κ. Figures 8 and 9 illustrates the relation between thickness of skin x and thermal dose Ω, which describes the extent of thermal damage or destruction of tissue in living bodies. It 's clear that thermal dose distribution remains constant at epidermis and dermis and through its transient from outer tissue to inner

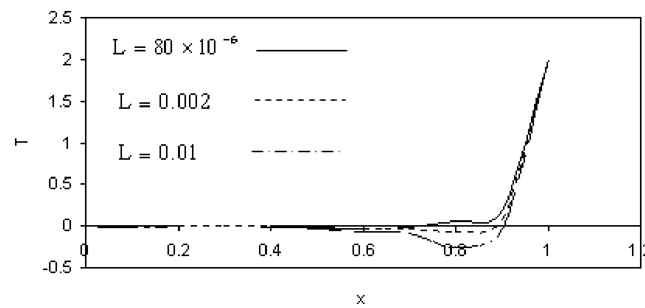


Figure 2. The temperature distribution plotted against space coordinates for different thickness of tissue skin, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \ \rho_b = 1060$.

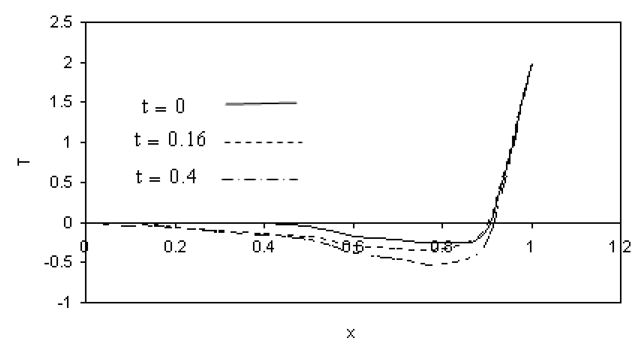


Figure 3. The temperature distribution plotted against space coordinates for different time steps, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \ \rho_b = 1060$.

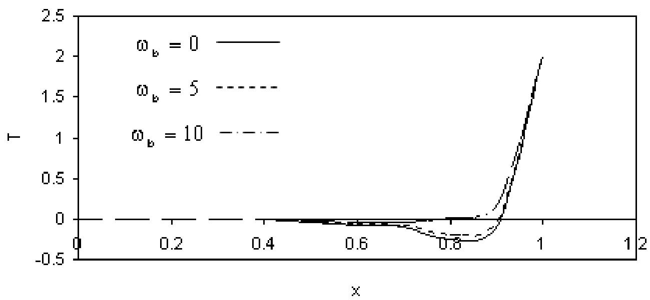


Figure 4. The temperature distribution plotted against space coordinates for blood perfusion rate, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \rho_b = 1060.$

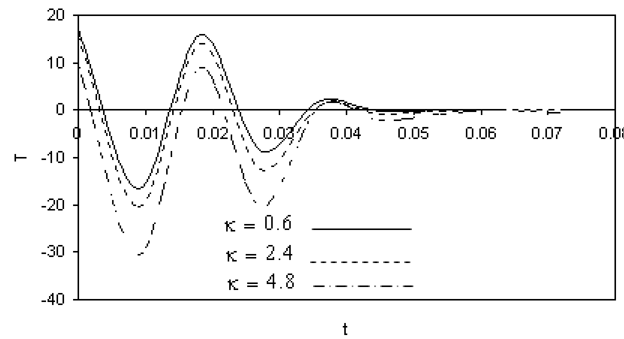


Figure 7. The temperature distribution plotted against time coordinates for thermal conductivity of tissue, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \rho_b = 1060.$

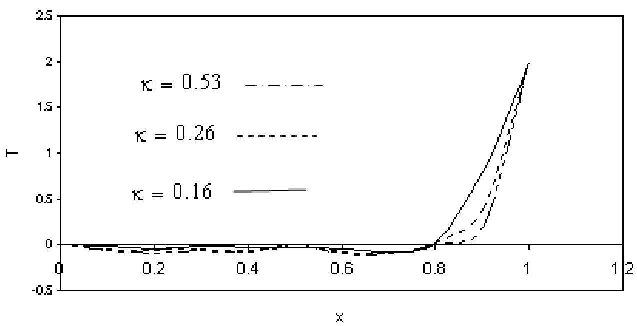
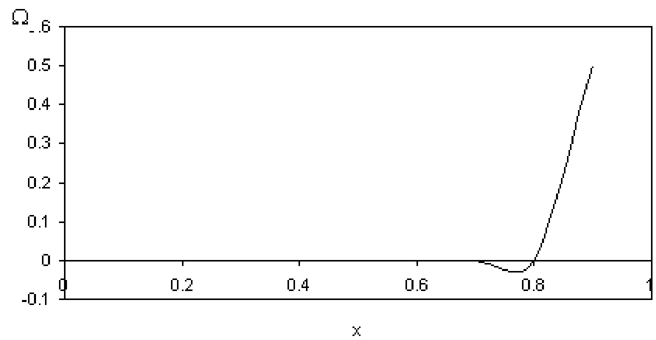


Figure 5. The temperature distribution plotted against space coordinates for thermal conductivity of tissue, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \rho_b = 1060.$



Figures 8 The thermal dose distribution for different thickness of tissue and time steps, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \rho_b = 1060.$

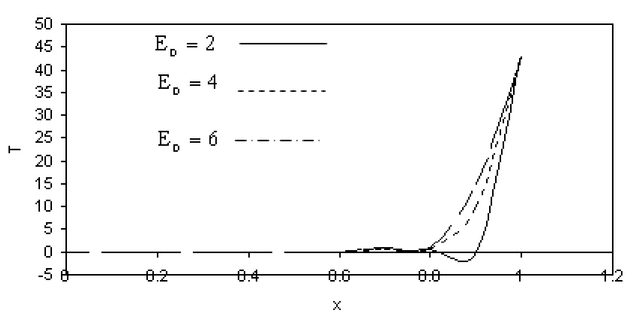
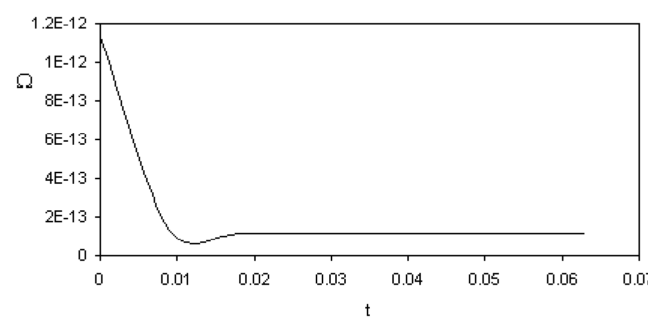


Figure 6. The temperature distribution plotted against space coordinates for different electric field in the free space upon the tissue, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \rho_b = 1060.$



Figures 9. The thermal dose distribution for different time steps, where $c_b = 3770, c_p = 3590, \omega_b = 0.00125, \kappa = 0.24, \rho = 1050, E_0 = 2, \rho_b = 1060.$

tissue this thermal dose damping and then increases at inner tissue. Figure 8 illustrates the thermal dose distribution Ω with time t, it is found that the thermal dose decreases as time t increases.

Finite difference method has been used to solve the system of partial differential equations which describe Maxwell's equations and bioheat equation in order to show the effect of microwave heating equations on the thermal states of biological tissues. The behaviors of temperature distributions for different thermal properties of tissue have been illustrated. The thermal dose distribution has been plotted for different thickness of tissue and time steps. The applications of this problem have a great benefit in various branches of science especially in cancer therapy using microwave devices such as the thermocouple.

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