



## Jump Resonance in Wind-Felled Plantains

Godwin Norensa Osarumwense Asemota<sup>1</sup>

<sup>1</sup>Electrical and Electronics Engineering Department  
School of Engineering, College of Science and Technology  
University of Rwanda, BP 3900, Kigali, Rwanda

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**Abstract.** In this paper, jump resonance was applied to wind-felled plantains, which budded on the plantain pseudo-stem side when guyed about  $60^\circ$  to the horizontal to obtain the jump function. Duffing's model, describing function and Chebyshev polynomials were used to obtain the best fit of approximants to the developed plantains jump function. Traditionally, wind-damaged plantains and banana pseudo-stems are cut for future vegetative growth, leading to heavy and perennial losses to individuals, households, communities, nations and even regions. The motivation for this study was the possibility of using jump resonance discontinuity as a plantain wind-damaged salvage process. Furthermore, it was to develop plantain growth equation and determine the harmonic content responsible for the observed stable switching mode for plantain side shoot outgrowth. The results show that the polynomial growth equation for plantains was of order 14 with 0.5 percent error. In addition, the first harmonic content in the plantain jump resonance function was absent and only the jump-up mode of the switching mechanism was stable, leading to plantain pseudo-stem side shoot outgrowth. Therefore, a sinusoid plus bias (steady or d.c.) signal input could be used to improve algorithm accuracy in future research.

**Key words:** Chebyshev polynomials; describing function; Duffing's model; harmonic; jump-up mode; switching

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\* Corresponding Author: [asemotaegno@gmail.com](mailto:asemotaegno@gmail.com)

## 1. INTRODUCTION

The importance of studying jump resonance phenomena in wind-felled plantains and banana lie in that fact that, plantain pseudo-stem side-bud outgrowth represents viable bananas and plantains damage recovery mechanism. In addition, many human livelihoods and animal life depend on the availability of bananas and plantains for their food security and economic well-being. It is expedient to seek for ways and means to stem whatever factors that could be responsible for their damage or destruction. This is especially so because, bananas and plantains are staple food for at least 400 million people (Schooffs *et al.*, 1999; CGIAR, 2005). About 10% of the more than 80 million tons produced annually is exported and serves as dessert for far more millions of people (Schooffs *et al.*, 1999; CGIAR, 2005). Furthermore, Faturoti *et al.* (2007) quoting FAO (2004), report that banana is the world's second most important fruit crop after oil palm, and that its world production stood at about 71 million metric tonnes in 2003 from 130 countries. They also state that plantain was grown in 52 countries in the world, with a total production of about 33 million metric tonnes. Although banana ranking as a major world food crop is as variable as the source, it is one of the least researched especially in developing countries (Faturoti *et al.*, 2007; Dankyi *et al.*, 2007). This is a consequence of their very complicated and time consuming breeding processes, resulting from their triploidy (Perea, 1998). It has been shown that (Robinson, 1996; Eckstein *et al.*, 1997; Asemota, 2004; Sikora and Pocasangre, 2004) wind-felled plantains, damage and sometimes complete loss of the crop to wind effects can be devastating to individuals, families, communities and even, nations. This is so because toppled, up-rooted and damaged plantain pseudo-stems were usually cut for future vegetative growth (Asemota, 2004). Today, the most serious climatic problems confronting all commercial producers of banana in Asia and the Pacific countries are tropical storms and typhoons that occur with predictable regularity (Molina and Valmayor, 2003). As a result, banana growers in Taiwan and Northern Philippines have adopted a planting calendar and plantain management practices that minimise damage arising from tropical winds and typhoons (Molina and Valmayor, 2003). Therefore, guided control method was used to demonstrate plantain side bud outgrowth employing fork-shaped props to enable guyed wind-felled plantain to exhibit jump resonance. Consequently, the system had three components: (1) a sensory mechanism to detect whether or not the organ was on course, (2) a reaction mechanism, which on receipt of the appropriate signal from the sensory mechanism that the organ was off-course, could initiate growth changes in the organ to ensure that the organ was brought back on course, and (3) a communication mechanism that conducted the signals from the sensory mechanism to the reaction mechanism (Wilkins, 1989), which in turn, led to jump resonance in wind-felled plantains. Generally, mechanical systems can be used to represent jump resonance commonly encountered in nature, and easily explained by the Duffing's model. The component parts of such mechanical systems are a nonlinear spring, mass, viscous friction, and damper. Experimental observations reveal that as the amplitude decreases, the frequency of the free oscillation either increases or decreases depending upon whether the spring constant, is positive or negative (Ogata, 1970; Atherton, 1982; McConnell, 1995). Whenever the spring constant is negative for low damping, resonant large amplitudes of oscillation result as the resonant frequency is approached. Similarly for an oscillating system of audio transducers that are injected with chirp signals, the peak amplitude of oscillation and frequency at jump resonance depends on the direction (increasing or decreasing frequency) of the chirp signal (Jabbari and Unruh,

