Afrika Statistika

ISSN 2316-090X

Afrika Statistika
Vol. 12 (3), 2017, pages 1447–1465.
DOI: http://dx.doi.org/10.16929/as/2017.1447.112

Generalized Stacy-Lindley Mixture Distribution

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Received June 10, 2017. Accepted October 22, 2017

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Abstract. . In this paper, we introduce a five parameter extension of mixture of two Stacy gamma distributions called generalized Stacy- Lindley mixture distribution. Several statistical properties are derived. Two types of estimation techniques are used for estimating the parameters. Asymptotic confidence interval is also calculated for these parameters. Finally, a real data application illustrates the performance of our proposed distribution.

Résumé. Dans cet article, nous introduisons une généralisation de la distributions mélange connue sous le nom de Gamma de Stacy, à une famille de cinq paramètres que nous nommons Distribution mélange de Stacy-Lindey. Plusieurs propriétés statistiques sont prouvées. Deux types de techniques d'estimation sont utilisés pour estimer les paramètres. Les intervalles de confiances asymptotiques sont également fournies. Enfin, une application de données réelles illustre la performance de la distribution proposée.

Key words: Lindley distribution; Stacy gamma distribution; Inequality measures; Uncertainty measures; Maximum likelihood estimation; Asymptotic confidence interval.
AMS 2010 Mathematics Subject Classification : 60E05; 62H12

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1. Introduction

Gamma distribution is a widely used distribution in many fields such as lifetime data analysis, reliability, hydrology, medicine, meteorology, etc. Many authors generalized the gamma distribution by various ways. Stacy (1962) has proposed a three parameter generalized

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gamma distribution. The probability density function of the generalized gamma distribution is given by

$$f(x;a,d,p) = \frac{(\frac{p}{a^d})x^{d-1}e^{-(\frac{x}{a})^p}}{\Gamma(\frac{d}{p})}. \tag{1}$$

This distribution includes many well known distributions as special cases such as the gamma, the exponential, the Weibull and the half-normal distributions. The generalized gamma distribution is appropriated for modeling data with dissimilar types of hazard rate. Cox *et al.* (2007) presented a taxonomy of the hazard functions of the generalized gamma family and applied the proposed taxonomy to study survival after a diagnosis of clinical AIDS during different eras of HIV therapy. Khodabin and Ahmadabadi (2010) discussed some properties of generalized gamma distribution.

The mixture distribution is defined as one of the most crucial ways to obtain new probability distributions in applied probability and several research areas. Lindley (1958) suggested an one parameter distribution to illustrate the difference between fiducial distribution and posterior distribution and has the following probability density function (pdf),

$$f(x;\theta) = \frac{\theta^2}{1+\theta}(1+x)e^{-\theta x};\ x > 0,\ \theta > 0. \tag{2}$$

This distribution is a mixture of exponential (θ) and gamma (2,θ) distributions.

Ghitany *et al.* (2008) developed different properties of Lindley distribution and showed that the Lindley distribution fits better than the exponential distribution based on the waiting times before service of the bank customers. Sankaran (1970) used Lindley distribution as the mixing distribution of a Poisson parameter and the resulting distribution is known as the Poisson-Lindley distribution. Zakerzadeh and Dolati (2009) have obtained a generalized Lindley distribution and discussed its various properties and applications.

Ghitany *et al.* (2013) and Nadarajah *et al.* (2011) have recently proposed two parameter extensions of the Lindley distribution named as the power Lindley and the the generalized Lindley distributions respectively. A discrete form of Lindley distribution was introduced by Gómez and Ojeda (2011) by discretizing the continuous Lindley distribution. Al-Mutairi *et al.* (2013) mentioned that the Lindley distribution belongs to an exponential family and it can be written as a mixture of an exponential and a gamma distribution with shape parameter 2. Location parameter extension of Lindley distribution is extensively discussed by Monsef (2015). Nedjar and Zeghdoudi (2016) introduced gamma Lindley distribution and studied some important properties of their proposed generalization. Zeghdoudi and Nedjar (2016) introduced pseudo Lindley distribution and studied some of its important properties.

To increase the flexibility for modeling purposes it will be useful to consider further generalizations of the existing models. Hence in this study we introduce a generalized Stacy- Lindley mixture distribution which was obtained by mixing two generalized (Stacy) gamma distributions. Some statistical and reliability properties, inequality measures and uncertainty measures of $GSLMD$ are derived in sections 3, 4 and 5. Different methods of estimation are carried out in section 6. Section 7 focuses attention on the asymptotic confidence interval of the parameters. Application of the introduced model is discussed in section 8.

2. Generalized Stacy- Lindley Mixture Distribution

Here we introduced a new mixed distribution, namely the Generalized Stacy- Lindley mixture distribution ($GSLMD$), which is obtained by mixing two generalized gamma distributions namely generalized gamma (α, β, θ) and generalized gamma (η, β, θ) with mixing probabilities $p_1 = \frac{\theta^\delta}{1+\theta^\delta}$ and $p_2 = \frac{1}{1+\theta^\delta}$ respectively, the corresponding pdf has the form

$$f(x;\theta,\alpha,\beta,\eta,\delta) = \frac{\beta e^{-\theta x^\beta}}{1+\theta^\delta}\left\{\frac{\theta^{\alpha+\delta}x^{\alpha\beta-1}}{\Gamma(\alpha)} + \frac{\theta^\eta x^{\eta\beta-1}}{\Gamma(\eta)}\right\}; x > 0, \tag{3}$$

θ, α, β, η, δ >0.

The cumulative distribution function (cdf) and survival function of $GSLMD$ are given by

$$F(x;\theta,\alpha,\beta,\eta,\delta) = \frac{1}{1+\theta^\delta}\left\{\frac{\theta^\delta\Big(\Gamma(\alpha) - \Gamma(\alpha,\theta x^\beta)\Big)}{\Gamma(\alpha)} + \frac{\Big(\Gamma(\eta) - \Gamma(\eta,\theta x^\beta)\Big)}{\Gamma(\eta)}\right\} \tag{4}$$

and

$$\overline{F}(x;\theta,\alpha,\beta,\eta,\delta) = \frac{1}{1+\theta^\delta}\left\{\frac{\theta^\delta\Gamma(\alpha,\theta x^\beta)}{\Gamma(\alpha)} + \frac{\Gamma(\eta,\theta x^\beta)}{\Gamma(\eta)}\right\}, \tag{5}$$

where $\Gamma(a,b) = \int\limits_b^\infty t^{a-1}e^{-t}dt$ is the upper incomplete gamma function.

