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AN IMPROVED MATHEMATICAL REPRESENTATION OF MOHR'S FAILURE CRITERION FOR BRITTLE MATERIALS

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ABSTRACT

Purpose: This paper addresses issues bearing on accuracy neglected by earlier failure theories such as Rankine's and Mohr's. The overall aim is thus to present a thorough analysis of Mohr's failure criterion and offer an improved model.

Design/Methodology/Approach: The foundation of the methodology is Mohr's criterion for predicting the failure of brittle isotropic homogeneous materials, built on the foundation of test results from three simple cases namely, pure tension, pure compression, and pure torsion. Thus the methodology involves first carrying out a critical analysis of Mohr's model, followed by encapsulation of Mohr's three simple monolithic cases in one generic equation of a circle whose parameters can be varied to match specific principal loading conditions more correctly. Experimental data are then used to validate the improved model.

Findings: The work's output is a material evaluation procedure that consists of a set of simple mathematical tests, any one of which predicting failure first, would then indicate the overall failure of the structural component under investigation. Results show clearly that this approach, i.e. using one parametric generic equation to represent material strength, is not only feasible but also robust. It offers an accurate method for predicting the failure of a brittle material under complex stresses.

Research Limitation: Improvised conditions for biaxial data collection were less than ideal.

Practical implication: The study recommended that other brittle materials beyond cast iron be included in any further studies to broaden the scope of applicability of the findings.

Social implication: The research adds new literature and findings to an old subject. With this new knowledge, bookmakers could shape the way brittle materials are used in engineering design.

Originality / Value: The value of the study lies in the fact that to date very few failure theories exist that cater fully satisfactorily to brittle materials. The rigour of the methodology confers potential for its application beyond brittle materials.

Keywords: *Brittle materials. design. mohr. rankine. tension.*

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INTRODUCTION

This paper intends to develop an improved criterion of failure for brittle homogeneous isotropic materials. The first credible failure criterion was proposed by Coulomb in the 1800s for isotropic materials under three-dimensional stress conditions. A hundred years later Mohr reformulated Coulomb's criterion into the Coulomb-Mohr form within the context of his famous "Mohr's Circle". Despite being easy to use, it is an unsatisfactory representation of the failure behaviour of engineering materials due to its failure to account for many physical effects.

After Coulomb-Mohr, there have been many attempts to develop criteria that apply to both ductile and brittle materials. These works include a two-parameter yield criterion in principal stress space by Yu and Wang (2019), a strain-energy-based criterion by Lazzarin, Campagnolo and Berto (2014); paraboloid criteria distinguishing between tension and compression effects (Gu & Chen, 2018a, 2018b); criteria involving convex failure surfaces by Giraldo-Londoño (2020) and Qu, Zhang, Zhang, Liu and Zhang (2016); failure prediction fusing size effects by Zheng, Wang, Jiang, Wan, and Meng (2022); a nonlinear criterion based on fracture (Wang, 2022; Zuo et al., 2021)); a failure initiation criterion (Yosibash and Mittelman, 2016; Yosibash, Mendelovich, Gilad & Bussiba, 2022); failure of pre-cracked materials by Vasiliev (2021); analysis of brittle materials via indentation by Wu et. al (2019); a unified finite strain continuum for quasi-brittle materials by Sun and Xiang (2022) and a 3-D failure initiation criterion for elastic brittle structures by Yosibash and Mittelman (2016). Jeong (2012) investigated a stress-based method applicable to mechanical structures, while Yu (2019) proposed a theory based on material configuration forces. Tiraviriyaporn and Aimmanee (2022) evolved a criterion for isotropic materials using energy formalism while Pereira, Costa, Anflor, Pardal, and Leiderman (2018) employed a method based on a comparison of numerical and experimental results.

In a further attempt to bridge the gap between failure criteria for ductile and brittle materials, Christensen published a series of papers (2016, 2018, 2019) developing a three-dimensional stress-based failure criterion for homogeneous isotropic materials, establishing strength relationships among shear (S), tension (T) and compression (C) and linking strength properties and Poisson's ratio through a concept of associated flow to assess failure potential of certain material-load regimes. His theory (Equation 1), built primarily on the von Mises criterion, states that failure occurs when the combined normalized stress effect exceeds 1. For brittle materials, Christensen's criterion seems more conservative (indicating the possibility of overdesign) for pure tensile stresses in the first quadrant than both the Maximum Normal Stress and the Coulomb-Mohr criteria. Under mixed (tension and compression) loadings it falls between these two criteria. For compression loadings a, large room exists for prediction errors.