2004). [Nguyen \(1988\)](#) explains jump resonance as a discontinuity of the plastic multiplier on the boundary in quasi-static evolution and bifurcation in standard plasticity and fracture. Also, [Moreau \(1988\)](#) describes jump resonance as a motion defining the state or position of the investigated system as a function of the real variable, time. [Moreau \(1988\)](#) and [Nguyen \(1988\)](#) showed that every real measure is uniquely decomposed into an atomic measure and a diffuse measure in non smooth mechanics. Therefore, any bounded variation (bv) function is atomic iff the differential measure equals the sum of a continuous bv function, unique up to an additive constant and a jump function ([Moreau, 1988](#); [Nguyen, 1988](#)). Nonlinear systems do not obey superposition principle. Analyses and measurements are carried out for different types of nonlinearities and magnitudes to discover their behaviours [Atherton \(1982\)](#). For given amplitude and frequency of an input sinusoid, a nonlinear system can have more than one stable periodic state (multimodal system). Therefore, slight changes in the input amplitude or frequency cause the system to change states, and this is called jump resonance ([Atherton, 1982](#)). Certain choices of linear element transfer functions for the nonlinear equations can exhibit multiple solutions over a range of input frequencies and amplitudes, resulting in jump resonance ([Ogata, 1970](#); [Atherton, 1982](#)). Experimental evidence has shown that jump resonance only occurs in lightly damped systems and also probably dependent on the open loop frequency response locus in the complex plane ([Ogata, 1970](#); [Atherton, 1982](#)). However, describing function methods applied to asymmetrical systems usually produce less accurate solutions due to neglect of harmonics, as the lowest, is normally the second rather than the third ([Ogata, 1970](#); [Atherton, 1982](#); [D’Azzo and Houpis, 1982](#); [Wilkins, 1989](#)). As expected for a random input, a system with two possible stable theoretical solution modes tend to move in a random fashion between the two states, based on short time averages of the nonlinearity input signal’s first solution and then the other ([Atherton, 1982](#)). A variety of analytic and computational techniques may be used to study chaotic systems like jump resonance. Fourier spectra and Poincare section are also applied to the driven pendulum in the jump resonance region. The pendulum is used as a primary timing device for clocks and to measure variations in the earth’s gravitational field ([Baker and Gollup, 1990](#)). It is, therefore, hypothesised in this study that the order 14 of jump resonance function of the wind-felled plantains is much higher than order 3 as postulated by the classical Duffing’s model. Consequently, plantains jump resonance function and growth equation, are Duffing’s counterexamples.

## 2. MATERIALS AND METHODS

The materials used for this research were wind-felled plantain pseudostems, fork-shaped supports and a fence wall. The wind-felled plantains slipped on forked wooden supports, to rest on fence wall and to later produce plantain side bud outgrowth when inclined at a specified angle to the horizontal. Duffing’s equation was used to model a mass-damper mechanical system as an analogue to wind-felled plantains pseudostems, for which jump resonance of order 3 have been known to be exhibited by soft springs ([Ogata, 1970](#); [Atherton, 1982](#)). While both [Ogata \(1970\)](#) and [Atherton \(1982\)](#), showed that mechanical systems displaying jump resonance discontinuity were for order 3, this work showed that wind-felled plantains jump resonance function are much higher in order of magnitude than the Duffing’s model. Therefore, frequency-amplitude dependence relationships, multi-valued responses to jump phenomena, describing functions methods of analyses, closed loop response control