3. Properties

If X has the $GSLMD$ with density function given in (3), then the r^{th} raw moment $\mu_r^{'}$ about origin is given by, for $r = 1, 2, ...$

$$\begin{aligned}\mu_r^{'} &= \int\limits_0^\infty x^r f(x;\theta,\alpha,\beta,\eta,\delta)dx \\ &= \frac{\theta^{-\frac{r}{\beta}}}{1+\theta^\delta}\left\{\frac{\theta^\delta}{\Gamma(\alpha)}\Gamma(\alpha+\frac{r}{\beta}) + \frac{\Gamma(\eta+\frac{r}{\beta})}{\Gamma(\eta)}\right\}.\end{aligned} \tag{6}$$

The mean and variance of $GSLMD$ are respectively given by

$$\mu = \frac{\theta^{-\frac{1}{\beta}}}{1+\theta^\delta}\left\{\frac{\theta^\delta}{\Gamma(\alpha)}\Gamma(\alpha+\frac{1}{\beta}) + \frac{\Gamma(\eta+\frac{1}{\beta})}{\Gamma(\eta)}\right\} \tag{7}$$

and

$$\sigma^2 = \frac{\theta^{\frac{-2}{\beta}}}{1+\theta^\delta}\Big\{\frac{\theta^\delta}{\Gamma(\alpha)}\Gamma(\alpha+\frac{2}{\beta})+\frac{\Gamma(\eta+\frac{2}{\beta})}{\Gamma(\eta)}\Big\} - \left(\frac{\theta^{-\frac{1}{\beta}}}{1+\theta^\delta}\Big\{\frac{\theta^\delta}{\Gamma(\alpha)}\Gamma(\alpha+\frac{1}{\beta})+\frac{\Gamma(\eta+\frac{1}{\beta})}{\Gamma(\eta)}\Big\}\right)^2. \quad (8)$$

The r^{th} conditional moment of $GSLMD$ is given by

$$\begin{aligned} M_r^{'} &= E(X^r|X>x) \\ &= \frac{\int_x^\infty t^r f(t;\theta,\alpha,\beta,\eta,\delta)dt}{\overline{F}(x;\theta,\alpha,\beta,\eta,\delta)} \\ &= \frac{\theta^{-\frac{r}{\beta}}\Big\{\theta^\delta\Gamma(\eta)\Gamma(\alpha+\frac{r}{\beta},\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta+\frac{r}{\beta},\theta x^\beta)\Big\}}{\theta^\delta\Gamma(\eta)\Gamma(\alpha,\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta,\theta x^\beta)}. \end{aligned} \quad (9)$$

Let X be a continuous random variable with p.d.f. $f(x)$ and c.d.f. $F(x)$. The hazard rate function , reversed hazard rate function, vitality function and mean residual life function of $GSLM$ random variable are respectively defined as

$$h_F(x) = \frac{\beta e^{-\theta x^\beta}\left\{\theta^{\alpha+\delta}x^{\alpha\beta-1}\Gamma(\eta)+\theta^\eta x^{\eta\beta-1}\Gamma(\alpha)\right\}}{\theta^\delta\Gamma(\eta)\Gamma(\alpha,\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta,\theta x^\beta)}, \quad (10)$$

$$\tau_F(x) = \frac{\beta e^{-\theta x^\beta}\left\{\theta^{\alpha+\delta}x^{\alpha\beta-1}\Gamma(\eta)+\theta^\eta x^{\eta\beta-1}\Gamma(\alpha)\right\}}{\theta^\delta\Gamma(\eta)\Big(\Gamma(\alpha)-\Gamma(\alpha,\theta x^\beta)\Big)+\Gamma(\alpha)\Big(\Gamma(\eta)-\Gamma(\eta,\theta x^\beta)\Big)}, \quad (11)$$

$$\begin{aligned} V(x) &= E(X|X>x) \\ &= \frac{\int_x^\infty t f(t;\theta,\alpha,\beta,\eta,\delta)dt}{\overline{F}(x;\theta,\alpha,\beta,\eta,\delta)} \\ &= \frac{\theta^{-\frac{1}{\beta}}\Big\{\theta^\delta\Gamma(\eta)\Gamma(\alpha+\frac{1}{\beta},\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta+\frac{1}{\beta},\theta x^\beta)\Big\}}{\theta^\delta\Gamma(\eta)\Gamma(\alpha,\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta,\theta x^\beta)}, \end{aligned} \quad (12)$$

which is immediate from (9) and

$$\begin{aligned} m(x) &= E(X-x|X>x) = E(X|X>x)-x \\ &= V(x)-x \\ &= \frac{\theta^{-\frac{1}{\beta}}\Big\{\theta^\delta\Gamma(\eta)\Gamma(\alpha+\frac{1}{\beta},\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta+\frac{1}{\beta},\theta x^\beta)\Big\}}{\theta^\delta\Gamma(\eta)\Gamma(\alpha,\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta,\theta x^\beta)} - x. \end{aligned} \quad (13)$$

The moment generating function and characteristic function of X are respectively given by

$$\begin{aligned} M_X(t) &= E(e^{tX}) \\ &= \sum_{r=0}^\infty \frac{t^r}{r!}\frac{\theta^{-\frac{r}{\beta}}}{1+\theta^\delta}\Big\{\frac{\theta^\delta}{\Gamma(\alpha)}\Gamma(\alpha+\frac{r}{\beta})+\frac{\Gamma(\eta+\frac{r}{\beta})}{\Gamma(\eta)}\Big\} \end{aligned} \quad (14)$$

and

$$\phi_X(t) = E(e^{itX}) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \frac{\theta^{-\frac{r}{\beta}}}{1+\theta^\delta} \Big\{ \frac{\theta^\delta}{\Gamma(\alpha)} \Gamma(\alpha + \frac{r}{\beta}) + \frac{\Gamma(\eta + \frac{r}{\beta})}{\Gamma(\eta)} \Big\}. \quad (15)$$