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Even though a large number of failure criteria now exist, the maximum distortion energy, the maximum shear stress, and the Coulomb–Mohr criteria (Yu, 2019; Barsanescu, 2017) are the most commonly used and referenced by engineers and scientists globally. Nevertheless, there exists a paucity of theories relating to brittle materials despite the prominent, long-standing *maximum principal stress* theory, by Rankine. Rankine's major issues are its failure to account for stress differences and significant dissimilarities in the tensile and compressive behaviour of brittle materials. A good attempt at addressing these issues was made by Mohr in his *modified theory for brittle materials*. Like Rankine, Mohr's theory treats shear stresses sub-optimally in the 2nd and 4th quadrants of the principal stress diagram.

Despite the extensive nature of the above works none has proven satisfactory enough regarding indicators for ductile-brittle differentiation. This paper intends to develop an improved criterion of failure for brittle homogeneous isotropic materials using Mohr's criterion for brittle materials as a starting point. The authors now turn to a detailed review of this theory (Figure 1).

Mohr's modified shear stress theory (the internal friction theory) for brittle materials

For brittle materials, Mohr proposed a three-circle construction based on his stress circle in the application of the maximum shear stress theory (Figure 1).

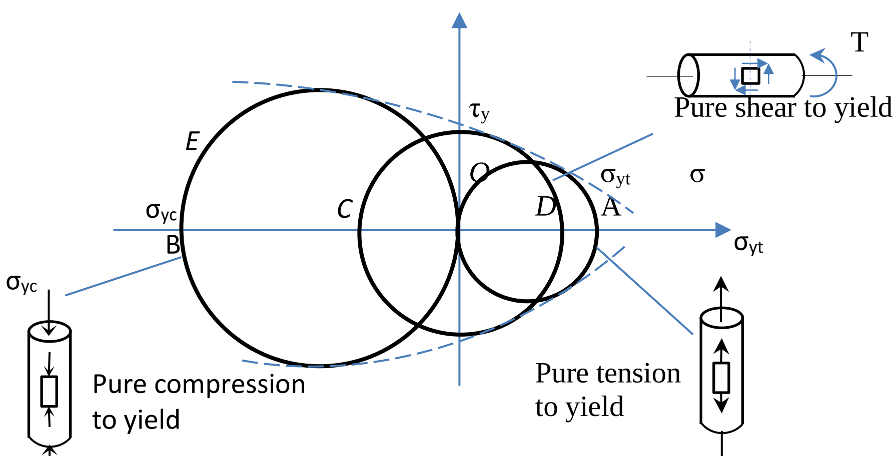
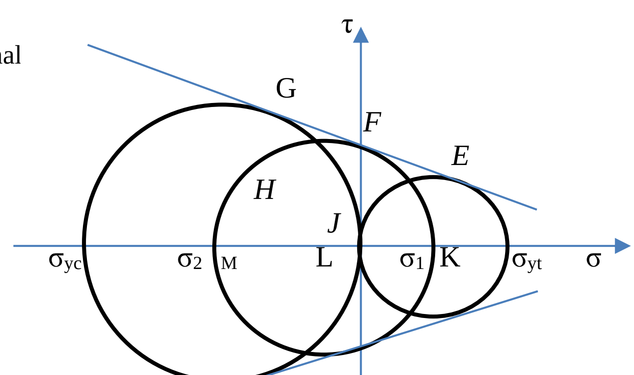


Figure 1. Mohr's theory on σ-τ axes

These circles are those for pure tension, pure compression, and pure shear, tests that can be easily performed in the laboratory. Failure is indicated when a loaded material's stress circle cuts into the dotted envelope. Figure 2 gives a visual impression of Mohr's modified criterion.