mechanisms, method of Hatanaka and Chebyshev polynomial approximations were used in this study to develop polynomial growth equation for plantains and plantain jump resonance function. Furthermore, MATLAB software was used to plot graphs of the exact equation and approximants of the Jump function, to determine the magnitude of errors between them. These graphs also provided pictorial explanation of the mechanics of the plantains side bud outgrowth in this study. We consider a free oscillation of a mechanical system made up of mass, dashpot and nonlinear spring. The differential equation of the dynamics of the system is of the form (Ogata, 1970):

$$mx'' + fx' + kx + k'x^3 \tag{1}$$

where  $x$  = mass displacement,  $m$  = mass,  $f$  = viscous friction coefficient of damper,  $kx + k'x^3$  = nonlinear spring force. The parameters  $m$ ,  $f$  and  $k$  are positive constants, while  $k'$  could be positive or negative. A positive  $k'$  value represents a hard spring; while a negative  $k'$  value, represents a soft spring (Ogata, 1970). The nonlinear equation (1) above is Duffing's equation. The solution of equation (1) is a damped oscillation if subjected to a non-zero initial condition. Experimentally, as the amplitude decreases, the frequency of the free oscillation either decreases or increases, depending on whether  $k' > 0$ , or  $k' < 0$ . If,  $k' = 0$ , however, the frequency of free oscillation remains the same as amplitude decreases (Ogata, 1970). The differential equation of the foregoing system with an externally imposed oscillating force is of the form (Ogata, 1970):

$$mx'' + fx' + kx + k'x^3 = P \cos(\omega t) \tag{2}$$

where  $P \cos(\omega t)$  =forcing function, which occurs at regular intervals. Thus, subharmonic, superharmonic oscillations and jump phenomena, from equation (2) could be observed (Ogata, 1970). Variation of frequency with amplitude  $P$  of the forcing function kept constant produces frequency response curves for hard springs and soft springs. These response curves show discontinuities (jumps), because a representative point on the curve follows different paths for increasing and decreasing frequencies (Ogata, 1970; Atherton, 1982). For jump resonance to take place it is essential for the damping term to be small, while the forcing function amplitude is large enough to drive the system into the nonlinear region. Additionally, a slight change in input amplitude or frequency can cause the system to change states, called jump resonance (Ogata, 1970; Atherton, 1982).

### 3. WIND-FELLED PLANTAINS JUMP RESONANCE

We consider a plantain pseudo-stem inclined at an angle to the horizontal. Taking a point  $P$  on the inclined plantain pseudo-stem, with coordinates  $x$  and  $y$ . Assuming that the base of the plantain, which is the lowest point to be the origin, (Mehta and Mehta, 2008). The two forces acting on the portion of plantain pseudo-stem are: (a) The weight  $wx$  of plantain pseudo-stem acting at distance  $x/3$  from  $O$ . The plantain pseudostem looks and tapers at the end more like a cone, than a cylinder. Therefore, assuming a right-angled plane triangular lamina to represent the inclined plantain pseudostem, and further taking the base distance of the incline to be  $x$ , then centre of area along the horizontal would be  $x/3$ . Similarly, the centre of area along the vertical or height of the incline would be represented by  $y/3$  (Rajput, 2001). The centroid or centre of area is defined as the point where the whole area of the figure is assumed to be concentrated. Hence, centroid can be taken as analogous to

centre of gravity when bodies have area only and not weight (Rajput, 2001). (b) Action ( $A = R$ ) acting at  $O$ , is equal and opposite to the reaction ( $T$ ), which can also be taken as the tension acting at  $O$ . For tension  $T$  acting at  $O$ , we have used the centroid value of a triangular object without proof in (a), and the empirical result of plantain shoot outgrowth inclined at about  $60^\circ$  to the horizontal, has equally strengthened this claim for plantains as reported in (Asemota, 2004, 2010). Equating moments of the above two forces about point  $O$  (for the  $T$  above point  $P$ ), we have