4. Inequality Measures

Bonferroni and Lorenz curves (see, Bonferroni (1930)) have been used in economics to study income and poverty. These curves have many applications in other fields such as demography, reliability, insurance and medicine and engineering. Zenga (2007) proposed an alternative curve based on the ratios of lower and upper means. In this section, we derive the Lorenz, Bonferroni and Zenga curves for $GSLM$ distribution.
The Lorenz, Bonferroni and Zenga curves are defined, respectively, by

$$L_F(x) = \frac{\int_0^x tf(t)dt}{E(X)},\ B_F(x) = \frac{\int_0^x tf(t)dt}{F(X)E(X)}$$

and $A_F(x) = 1 - \frac{\mu^-(x)}{\mu^+(x)}$, where $\mu^-(x) = \frac{\int_0^x tf(t)dt}{F(X)}$ and $\mu^+(x) = \frac{\int_x^\infty tf(t)dt}{\overline{F}(X)}$ are the lower and upper means, respectively.

Lorenz, Bonferroni and Zenga curves for $GSLM$ random variable are respectively given by

$$L_F(x) = \frac{\int_0^x tf(t;\theta,\alpha,\beta,\eta,\delta)dt}{E(X)} = \frac{\theta^\delta\Gamma(\eta)\Big\{\Gamma(\alpha+\frac{1}{\beta}) - \Gamma(\alpha+\frac{1}{\beta},\theta x^\beta)\Big\} + \Gamma(\alpha)\Big\{\Gamma(\eta+\frac{1}{\beta}) - \Gamma(\eta+\frac{1}{\beta},\theta x^\beta)\Big\}}{\Gamma(\eta)\theta^\delta\Gamma(\alpha+\frac{1}{\beta}) + \Gamma(\alpha)\Gamma(\eta+\frac{1}{\beta})},$$

$$B_F(x) = \frac{\int_0^x tf(t;\theta,\alpha,\beta,\eta,\delta)dt}{F(X)E(X)} = \frac{(1+\theta^\delta)\Gamma(\alpha)\Gamma(\eta)\Big(\theta^\delta\Gamma(\eta)\Big\{\Gamma(\alpha+\frac{1}{\beta}) - \Gamma(\alpha+\frac{1}{\beta},\theta x^\beta)\Big\} + \Gamma(\alpha)\Big\{\Gamma(\eta+\frac{1}{\beta}) - \Gamma(\eta+\frac{1}{\beta},\theta x^\beta)\Big\}\Big)}{\Big\{\Gamma(\eta)\theta^\delta\Gamma(\alpha+\frac{1}{\beta}) + \Gamma(\alpha)\Gamma(\eta+\frac{1}{\beta})\Big\}\Big\{\Gamma(\eta)\theta^\delta\Big(\Gamma(\alpha) - \Gamma(\alpha,\theta x^\beta)\Big) + \Gamma(\alpha)\Big(\Gamma(\eta) - \Gamma(\eta,\theta x^\beta)\Big)\Big\}}$$

and

$$A_F(x) = 1 - \frac{\mu^-(x)}{\mu^+(x)},$$

where

$$\mu^-(x) = \frac{\int_0^x tf(t)dt}{F(X)}$$
$$= \frac{\theta^{-\frac{1}{\beta}}\left(\theta^\delta\Gamma(\eta)\left\{\Gamma(\alpha+\frac{1}{\beta})-\Gamma(\alpha+\frac{1}{\beta},\theta x^\beta)\right\}+\Gamma(\alpha)\left\{\Gamma(\eta+\frac{1}{\beta})-\Gamma(\eta+\frac{1}{\beta},\theta x^\beta)\right\}\right)}{\Gamma(\eta)\theta^\delta\left(\Gamma(\alpha)-\Gamma(\alpha,\theta x^\beta)\right)+\Gamma(\alpha)\left(\Gamma(\eta)-\Gamma(\eta,\theta x^\beta)\right)}$$

and

$$\mu^+(x) = \frac{\int_x^\infty tf(t)dt}{\overline{F}(X)}$$
$$= \frac{\theta^{-\frac{1}{\beta}}\left(\theta^\delta\Gamma(\eta)\Gamma(\alpha+\frac{1}{\beta},\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta+\frac{1}{\beta},\theta x^\beta)\right)}{\Gamma(\eta)\theta^\delta\Gamma(\alpha,\theta x^\beta)+\Gamma(\alpha)\Gamma(\eta,\theta x^\beta)}.$$

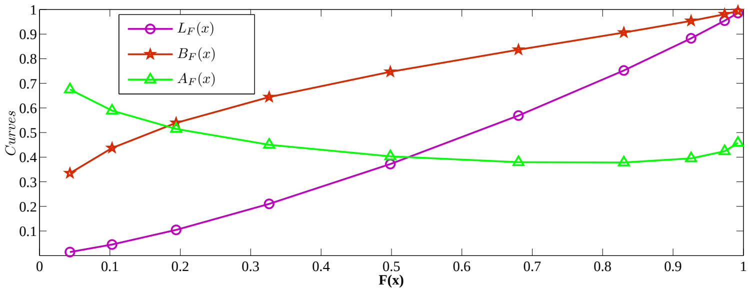

Fig. 1. Plots of Lorenz, Bonferroni and Zenga curves

5. Uncertainty Measures

The concept of entropy was introduced and extensively studied by Shannon (1948). Let X be a non-negative random variable admitting an absolutely continuous cdf $F(x)$ and with pdf $f(x)$. Then the Shannon's entropy associated with X is defined as $H(X) = -\int_0^\infty f(x)\ \ln f(x)\ dx$. It gives the expected uncertainty contained in $f(x)$ about the predictability of an outcome of X. The Shannon's entropy finds immense applications in several branches of learning. In Communication theory, an aspect of interest is the flow of information in some network where information is carried from a transmitter to receiver. Another field of application of Shannon's entropy is Economics. Hart (1971) discussed the entropy and other measures of concentration in the context of Economics and Business concentration. The usefulness of this concept in statistical problems has been examined by several authors, including Kullback and Leibler (1951)and Lindley (1957). Kapur (1968) used the technique of dynamic programming to solve some optimization problems relating to entropy in operation research. In data mining, the entropy is used to define an error function as part of the learning of weights in multilayer preceptrons in neural networks. The entropy error function is then minimized to determine the optimal weights (see, Giudici (2005)).