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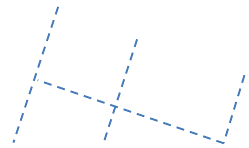

Figure 2. The theoretical expression of Mohr's theory

Equations (1) and (2) represent the mathematical expression of Mohr's modified criterion for a point in the 4th and 2nd quadrants.

$$\frac{|\sigma_1|}{\sigma_{ft}} + \frac{|\sigma_2|}{\sigma_{fc}} < 1 \qquad (1) \qquad\qquad \frac{|\sigma_1|}{\sigma_{fc}} + \frac{|\sigma_2|}{\sigma_{ft}} < 1 \qquad (2)$$

Figure 3 portrays some critical stress scenarios relating to Mohr's theory.

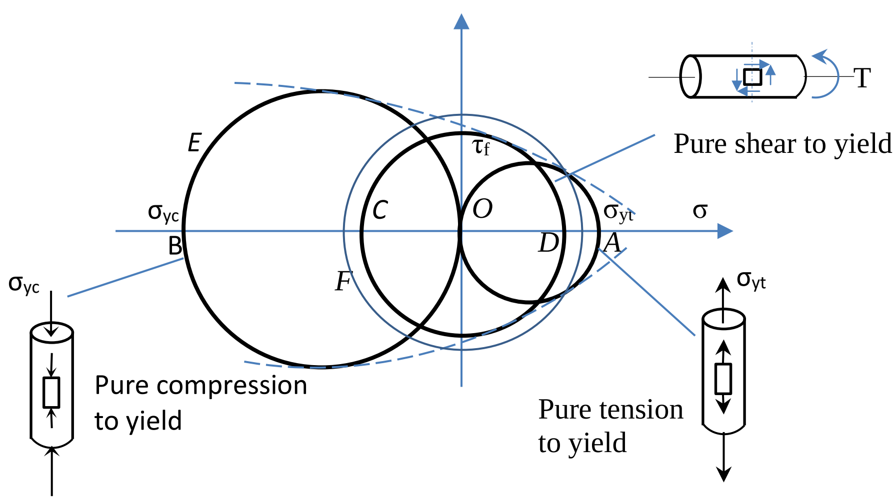

Figure 3. Possible failure by shear (Circle *F*, superimposed), not considered by Mohr

Mohr's theory, while certainly an improvement on the maximum principal stress criterion, suffers from the following issues:

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1. Equations 1 and 2 lack rigour and accuracy. Owing to simplifying assumptions in their derivation, the model excludes certain stress combinations that could cause failure. The line *GFE*, if it were superimposed on the original plot in Figure 1, would exclude a portion of the pure-shear circle, which supposedly must be fully contained within the failure envelope. Mohr's presupposition of tangents is not necessarily true.
2. Equations 1 and 2 may fail to predict failure under some specific loading conditions, especially when the principal stresses are equal, or very nearly equal in magnitude but carry opposite signs (Circle *F* in Figure 3).
3. It does not consider the limits of the application of Mohr's stress circle, whose mathematical representation can best be described as a theoretical calculating formula. The maximum shear stress corresponding to Circle *E* in Figure 3 violates physical reality since it indicates that τ_{max} can be greater than the material's strength in pure shear as depicted by circle *C*. Likewise, in the tension domain τ_{max} ($= \sigma_{ft}/2$) is often unequal to the material's strength in pure shear. A more realistic representation of the three monolithic strengths would be an ellipsoid.
4. It does not consider the irregular behaviour of brittle materials regarding the τ_{max} indicated by Mohr's stress circle and the actual shear strength, τ_f of the material.

Thus the absence of a satisfactorily rigorous theory for brittle materials taking full account of shearing effects and the unrealities of Mohr's circle has been demonstrated and the need for a viable representation is established. The purpose of this work is to develop an improved procedure which, while offering failure prediction of brittle materials with a fair level of accuracy, addresses the above issues of Mohr's criterion as well in the process. This is done by evolving a suitable representation of the stress state of the complex system and comparing it with Mohr's model using experimental data to reveal the improvements.

The next sections deal with the methodology, failure modelling, results, discussion of results, and conclusions drawn from the results.

METHODOLOGY

In the methodology for this work, a critical analysis of Mohr's model is first carried out by encapsulating Mohr's three simple monolithic cases in one generic equation of a circle whose parameters are then varied to suit specific principal-stress loading conditions. This leads to the development of a procedure involving a set of tests to be performed on several equations to determine the failure status of the material. Experimental data are then used to validate the proposed model which, as a double check, is then compared with the Mohr criterion.