$$Ty = wx(x/3).$$

Hence,

$$T = \frac{wx^2}{3y},$$

where  $w$  = weight per unit length,  $T$  = Tension or reacting force,  $x$  = horizontal distance,  $y$  = vertical distance. If we express the coordinates of point  $P$  in terms of plantain pseudostem length ( $l$ ), then equation (4) could be expressed as:

$$T = \left[ \frac{wl \cot \theta}{3} \right] \cos \theta \tag{3}$$

It can be seen that Equation (5) is both a cosine and cotangent function. In order for the wind-felled plantain pseudostem to be able to respond to external stimuli, after being knocked down by wind in bad weather and also able to carry out the necessary growth initiating processes, there must be a sensing device, a communications mechanism as well as feedback loop to check the differences between input and output response signals (Wilkins, 1989) for the necessary jump resonance growth mechanism. Therefore, a closed loop response model was invoked to help our analyses for determining the wind-felled plantains jump resonance function and plantains growth equation determined in this study. It can be seen that over certain ranges of amplitude and frequency of an input harmonic signal, to certain nonlinearity system, three analytical solutions are possible, for which only two are stable Atherton (1982). Obviously, the Hatanaka method can be used to obtain conditions for the existence of the unstable third solution (Atherton, 1982). For  $b$  of frequency  $\omega_f$  (Atherton, 1982):  $N_{ib}(a) = N + (a/2)(dN/da)(1 + e^{-j2\phi})$  and for a single valued nonlinearity (SVNL), the maximum phase shift of  $N_{ib}(a)$  for a given  $a$ , is  $\alpha$  where

$$\sin \alpha = \frac{|(a/2)(dN/da)|}{N + (a + 2)(dN/da)}. \tag{4}$$

If  $\alpha_m$  is the maximum possible value of  $\alpha$  for all  $a$ , then no unstable forced solution can occur if the open loop  $G(j\omega)$  locus does not enter the region of the Nyquist diagram bounded by the radial lines  $(180^\circ + \alpha_m)$ . For a polynomial nonlinearity of the form  $n(x) = a |x|^n \sin x$ , it can be shown that (Atherton 1982)

$$\alpha = \tan^{-1} \left( \frac{1 - \mu}{2\mu^{\frac{1}{2}}} \right), \tag{5}$$

which is independent of  $a$ , so that  $\alpha_m = \alpha$ , for any  $a$ . It follows that, whenever the  $G(j\omega)$  locus enters the region bounded by the lines  $(180^\circ + \alpha_m)$ , jump resonance may occur in

the frequency response. Fortunately, jump resonance occurred for wind-felled plantains supported on a fence wall when inclined at  $60^\circ$  to the horizontal (Asemota, 2004, 2010). The major contribution of this study is that realisation that jump resonance of a soft spring is not limited to order 3 as postulated by the Duffing’s model. Indeed, much higher orders of magnitude are possible as shown in the ensuing analyses. Upon using the polynomial representation of equation (7) for the jump resonance in plantain pseudo-stem lying at  $60^\circ$  to the horizontal, which resulted in plantain side bud outgrowth, we have ( $\alpha = 60^\circ$ )

$$\tan 60^\circ = \frac{1 - \mu}{2\mu^{\frac{1}{2}}} \tag{6}$$

Upon substituting the values for  $\tan 60^\circ$  and evaluating, we have:  $\sqrt{3} = \frac{1-\mu}{2\mu^{\frac{1}{2}}}$  and

$$\mu^2 - 14\mu + 1 = 0 \tag{7}$$

with solutions

$$\mu = 13.93 \text{ or } \mu = 0.07 \tag{8a}$$

and

$$n(x) = x^{13.93} \text{ or } n(x) = x^{0.07}. \tag{8b}$$

Since assumed order of polynomial is integer, the nearest integer value is:  $\mu = 14$  with (-0.5%) error. Therefore,

$$n(x) \approx x^{14}. \tag{9}$$

Consequently, any even power of x can be represented as a series in Chebyshev polynomials, so that orthogonal properties of the polynomial can be used to obtain coefficients  $b_s$  without evaluating the integral (Atherton, 1982; Gerald and Wheatley, 1999)

$$b_s = (4/a\pi) \int_0^a n_q(x)U_{s-1}(x/a)dx \tag{10}$$

Considering (Atherton, 1982; Gerald and Wheatley, 1999),

$$n(x) = \sum_{k=1(2)}^{\infty} c_k T_k(x/a), \tag{11}$$

then (Atherton, 1982; Gerald and Wheatley, 1999)

$$a_s = 2 \int_{-a}^a \sum_{k=1(2)}^{\infty} c_k T_k(x/a) T_s(x/a) r(x) dx. \tag{12}$$