Rényi's entropy

Several generalizations of Shannon's entropy have been put forward by researchers. A generalization which has received much attention subsequently is due to Rényi (1959). The Rényi's entropy of order ν is defined as $H^{\nu}(X) = \frac{1}{1-\nu} \ln \int\limits_0^{\infty} f^{\nu}(x)\, dx,$ for $\nu > 0, \nu \neq 1$.

Rényi entropy for $GSLM$ random variable is given by

$$H^{\nu}(X) = \frac{1}{(1-\nu)} \ln \int\limits_0^{\infty} f^{\nu}(x;\theta,\alpha,\beta,\eta,\delta)dx; \text{ for } \nu > 0, \nu \neq 1.$$

Now

$$\int\limits_0^{\infty} f^{\nu}(x;\theta,\alpha,\beta,\eta,\delta)dx = \left(\frac{\beta}{1+\theta^{\delta}}\right)^{\nu} \left(\frac{\theta^{\alpha+\delta}}{\Gamma(\alpha)}\right)^{\nu} \sum_{j=0}^{\nu} \frac{1}{\beta}\binom{\nu}{j} \left(\frac{\theta^{\eta}\Gamma(\alpha)}{\theta^{\alpha+\delta}\Gamma(\eta)}\right)^{j} \frac{\Gamma\left(\frac{\nu(\alpha\beta-1)+j\beta(\eta-\alpha)+1}{\beta}\right)}{(\theta\nu)^{\frac{\nu(\alpha\beta-1)+j\beta(\eta-\alpha)+1}{\beta}}}. \tag{16}$$

Therefore,

$$H^{\nu}(X) = \frac{1}{1-\nu} \left\{ \ln \left(\frac{\beta^{\frac{\nu-1}{\nu}} \theta^{\frac{1}{\beta}-\frac{1}{\beta\nu}+\delta} \nu^{\frac{1}{\beta}-\frac{1}{\beta\nu}-\alpha}}{(1+\theta^{\delta})\Gamma(\alpha)} \right)^{\nu} + \ln\left(\sum_{j=0}^{\nu} \binom{\nu}{j} \left(\frac{\theta^{-\delta}\Gamma(\alpha)\nu^{\alpha-\eta}}{\Gamma(\eta)}\right)^{j} \Gamma\Big(\frac{\nu(\alpha\beta-1)+j\beta(\eta-\alpha)+1}{\beta}\Big) \right) \right\}.$$

Havrda-Charvát-Tsallis entropy

Another important generalization of Shannon's entropy is the Havrda-Charvát-Tsallis (HCT) entropy. It was introduced by Havrda and Charvát (1967) and further developed by Tsallis (1988) and Gell-Mann and Tsallis (2004) and is defined as $H^{\rho}(X) = \frac{1}{\rho-1}\left(1 - \int\limits_0^{\infty} f^{\rho}(x)\, dx\right)$, for $\rho > 0, \rho \neq 1$.

Havrda-Charvát-Tsallis entropy for $GSLM$ random variable is given by

$$H^{\rho}(X) = \frac{1}{(\rho-1)}\left(1 - \int_0^{\infty} f^{\rho}(x;\theta,\alpha,\beta,\eta,\delta)dx\right);\ \text{for } \rho > 0, \rho \neq 1.$$

From (16)

$$\int_0^{\infty} f^{\rho}(x;\theta,\alpha,\beta,\eta,\delta)dx = \left(\frac{\beta^{\frac{\rho-1}{\rho}}\theta^{\frac{1}{\beta}-\frac{1}{\beta\rho}+\delta}\rho^{\frac{1}{\beta}-\frac{1}{\beta\rho}-\alpha}}{(1+\theta^{\delta})\Gamma(\alpha)}\right)^{\rho}\sum_{j=0}^{\rho}\binom{\rho}{j}\left(\frac{\theta^{-\delta}\Gamma(\alpha)\rho^{\alpha-\eta}}{\Gamma(\eta)}\right)^{j} \times\Gamma\Big(\frac{\rho(\alpha\beta-1)+j\beta(\eta-\alpha)+1}{\beta}\Big).$$

Therefore

$$H^{\rho}(X) = \frac{1}{(\rho-1)}\left\{1 - \left(\frac{\beta^{\frac{\rho-1}{\rho}}\theta^{\frac{1}{\beta}-\frac{1}{\beta\rho}+\delta}\rho^{\frac{1}{\beta}-\frac{1}{\beta\rho}-\alpha}}{(1+\theta^{\delta})\Gamma(\alpha)}\right)^{\rho}\sum_{j=0}^{\rho}\binom{\rho}{j}\left(\frac{\theta^{-\delta}\Gamma(\alpha)\rho^{\alpha-\eta}}{\Gamma(\eta)}\right)^{j} \times\Gamma\Big(\frac{\rho(\alpha\beta-1)+j\beta(\eta-\alpha)+1}{\beta}\Big)\right\}.$$

6. Different Methods of Estimation

Method of Moments

From (6), the r^{th} raw moment about origin for the $GSLM$ random variable is given by

$$\mu_r^{'} = \frac{\theta^{-\frac{r}{\beta}}}{1+\theta^{\delta}}\Big\{\frac{\theta^{\delta}}{\Gamma(\alpha)}\Gamma(\alpha+\frac{r}{\beta}) + \frac{\Gamma(\eta+\frac{r}{\beta})}{\Gamma(\eta)}\Big\}.$$