Data for model validation came from uniaxial and bi-axial tests, performed using a standard universal testing machine (INSTRON 8801) improvised to enable bi-axial recordings.

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Several materials were employed: ASTM 30, ASTM 40, ASTM 50, Grey Cast Iron ASTM, 48 and BS 1452 Grade 250 continuous-cast grey cast iron with the following properties from the supplier: E, Young's Modulus, = 39 000 MPa and $\sigma_{UT,}$ ultimate tensile strength, = 290 MPa. Regarding the last material, outputs from the lab experiments (some for confirmation) include Young's Modulus, ultimate tensile strength, maximum load, tensile strain at rupture, and shear strength. Uniaxial tests are natural to the machine and easy to perform but the biaxial tests required improvisation to adapt the machine to enable measurement of strains in the second (lateral) dimension with strain gauges.

The test samples were prepared square in shape 12 mm x 12 mm and the material was loaded biaxially in two perpendicular directions to obtain two principal strains. Principal stresses σ_1 and σ_2 stress were then calculated from $\sigma = E\varepsilon$ using the average value of E outputted by the machine during the strength determinations. Table 1 lays out partial data for the biaxial cases, while Figure 4 displays graphically an example of loading in this regime.

Table 1: Biaxial loading cases (with exception of row 1)

Case	**Specimen**	**Strain ε_2**	**Stress σ_2 (MPa)**	**Stress σ_1 (MPa)**
1	3	0	0	328
2	7	10×10^{-6}	0.43	316
3	8	420×10^{-6}	16.8	219
4	9	170×10^{-6}	7.3	288
5	10	330×10^{-6}	14.1	297

Data source: Experimental tests (Biaxial)

MODELLING AND RESULTS

Analysis of Different Principal Stress Systems and Comparison with Mohr's Model

We begin this section by considering that since the three circles forming the basis of Mohr's approach depict, in effect, material strengths, we would represent them with one generic equation of a circle containing parameters that can be adjusted to suit specific loading conditions.

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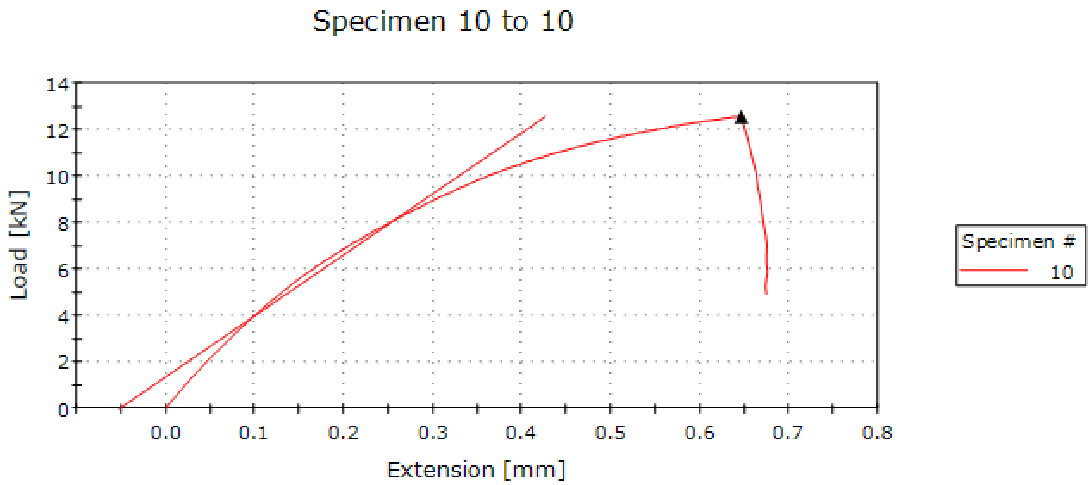

Figure 4: Biaxial tension-compression loading results (Specimen 10)