Using the orthogonal relationship (Atherton, 1982; Gerald and Wheatley, 1999):

$$\int_{-a}^a T_k(x/a) T_s(x/a) r(x) dx = \delta_{sk} / (\epsilon_s \epsilon_k)^{1/2} \tag{13}$$

we obtain  $a_s = c_s$  and with  $n(x) = x^{14}$ ,

$$x^{14} = a_2T_2(x/a) + a_4T_4(x/a) + a_6T_6(x/a) + a_8T_8(x/a) + a_{10}T_{10}(x/a) + a_{12}T_{12}(x/a) + a_{14}T_{14}(x/a) \tag{14}$$

Upon substituting for  $T_2, T_4, T_6, T_8, T_{10}, T_{12}, T_{14}$  and comparing the coefficients of  $x^2, x^4, x^6, x^8, x^{10}, x^{12}, x^{14}$ , we have

$$\begin{aligned} T_0 &= 1, \\ T_1 &= x, \\ T_2 &= 2x^2 - 1, \\ T_4 &= 8x^4 - 8x^2 + 1, \\ T_6 &= 32x^6 - 48x^4 + 18x^2 - 1, \\ T_8 &= 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1, \\ T_{10} &= 512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2 - 1, \\ T_{12} &= 2048x^{12} - 6144x^{10} + 6912x^8 - 3584x^6 + 856x^4 - 72x^2 + 1, \\ T_{14} &= 8192x^{14} - 28672x^{12} + 39424x^{10} - 26880x^8 + 9472x^6 - 1600x^4 + 98x^2 - 1, \end{aligned}$$

Equating coefficients in the above recursive terms and replacing  $x$  with  $a$ , we have

$$8192a_{14}/a^{14} = 1, \tag{15}$$

$$-28672a_{14}/a^{12} + 2048a_{12}/a^{12} = 0, \tag{16}$$

$$39424a_{14}/a^{10} - 5632a_{12}/a^{10} + 512a_{10}/a^{10} = 0, \tag{17}$$

$$-26880a_{14}/a^8 + 6912a_{12}/a^8 - 1280a_{10}/a^8 + 128a_8/a^8 = 0, \tag{18}$$

$$9472a_{14}/a^6 - 3584a_{12}/a^6 + 1120a_{10}/a^6 - 256a_8/a^6 + 32a_6/a^6 = 0, \tag{19}$$

$$-1600a_{14}/a^4 + 856a_{12}/a^4 - 400a_{10}/a^4 + 160a_8/a^4 - 48a_6/a^4 = 0, \tag{20}$$

$$98a_{14}/a^2 - 72a_{12}/a^2 + 50a_{10}/a^2 - 32a_8/a^2 + 18a_6/a^2 - 8a_4/a^2 + 2a_2/a^2 = 0. \tag{21}$$

But,  $a_0 = 1$

$$a_{14} = a^{14}/8192, \quad a_{12} = 14a_{14}, \quad a_{10} = 77a_{14}, \tag{22a}$$

$$a_8 = 332a_{14}, \quad a_6 = 1233a_{14}, \quad a_4 = 3397.5a_{14}. \tag{22b}$$

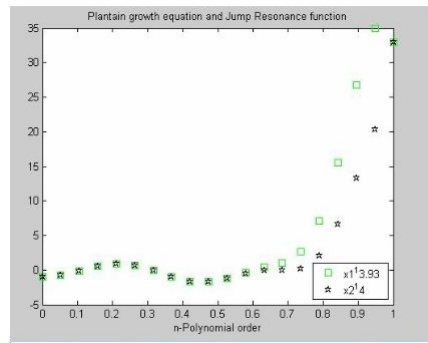
$$a_2 = 6335a_{14} = (a^{14}/8192)6335 = 0.773315429a^{14}. \tag{23}$$