Put $r = 1,2,3,4$ and 5, the first five raw moments are obtained. Equating this raw moments to the corresponding sample moments; say $m_1^{'}$, $m_2^{'}$, $m_3^{'}$, $m_4^{'}$ and $m_5^{'}$, where $m_r^{'} = \frac{1}{n}\sum_{i=1}^{n} X_i^r$, we get

$$\frac{\theta^{-\frac{1}{\beta}}}{1+\theta^{\delta}}\Big\{\frac{\theta^{\delta}}{\Gamma(\alpha)}\Gamma(\alpha+\frac{1}{\beta}) + \frac{\Gamma(\eta+\frac{1}{\beta})}{\Gamma(\eta)}\Big\} = m_1^{'}, \tag{17}$$

$$\frac{\theta^{-\frac{2}{\beta}}}{1+\theta^{\delta}}\Big\{\frac{\theta^{\delta}}{\Gamma(\alpha)}\Gamma(\alpha+\frac{2}{\beta}) + \frac{\Gamma(\eta+\frac{2}{\beta})}{\Gamma(\eta)}\Big\} = m_2^{'}, \tag{18}$$

$$\frac{\theta^{-\frac{3}{\beta}}}{1+\theta^{\delta}}\Big\{\frac{\theta^{\delta}}{\Gamma(\alpha)}\Gamma(\alpha+\frac{3}{\beta}) + \frac{\Gamma(\eta+\frac{3}{\beta})}{\Gamma(\eta)}\Big\} = m_3^{'}, \tag{19}$$

$$\frac{\theta^{-\frac{4}{\beta}}}{1+\theta^{\delta}}\Big\{\frac{\theta^{\delta}}{\Gamma(\alpha)}\Gamma(\alpha+\frac{4}{\beta})+\frac{\Gamma(\eta+\frac{4}{\beta})}{\Gamma(\eta)}\Big\}=m_4^{'} \tag{20}$$

and

$$\frac{\theta^{-\frac{5}{\beta}}}{1+\theta^{\delta}}\Big\{\frac{\theta^{\delta}}{\Gamma(\alpha)}\Gamma(\alpha+\frac{5}{\beta})+\frac{\Gamma(\eta+\frac{5}{\beta})}{\Gamma(\eta)}\Big\}=m_5^{'}. \tag{21}$$

Since the above system of equations are nonlinear, the numerical solutions are obtained using iteration procedure. From the practical point of view, we consider a real life data set which is given in section 8 and find out first five sample moments. These moments are equated to the corresponding moments of the population and solve these system of equations iteratively using statistical softwares like *MATHCAD, MATHEMATICA* and *R*, we get the moment estimators of the parameters of $GSLMD$ and are given in Table 1.

Method of Maximum Likelihood

Let $X_1, X_2, ..., X_n$ be a random sample of size n from $GSLMD$ with unknown parameter vector $\Theta = \Big(\theta, \alpha, \beta, \eta, \delta\Big)$. The likelihood function for Θ is

$$\begin{aligned} l(\Theta) &= \prod_{i=1}^{n} f_i\Big(x; \theta, \alpha, \beta, \eta, \delta\Big) \\ &= \left(\frac{\beta}{(1+\theta^{\delta})\Gamma(\alpha)\Gamma(\beta)}\right)^n e^{-\theta\sum_{i=1}^{n} x_i^{\beta}} \prod_{i=1}^{n}\Big\{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1}+\theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}\Big\}. \end{aligned}$$

The partial derivatives of $\log l(\Theta)$ with respect to the parameters are

$$\begin{aligned} \frac{\partial \log l}{\partial \theta} = \frac{-n\delta\theta^{\delta-1}}{(1+\theta^{\delta})} - \sum_{i=1}^{n} x_i^{\beta} + \\ \sum_{i=1}^{n}\left(\frac{(\alpha+\delta)\theta^{\alpha+\delta-1}\Gamma(\eta)x_i^{\alpha\beta-1}+\eta\theta^{\eta-1}\Gamma(\alpha)x_i^{\eta\beta-1}}{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1}+\theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}}\right), \end{aligned} \tag{22}$$

$$\frac{\partial \log l}{\partial \alpha} = -n\psi(\alpha) + \sum_{i=1}^{n}\frac{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1}\log(\theta x_i^{\beta})+\theta^{\eta}\Gamma^{'}(\alpha)x_i^{\eta\beta-1}}{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1}+\theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}}, \tag{23}$$

$$\frac{\partial \log l}{\partial \beta} = \frac{n}{\beta} - \theta\sum_{i=1}^{n} x_i^{\beta}\log(x_i) + \sum_{i=1}^{n}\left(\frac{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1}\log(x_i^{\alpha})+\theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}\log(x_i^{\eta})}{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1}+\theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}}\right), \tag{24}$$

$$\frac{\partial \log l}{\partial \eta} = -n\psi(\eta) + \sum_{i=1}^{n} \left(\frac{\theta^{\alpha+\delta}\Gamma'(\eta)x_i^{\alpha\beta-1} + \theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}\log(\theta x_i^{\beta})}{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1} + \theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}} \right) \tag{25}$$

and

$$\frac{\partial \log l}{\partial \delta} = \frac{-n\theta^{\delta}\log(\theta)}{1+\theta^{\delta}} + \sum_{i=1}^{n} \left(\frac{\theta^{\alpha+\delta}\log(\theta)\Gamma(\eta)x_i^{\alpha\beta-1}}{\theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1} + \theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1}} \right). \tag{26}$$

As in the case of solving moment equations. The MLE of the parameters $\Theta = \left(\theta, \alpha, \beta, \eta, \delta\right)$ say $\widehat{\Theta} = \left(\widehat{\theta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\eta}, \widehat{\delta}\right)$ are obtained by solving the equations $\frac{\partial \log l}{\partial \theta} = 0$, $\frac{\partial \log l}{\partial \alpha} = 0$, $\frac{\partial \log l}{\partial \beta} = 0$, $\frac{\partial \log l}{\partial \eta} = 0$ and $\frac{\partial \log l}{\partial \delta} = 0$. Here first we set moment estimates obtained from the real life data set as the initial solution of the parameters and then solving the above nonlinear system of likelihood equations using statistical softwares like MATHCAD, MATHEMATICA and R, we get the maximum likelihood estimates of the parameters of $GSLMD$ and are given in Table 1. From this table, it is observed that chi-square statistic for the $GSLMD$ is lower than those of competing models showing that our model satisfactorily fits better.