In what follows, τ_{max} is the maximum shear stress indicated by Mohr's stress circle, and a is the variable centre of this circle. Then the generic equation representing the three simple cases in Figure 3 is:

$$\tau^2 + (\sigma \pm a)^2 = \tau_{max}^2 \qquad (3)$$

and a is given as

$$a = \frac{(\sigma_1 + \sigma_2)}{2} \qquad (4)$$

It is easily observed that as a approaches zero (i.e. the σ-τ axes origin), torsional effects become overriding and important, especially if σ_1 approaches the failure stress, σ_{ft}, at the same time. Various load regimes are now considered under different categories as follows:

Category 1: Monolithic Loading Conditions (pure shear or tension or compression)
Case 1.1(pure shear)

If a = 0 (pure shear) then $\sigma_1 = -\sigma_2 = \tau_{max}$, and the failure criterion must be

$$\sigma_1 = -\sigma_2 = \tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \geq \tau_f \qquad \text{if } \sigma_1 = < \sigma_{ft}$$

$$\text{or} \quad \sigma_1 \geq \sigma_{ft} \text{ or } \sigma_2 \geq \sigma_{fc}, \quad \text{otherwise;}$$

where σ_{fc} and τ_f are the ultimate strengths of the be material in pure compression, and pure shear, respectively. A possibility remains that σ_1 could be less than σ_{ft} (Region *DA* in Figure 3), and yet failure would still occur on account of shear stresses.

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Case 1.2 (pure tension)

If $a \neq 0$, but $\sigma_1 > 0$ and $\sigma_2 = 0$ (pure tension), then and $\tau_{max} = \sigma_1/2 = \sigma_{ft}/2$, and the failure criterion must be

$$\sigma_1 \geq \sigma_{ft}$$

Case 1.3 (pure compression)

If $a \neq 0$, but $\sigma_2 < 0$ and $\sigma_1 = 0$, (pure compression), then ignoring the unrealities of Mohr's stress circle regarding the shear stress by noting that the indicated τ_{max} in Mohr's stress circle cannot be greater than τ_f, the failure criterion is

$$|\sigma_2| \geq |\sigma_{fc}| \quad \text{or} \quad \tau_{max} = |\sigma_2|/2 \geq \tau_f,$$

whichever happens first.

Category 2: τ_{max} out of a plane

Case 2.1 (Non-zero principal stresses having positive sign)

If $a \neq 0$, but then $\sigma_1, \sigma_2 > 0$, then the failure criterion, with σ_1 assumed greater in magnitude, is

$$\sigma_1 = \sigma_{ft} \text{ or } \tau_{max} = \sigma_1/2 \geq \tau_f$$

Case 2.2 (Non-zero principal stresses having negative sign)

If $a \neq 0$ and $\sigma_1, \sigma_2 < 0$, then the failure criterion (with σ_2 assumed greater in magnitude) is

$$|\sigma_2| \geq |\sigma_{fc}| \quad \text{or} \quad \tau_{max} = |\sigma_2|/2 \geq \tau_f$$

Category 3: Principal stresses unequal, with opposite signs (Second/ fourth quadrants)

To overcome the shortcomings of the Coulomb-Mohr failure criterion in the 2nd and 4th quadrants, we adopt a different approach than we did above. We will adopt the expression

$$\sigma_{1eq} = \sigma_1 + \nu\sigma_2 \qquad (5)$$

to create the equivalent unilateral stress in the direction of σ_1. Here ν is the Poisson's ratio for the material. We now formally state the proposed criterion for this category as follows:
If $a \neq 0$ but $\sigma_1 > 0$ and $\sigma_2 < 0$, or $\sigma_1 < 0$ and $\sigma_2 > 0$, then if $|\sigma_2| < |\sigma_1|$, the failure criterion is

$$\sigma_{1eq} \geq \sigma_{ft} \quad \text{or} \quad \tau_{max} = \frac{(\sigma_1 - \sigma_2)}{2} \geq \tau_f.$$

But if $|\sigma_2| > |\sigma_1|$ then it is

$$|\sigma_{2eq}| = |\sigma_{fc}| \quad \text{or} \quad \tau_{max} = \frac{(\sigma_1 - \sigma_2)}{2} \geq \tau_f$$

ISSN: 2408-7920

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This gives

$$\sigma_{1eq} = \sigma_1 + \nu\sigma_2 \geq \sigma_{ft} \qquad (6)$$

or, considering the lower limit,

$$\sigma_{2P} = (\sigma_{ft} - \sigma_1)/\nu \qquad (7)$$

Equation (7) is then employed to determine the value of lateral stress that would mark the lower failure boundary, similar to Mohr's polygon boundary value, σ_{2M}. In tables 2 and 3 this is designated as σ_{2P} in a column heading.