Therefore,

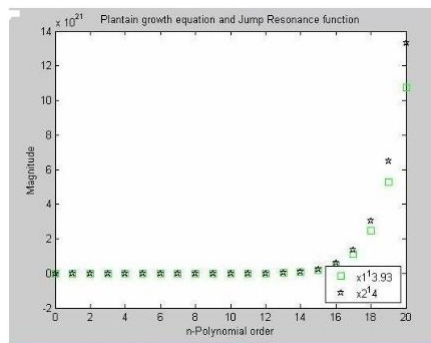
$$N(a) = 6335a^{12}/8192. \tag{24}$$

The jump-up plantain growth equation becomes

$$y = 8192x^{14} - 28672x^{12} + 39424x^{10} - 26880x^8 + 9472x^6 - 1600x^4 + 98x^2 - 1. \quad (25)$$



**Fig. 1.** Microscopic exact and approximate region of jump resonance.



**Fig. 2.** Plantains exact and approximate growth equations

#### 4. RESULTS AND DISCUSSIONS

The describing function and Hatanaka methods were used to analyse the wind-felled plantain jump resonance phenomena in this work. The starting point for modelling the inclined plantain pseudo-stem was the Duffing's equation. The model showed it to be both a cosine function as well as a cotangent function. We analysed the various shapes encountered for systems that exhibit jump resonance phenomena. Both the soft-spring and hard-spring configurations were considered and it was shown that only the soft spring nonlinear systems exhibit jump resonance phenomena (Ogata, 1970; Atherton, 1982). The analyses resulted in polynomial representation of the wind-felled plantains system and the integral of the resulting equation was assumed to be of the integer power form. The Chebyshev polynomials



were used to evaluate the integral because they are best fit approximants of any function (Zheng, 2010), and especially as they provide us with functions of minimum error (Gerald and Wheatley, 1999). The Chebyshev polynomial of the first kind was used for the evaluation of the resulting polynomial function because it is convenient to use and avoids the tedium of evaluating the integral itself. The coefficients evaluation of the Chebyshev polynomial is laborious. Nonetheless, Chebyshev polynomials are much faster and easier to use for obtaining (rather) “accurate approximants” of the solution to the integral (Gerald and Wheatley, 1999; Zheng, 2010). The results show that the resulting polynomial was of order 14, with about 0.5% error. This was so because the quadratic equation used to evaluate the order of the polynomial gave  $n = 13.93$  and  $n = 0.07$ , respectively. We have used the integer value of order 14, since there would be no much loss of generalisation as the error was only about 0.5% of the obtained value of the order of polynomial used in this study. Additionally, the above resulting integral equally represents the growth equation of the wind-felled plantain shoot (side bud outgrowth)

$$y = 8192x^{14} - 28672x^{12} + 39424x^{10} - 26880x^8 + 9472x^6 - 1600x^4 + 98x^2 - 1.$$