7. Asymptotic Confidence Interval

In this section, we present the asymptotic confidence intervals for the parameters of the $GSLMD$. Let $\widehat{\Theta} = \left(\widehat{\theta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\eta}, \widehat{\delta}\right)$ be the maximum likelihood estimator of $\Theta = \left(\theta, \alpha, \beta, \eta, \delta\right)$. To obtain the asymptotic confidence intervals for these parameters, we can use the large sample property of MLE. Therefore $\left(\widehat{\theta} - \theta,\ \widehat{\alpha} - \alpha,\ \widehat{\beta} - \beta,\ \widehat{\eta} - \eta,\ \widehat{\delta} - \delta\right)^{\tau}$ is asymptotically normally distributed with mean vector $\underline{0} = \left(0, 0, 0, 0, 0\right)^{\tau}$ and estimated variance covariance matrix Δ^{-1}, where Δ is the observed Fisher's information matrix given by

$$\Delta = \begin{pmatrix} -\frac{\partial^2 \log l}{\partial \theta^2} & -\frac{\partial^2 \log l}{\partial \theta \partial \alpha} & -\frac{\partial^2 \log l}{\partial \theta \partial \beta} & -\frac{\partial^2 \log l}{\partial \theta \partial \eta} & -\frac{\partial^2 \log l}{\partial \theta \partial \delta} \\ -\frac{\partial^2 \log l}{\partial \alpha \partial \theta} & -\frac{\partial^2 \log l}{\partial \alpha^2} & -\frac{\partial^2 \log l}{\partial \alpha \partial \beta} & -\frac{\partial^2 \log l}{\partial \alpha \partial \eta} & -\frac{\partial^2 \log l}{\partial \alpha \partial \delta} \\ -\frac{\partial^2 \log l}{\partial \beta \partial \theta} & -\frac{\partial^2 \log l}{\partial \beta \partial \alpha} & -\frac{\partial^2 \log l}{\partial \beta^2} & -\frac{\partial^2 \log l}{\partial \beta \partial \eta} & -\frac{\partial^2 \log l}{\partial \beta \partial \delta} \\ -\frac{\partial^2 \log l}{\partial \eta \partial \theta} & -\frac{\partial^2 \log l}{\partial \eta \partial \alpha} & -\frac{\partial^2 \log l}{\partial \eta \partial \beta} & -\frac{\partial^2 \log l}{\partial \eta^2} & -\frac{\partial^2 \log l}{\partial \eta \partial \delta} \\ -\frac{\partial^2 \log l}{\partial \delta \partial \theta} & -\frac{\partial^2 \log l}{\partial \delta \partial \alpha} & -\frac{\partial^2 \log l}{\partial \delta \partial \beta} & -\frac{\partial^2 \log l}{\partial \delta \partial \eta} & -\frac{\partial^2 \log l}{\partial \delta^2} \end{pmatrix}$$

The elements of Δ are given by

$$\begin{aligned} \Delta_{11} =& \frac{n\delta\theta^{\delta-2}(\delta - 1 - \theta^{\delta})}{(1+\theta^{\delta})^2} - \frac{1}{\theta^2}\sum_{i=1}^{n} \frac{1}{(U_i+V_i)^2} \Bigg\{ (U_i+V_i)\Big((\alpha+\delta)(\alpha+\delta-1)U_i \\ & + \eta(\eta-1)V_i\Big) - \Big((\alpha+\delta)U_i + \eta V_i\Big)^2 \Bigg\}, \end{aligned} \tag{27}$$

$$\Delta_{12} = \Delta_{21} = -\frac{1}{\theta}\sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big\{U_i\Big(1+(\alpha+\delta)\log(\theta x_i^{\beta})\Big)+\eta\psi(\alpha)V_i\Big\} - \Big((\alpha+\delta)U_i+\eta V_i\Big)\Big(U_i\log(\theta x_i^{\beta})+V_i\psi(\alpha)\Big)\Bigg\}, \tag{28}$$

$$\Delta_{13} = \Delta_{31} = \sum_{i=1}^{n} x_i^{\beta}\log x_i - \frac{1}{\theta}\sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big\{(\alpha+\delta)\log(x_i^{\alpha})U_i + \eta\log(x_i^{\eta})V_i\Big\} - \Big((\alpha+\delta)U_i+\eta V_i\Big)\Big(U_i\log(x_i^{\alpha})+V_i\log(x_i^{\eta})\Big)\Bigg\}, \tag{29}$$

$$\Delta_{14} = \Delta_{41} = -\frac{1}{\theta}\sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big\{(\alpha+\delta)\psi(\eta)U_i+(1+\eta\log(\theta x_i^{\beta}))V_i\Big\} - \Big((\alpha+\delta)U_i+\eta V_i\Big)\Big(\psi(\eta)U_i+\log(\theta x_i^{\beta})V_i\Big)\Bigg\}, \tag{30}$$

$$\Delta_{15} = \Delta_{51} = \frac{n\theta^{\delta-1}(1+\delta\log\theta+\theta^{\delta})}{(1+\theta^{\delta})^2} - \frac{1}{\theta}\sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big(1+(\alpha+\delta)\log(\theta)U_i\Big) - \Big((\alpha+\delta)U_i+\eta V_i\Big)U_i\log(\theta)\Bigg\}, \tag{31}$$