Application

To apply the proposed criterion after determining the principal stresses,

1. Calculate a is the centre of the prevailing circle
2. Determine if $\sigma_1 = \sigma_2$ or $\sigma_1 > \sigma_2$ or $|\sigma_1| < |\sigma_2|$
3. Select the appropriate category from the above case list
4. Apply the remainder of the test procedure to determine failure status or potential.

Model Validation

The proposed model is now validated using standard data.
The mathematical representation of Mohr's criterion can be expressed as:

$$\sigma_{2M} = \left|\frac{\sigma_{fc}}{\sigma_{ft}}\right|\sigma_1 - |\sigma_{fc}| \qquad (6) \qquad \text{fourth quadrant,}$$

$$\sigma_{2M} = \left|\frac{\sigma_{ft}}{\sigma_{fc}}\right|\sigma_1 - |\sigma_{ft}| \qquad (7) \qquad \text{second quadrant.}$$

A point in the second or fourth quadrant will fall outside Mohr's hexagon (not shown), indicating failure, if its absolute value is greater than σ_{2M} in equation (5) or (6), respectively. Table 1 displays results for various brittle materials of measured tensile, compressive, and, in some cases, shear strengths under specific principal stress loading conditions. The values in column 7 are determined from Equation (5) or (6), whichever applies.

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African Journal of Applied Research
Vol. 8, No. 2 (2022), pp. 248-263
http://www.ajaronline.com
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Table 2. Indications of the possibility of failure under specific principal stress load regimes of selected materials of measured tensile, compressive, and/or shear strength (C: compression basis; T: tension basis; S: shear basis)

	Material	**Ultimate Strengths (MPa)**			**Principal stresses**	**Max. shear stress**	**σ_{2M}, Mohr polygon boundary (MPa)**	**σ_{2P} from proposed model (MPa)**	**Experimental test results with σ_1 and σ_2**	**Failure Predicted? Yes(Y)/No(N)**	
		σ_{ft}	**σ_{fc}**	**τ_f**	**σ_1, σ_2 (MPa)**	**τ_{max} (MPa)**				**Mohr**	**Improved model**
1	**Gray Cast Iron 4.5% C, ASTM A-48**	80	-200	-	$\sigma_1 = 32$ $\sigma_2 = -112$	72	-119.7	-165.5	No failure	N	N
		170	-655	240	$\sigma_1 = 100$ $\sigma_2 = -300$	200	-269.7	-241.4	Failure	Y	Y
		80	-200	-	$\sigma_1 = 68$ $\sigma_2 = -38$	53	-29.6	-41.4	No failure	Y	N
		60	-120	-	$\sigma_1 = 40$ $\sigma_2 = -70$	55	-40	-69	Failure	Y	Y
2	**ASTM 30**	210	-750	276	$\sigma_1 = 150$ $\sigma_2 = -300$	225	-214.3	-206.9	Failure	Y	Y
4	**ASTM 40**	280	-970	393	$\sigma_1 = 200$ $\sigma_2 = -300$	250	-277.1	-275.9	Failure	Y	Y
5	**ASTM 50**	345	-1130	503	$\sigma_1 = 214$ $\sigma_2 = -400$	307	-425.8	-451.7	No failure	N	N

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DISCUSSION

Of the large number of failure criteria now exist, the maximum distortion energy, the maximum shear stress, and the Coulomb–Mohr criteria are the most commonly used and referenced by engineers and scientists globally (Yu, 2019; Barsanescu, 2017).

Examining the results in Table 1, it can be seen from rows 3, 5, and 7, that τ_{max} or σ_1 or σ_2 do not have to equal or exceed the material's respective strengths before failure can occur. It is commonly acknowledged that failure depends not only on the properties of the material but also on the stress system to which it is subjected (Injeti et al, 2019). This is the essence of quadrants 2 and 4 of the principal stress diagram, where Mohr's modified criterion for brittle materials performs sub-optimally (Gu & Chen, 2018). A recent model developed by Christensen (2018) is more consistent in these quadrants than Mohr but is prone to error regarding quadrant 1, probably due to hydrostatic complications. Christensen's (2016) theory states that failure occurs when the combined normalized stress effect exceeds 1, with an additional constraint for brittle materials that says no principal stress shall exceed the tensile strength of the material. The inference to be drawn is that whereas Christensen's criterion represents an improvement on the Coulomb-Mohr form, the proposed criterion, in turn, improves upon Christensen's since it covers all four quadrants.