We have used the jump resonance phenomena region of occurrence to get the plantain growth equation. This was so because it has been shown that switching operations at discontinuities are responsible for only two stable states (Atherton, 1982). Whereas two switching operations are usually observed in jump resonance phenomena (jump-down and jump-up modes), only the jump-up process (Ogata, 1970; Atherton, 1982) for which the switching was stable could be seen to result in plantain side bud outgrowth. The above can also be seen from the solution to the polynomial equation of the wind-felled plantain system, mainly because only even functions were recorded with the odd coefficients being respectively, zero. The foregoing was revealed from the resulting analyses of the growth equation that the first harmonic content was absent and the second harmonic content of the jump resonance phenomena was determined to be of order 12. Additionally, two visible inputs of gravity and sunlight, amongst several others were also seen to apply most to jump resonance discontinuity to produce plantain side bud outgrowths for plantains knocked down in bad weather (Wilkins, 1989; Asemota, 2004, 2010), as a plantains salvage mechanism. Micro-systems in the wind-felled plantains were able to sense that the plantain had been knocked off-course. Signals were then sent to the appropriate growth initiation centres in a self-organising fashion to ensure that plantain pseudo-stem is brought back to the a priori upright position before it was knocked over by wind (Wilkins, 1989). Mainly because the reaction and growth initiation systems were to set the plantain in its usual or a priori growth position as preset by the processes of adaptation, a side bud ensued in the appropriate inclination angle (Asemota, 2004, 2010) that represents the region of its jump resonance (Atherton, 1982), which was about  $60^\circ$  to the horizontal. The explanation that could readily be adduced is that as soon as the plantain pseudo-stem was knocked off-course, the sensory and reaction signals communicated with each other and growth mechanisms were set in motion. The phase shift of the appropriate magnitude angle ( $60^\circ$ ) was required to cause plantain shoot outgrowth, as determined experimentally by (Asemota, 2004). The sinusoids produced further reveal that a crest was first attained, followed by a trough before the steep ascents showing the up-shoot growth pattern, thereafter. The trough before the ascent represents the force (energy) required for the growing shoot to “tear” the plantain pseudo-stem open for it to access sunlight and present itself once more as a living entity, which is capable of exhibiting self-

organising features . There are large regions of agreement between the polynomial orders of the approximant and exact solutions. The exact solution of polynomial reveals greater ascent and steeper tangent than the approximation of order 14. The divergence between the exact solution of polynomial and its approximant is so wide for large polynomial orders that the errors introduced into the plantain growth equation makes the assumption rather too simplistic: such that there was only good agreement at lower polynomial orders. Similarly, a macroscopic illustration of the two relations shows a very good agreement along most of their two curves. Marked differences began to show from polynomial order of 16. Consequently, and while we chose the exact solution of order 13.93 to be a better approximation to the plantain growth equation and jump resonance function, the approximation of order 14, enabled us to readily determine the nature and form of the plantain growth equation employing Chebyshev polynomials and avoiding the difficulty and complications of evaluating inexact integrals.

## 5. CONCLUSION

We used jump resonance phenomena methodologies of describing functions, Duffing, Hatanaka and Chebyshev polynomials for wind-felled plantain pseudo-stems guyed at a specified angle, which subsequently budded and enabled us to determine plantain jump-up growth equations and harmonic content in this study. In addition, the approximate jump-up plantain polynomial growth equation developed using the describing function approach was of order 14, with about 0.5% error. Also, Chebyshev polynomial coefficients evaluation was used to obtain the polynomial equation of the plantain in the region of its jump resonance to minimise the problems of inexact integral computation. The foregoing procedure was employed, using the benefits of hindsight, that only lightly damped systems exhibit jump resonance, coupled with that fact that, plantain side bud outgrowth represents jump resonance and that the  $60^\circ$  inclination is in the region of jump resonance. This study also established that plantains exhibited even nonlinearity in their jump resonance phenomena, because the first harmonic was absent from results obtained using the describing function method. The presence of the second harmonic in wind-felled plantains jump resonance phenomena is also characteristic of asymmetrical or symmetrical nonlinear systems when subjected to bias inputs, either as constant reference signals or disturbances, where the lowest harmonic is normally the second rather than the third. It was also discovered that only the jump-up mode of the jump resonance phenomena in wind-felled plantains produced the desired results for plantain salvage mechanism, which has applicability to plantain and banana agriculture, loss reduction strategies, investment enhancements and sustainable development. Since it has been established that only soft springs readily exhibit jump resonance phenomenon, it therefore, follows that plantains consist of soft springs. It was also visible from this work that plantain shoot growth equations follow very closely both the exact and approximate solution patterns up to polynomial order, 16. But, thereafter, there was divergence between the approximate and exact solutions, with the approximant revealing a much steeper slope for the plantain growth equation model. Conclusively, we can say that there was good agreement between the two equations at lower orders mainly because the major contribution to the divergence in the two equations was in their last terms of the highest polynomial order. It is, recommended that a sinusoid plus bias (steady or d.c.) signal input be used to modify and subsequently to replace the original characteristic equation when the bias becomes

slowly varying or other signal is used in future research to improve algorithm accuracy of the jump resonance nonlinearity in plantains.

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