$$\Delta_{22} = \frac{n}{(\Gamma(\alpha))^2}\Big\{\Gamma(\alpha)\Gamma^{''}(\alpha)-(\Gamma^{'}(\alpha))^2\Big\} - \sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big(U_i(\log(\theta x_i^{\beta}))^2+\frac{\Gamma^{''}(\alpha)}{\Gamma(\alpha)}V_i\Big) - \Big(U_i\log(\theta x_i^{\beta})+\psi(\alpha)V_i\Big)^2\Bigg\}, \tag{32}$$

$$\Delta_{23} = \Delta_{32} = -\sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big\{U_i\log(x_i)\Big(1+\alpha\log(\theta x_i^{\beta})\Big)+ V_i\psi(\alpha)\log(x_i^{\eta})\Big\} - \Big(U_i\log(\theta x_i^{\beta})+\psi(\alpha)V_i\Big)\Big(U_i\log(x_i^{\alpha})+V_i\log(x_i^{\eta})\Big)\Bigg\}, \tag{33}$$

$$\Delta_{24} = \Delta_{42} = -\sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Bigg\{\psi(\eta)U_i\log(\theta x_i^{\beta})+\psi(\alpha)V_i\Big(\log(\theta)+\log(x_i^{\beta})\Big)\Bigg\} - \Big(U_i\log(\theta x_i^{\beta})+\psi(\alpha)V_i\Big)\Big(\psi(\eta)U_i+V_i\Big(\log(\theta)+\log(x_i^{\beta})\Big)\Big)\Bigg\}, \tag{34}$$

$$\Delta_{25} = \Delta_{52} = -\log(\theta)\sum_{i=1}^{n}\frac{1}{(U_i+V_i)^2}\Bigg\{U_i(U_i+V_i)\log(\theta x_i^{\beta}) - U_i\Big(U_i\log(\theta x_i^{\beta})+\psi(\alpha)V_i\Big)\Bigg\}, \tag{35}$$

$$\Delta_{33} = \frac{n}{\beta^2} + \theta \sum_{i=1}^{n} x_i^{\beta}\Big(\log(x_i)\Big)^2 - \sum_{i=1}^{n} \frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big(U_i\Big(\log(x_i^{\alpha})\Big)^2 + V_i\Big(\log(x_i^{\eta})\Big)^2\Big) - \Big(U_i\log(x_i^{\alpha}) + V_i(\log(x_i^{\eta})\Big)^2\Bigg\}, \tag{36}$$

$$\Delta_{34} = \Delta_{43} = -\sum_{i=1}^{n} \frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big\{\psi(\eta)\log(x_i^{\alpha})U_i + V_i\log(x_i)\Big(1+\eta\log(\theta)+\eta\log(x_i^{\beta})\Big)\Big\} - \Big(U_i\log(x_i^{\alpha}) + V_i\log(x_i^{\eta})\Big)\Big(\psi(\eta)U_i + V_i\Big(\log(x_i^{\beta}) + \log(\theta)\Big)\Big)\Bigg\}, \tag{37}$$

$$\Delta_{35} = \Delta_{53} = -\log(\theta)\sum_{i=1}^{n} \frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)U_i\log(x_i^{\alpha}) - U_i\Big(U_i\log(x_i^{\alpha}) + V_i\log(x_i^{\eta})\Big)\Bigg\}, \tag{38}$$

$$\Delta_{44} = \frac{n}{(\Gamma(\eta))^2}\Big\{\Gamma(\eta)\Gamma''(\eta) - (\Gamma'(\eta))^2\Big\} - \sum_{i=1}^{n} \frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Bigg(\frac{\Gamma''(\eta)}{\Gamma(\eta)}U_i + V_i\log(\theta x_i^{\beta})\Big(\log(\theta)+\log(x_i^{\beta})\Big)\Bigg) - \Big(\psi(\eta)U_i + V_i\log(\theta x_i^{\beta})\Big)\Big(\psi(\eta)U_i + V_i\Big(\log(\theta)+\log(\theta x_i^{\beta})\Big)\Big)\Bigg\}, \tag{39}$$

$$\Delta_{45} = \Delta_{54} = -\sum_{i=1}^{n} \frac{1}{(U_i+V_i)^2}\Bigg\{(U_i+V_i)\Big(\log(\theta)\psi(\eta)U_i\Big) - \Big(\psi(\eta)U_i + \log(\theta x_i^{\beta})V_i\Big)U_i\log(\theta)\Bigg\} \tag{40}$$

and

$$\Delta_{55} = \frac{n\theta^{\delta}(\log(\theta))^2}{(1+\theta^{\delta})^2} - \sum_{i=1}^{n} \frac{1}{(U_i+V_i)^2}\Bigg\{\Big(\log(\theta)\Big)^2 U_i(U_i+V_i) - \Big(\log(\theta)\Big)^2 U_i^2\Bigg\}, \tag{41}$$

where

$$U_i = \theta^{\alpha+\delta}\Gamma(\eta)x_i^{\alpha\beta-1},$$

$$V_i = \theta^{\eta}\Gamma(\alpha)x_i^{\eta\beta-1},$$

$$\psi(a) = \frac{\Gamma'(a)}{\Gamma(a)}$$

and

$$\Gamma^{(n)}(a) = \int_0^\infty t^{a-1}\Big(\log(t)\Big)^n e^{-t}dt$$

is the n^{th} order derivative of gamma function.