Examining Table 1 once again, and judging from the data in row number 3, it is difficult to identify the prevailing mode of failure since all indicated stress values are significantly below the material's strength; however, since the least Factor of Safety (FS) among the three entries in this category is (-241.4)/(-300) = 0.805, one might be justified to conclude that the failure mode is tensile. Any issues stemming from hydrostatic tension and compression effects on brittle materials as discussed by Gu and Chen (2018a, 2018b) are addressed by the proposed theory since its results apply to all four quadrants of the principal stress diagram.

The outputs from both the proposed and Mohr models correlate well with the experimental test results, but the proposed model is cast strongly as a more accurate predictor of failure. If one were to allow a small margin of tolerance, say ± 2 MPa on the magnitude of the principal stress, then the -41.4 MPa stress in row number 4 would be seen to fall within this margin of error if compared with the corresponding principal stress value of 38 MPa. If this was done, the proposed model would turn out 100% in agreement with the test results.

Finally, Table 3 isolates grey cast iron 4.5% ASTM A-48 for further analysis under a different loading regime. Two parameters are considered as follows:

Case 1: Extent of correlation with test data

ISSN: 2408-7920

Referring to the last three columns of Table 3 there are strong indications that the proposed model correlates well with the test data. Whereas Mohr's model is at variance with the test results in three places, the proposed model delivers 100% correlation with them.

Case 2: Efficiency

It can be easily noticed that the proposed model tends toward conservative results, indicating greater accuracy; and where both models correctly predict failure, the proposed criterion does so with a smaller margin of error, a fact which renders it more efficient.

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Table 3. Indications of the possibility of failure under various principal stress load regimes for one material of measured tensile, compressive, and shear strength (C: compression basis; T: tension basis; S: shear basis)

Material		**Ultimate Strengths (MPa)**			**Principal stress**	**Max. shear stress**	**σ_{2M}, Mohr polygon boundary point (MPa)**	**σ_{2P} from improved model (MPa)**	**Experimental test results with σ_1 and σ_2**	**Failure Predicted? Yes(Y)/No(N)**	
		σ_{ft}	σ_{fc}	τ_f	σ_1, σ_2 (MPa)	τ_{max} (MPa)				**Mohr**	**Improved model**
A	Gray Cast Iron 4.5% C, ASTM A-48	170	-655	240	$\sigma_1 = 50$ $\sigma_2 = -250$	150	-462.4	-413.8	N	N	N
B					$\sigma_1 = 75$ $\sigma_2 = -200$	137.5	-366.0	-327.6	N	N	N
C					$\sigma_1 = 100$ $\sigma_2 = -250$	175	-269.7	-241.4	Y	N	Y
D					$\sigma_1 = 125$ $\sigma_2 = -150$	212.5	-173.4	-155.2	N	Y	N
E					$\sigma_1 = 150$ $\sigma_2 = -150$	150	-77.1	-69.0	Y	Y	Y
F					$\sigma_1 = 200$ $\sigma_2 = -110$	150	-115.6	-103.4	Y	N	Y

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CONCLUSION

Several issues were raised with Mohr's method of failure prediction for brittle materials, and the proposed failure prediction criterion, more transparent and rigorous, has been shown to contribute to correcting these issues.

In summary, the proposed model is a procedure consisting of a set of tests applied in turn until failure is indicated by one of them. Variations between the proposed model and Mohr's criterion are in respect of the mode of representing and handling primary data and the adjustments in the relationships among the three fundamental stress circles necessary to accommodate the peculiarities of brittle material under specific principal loading conditions. The irregular behaviour of cast iron and other brittle materials means complete test data for shear strength must be available before a full evaluation of the failure potential of such materials under complex loading can be made. This would add more rigour to the investigation as both shear and direct stress effects are taken into consideration.

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