The estimated variance-covariance matrix of the parameters $\widehat{\theta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\eta}$ and $\widehat{\delta}$ can be calculated by

$$\begin{bmatrix} \widehat{Var(\theta)} & \widehat{Cov(\theta,\alpha)} & \widehat{Cov(\theta,\beta)} & \widehat{Cov(\theta,\eta)} & \widehat{Cov(\theta,\delta)} \\ \widehat{Cov(\alpha,\theta)} & \widehat{Var(\alpha)} & \widehat{Cov(\alpha,\beta)} & \widehat{Cov(\alpha,\eta)} & \widehat{Cov(\alpha,\delta)} \\ \widehat{Cov(\beta,\theta)} & \widehat{Cov(\beta,\alpha)} & \widehat{Var(\beta)} & \widehat{Cov(\beta,\eta)} & \widehat{Cov(\beta,\delta)} \\ \widehat{Cov(\eta,\theta)} & \widehat{Cov(\eta,\alpha)} & \widehat{Cov(\eta,\beta)} & \widehat{Var(\eta)} & \widehat{Cov(\eta,\delta)} \\ \widehat{Cov(\delta,\theta)} & \widehat{Cov(\delta,\alpha)} & \widehat{Cov(\delta,\beta)} & \widehat{Cov(\delta,\eta)} & \widehat{Var(\delta)} \end{bmatrix} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24} & \Delta_{25} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \Delta_{34} & \Delta_{35} \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & \Delta_{44} & \Delta_{45} \\ \Delta_{51} & \Delta_{52} & \Delta_{53} & \Delta_{54} & \Delta_{55} \end{bmatrix}^{-1}$$

The diagonal elements $\widehat{Var(\theta)}$, $\widehat{Var(\alpha)}$, $\widehat{Var(\beta)}$, $\widehat{Var(\eta)}$ and $\widehat{Var(\delta)}$ are the asymptotic variances of the estimators of $\theta, \alpha, \beta, \eta$ and δ respectively. The approximate $100(1-\varphi)\%$ two-sided confidence intervals for $\theta, \alpha, \beta, \eta$ and δ are $\widehat{\theta} \pm Z_{\frac{\varphi}{2}}\sqrt{\widehat{Var(\theta)}}$, $\widehat{\alpha} \pm Z_{\frac{\varphi}{2}}\sqrt{\widehat{Var(\alpha)}}$, $\widehat{\beta} \pm Z_{\frac{\varphi}{2}}\sqrt{\widehat{Var(\beta)}}$, $\widehat{\eta} \pm Z_{\frac{\varphi}{2}}\sqrt{\widehat{Var(\eta)}}$ and $\widehat{\delta} \pm Z_{\frac{\varphi}{2}}\sqrt{\widehat{Var(\delta)}}$ respectively, where $Z_{\frac{\varphi}{2}}$ is the upper $\frac{\varphi}{2}^{th}$ percentile of a standard normal distribution.

* Here we use the likelihood ratio (LR) test to compare our model with its sub models for a given data. For testing $\eta = \delta = 1$, the LR statistic is given by

$$\omega = 2\Big(\log l(\widehat{\theta}, \widehat{\alpha}, \widehat{\beta}, \widehat{\eta}, \widehat{\delta}) - \log l(\tilde{\theta}, \tilde{\alpha}, \tilde{\beta}, 1, 1)\Big),$$

where $\widehat{\theta}$, $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\eta}$ and $\widehat{\delta}$ are the unrestricted estimates and $\tilde{\theta}$, $\tilde{\alpha}$ and $\tilde{\beta}$ are the restricted estimates. The LR test rejects the null hypothesis if $\omega > \chi^2_d$, where χ^2_d denote the upper $100d\%$ point of the χ^2 distribution with 2 degrees of freedom.

8. Application

In this section, we demonstrate the usefulness of the *GSLMD* by fitting a real data set. The data set was given by Bjerkedal (1960). The data are the survival times of guinea pigs injected with different doses of tubercle bacilli. We used the data set obtained under the regimen 6.6. Guinea pigs are known to have high susceptibility to human tuberculosis. Even an infection initiated with a few virulent tubercle bacilli will lead to progressive disease and death. The data set consists of 72 observations and are listed below:

Survival times of 72 guinea pigs under regimen 6.6

0.12, 0.15, 0.22, 0.24, 0.24, 0.32, 0.32, 0.33, 0.34, 0.38, 0.38,0.43, 0.44, 0.48, 0.52, 0.53, 0.54, 0.54, 0.55, 0.56, 0.57, 0.58, 0.58, 0.59, 0.60, 0.60, 0.60, 0.60, 0.61, 0.62, 0.63, 0.65, 0.65, 0.67, 0.68, 0.70, 0.70, 0.72, 0.73, 0.75, 76, 0.76, 0.81, 0.83, 0.84, 0.85, 0.87, 0.91, 0.95, 0.96, 0.98, 0.99, 1.09, 1.10, 1.21, 1.27, 1.29, 1.31, 1.43, 1.46, 1.46, 1.75, 1.75, 2.11, 2.33, 2.58, 2.58, 2.63, 2.97, 3.41, 3.41, 3.76.

We fit the density functions of the generalized Stacy- Lindley mixture Distribution ($GSLMD$), Lindley distribution (LD_1) ((Lindley (1958)), two-parameter Lindley distribution (LD_2) (Shanker *et al.* (2013)), generalized Lindley distribution (GLD_1)(Zakerzadeh and Dolati (2009)) and a new generalized Lindley distribution (GLD_2) (Abouammoh *et al.* (2015)). The pdfs of the LD_1, LD_2, GLD_1 and GLD_2 distributions are respectively given as

$$f_1(x;\theta) = \frac{\theta^2}{1+\theta}(1+x)e^{-\theta x};\ x>0,\ \theta>0,$$

$$f_2(x;\alpha,\theta) = \frac{\theta^2}{\theta+\alpha}(1+\alpha x)e^{-\theta x};\ x>0,\ \theta>0,\ \alpha>-\theta,$$

$$f_3(x;\alpha,\theta,\gamma) = \frac{\theta^2(\theta x)^{\alpha-1}(\alpha+\gamma x)}{(\gamma+\theta)\Gamma(\alpha+1)}e^{-\theta x};\ x>0,\ \alpha,\theta,\gamma>0$$

and

$$f_4(x;\alpha,\theta) = \frac{\theta^{\alpha}x^{\alpha-2}}{(\theta+1)\Gamma(\alpha)}(x+\alpha-1)e^{-\theta x}; x>0, \theta\geq 0, \alpha\geq 1.$$

Maximum likelihood estimates and moment estimates of the parameters of the distributions and their χ^2 values are given in Table 1. They indicate that $GSLMD$ fits the data set better than the other distributions. Figure 2 shows the plots of the fitted densities. Red line represents $GSLMD$ and all other lines represent LD_1, LD_2, GLD_1 and GLD_1.

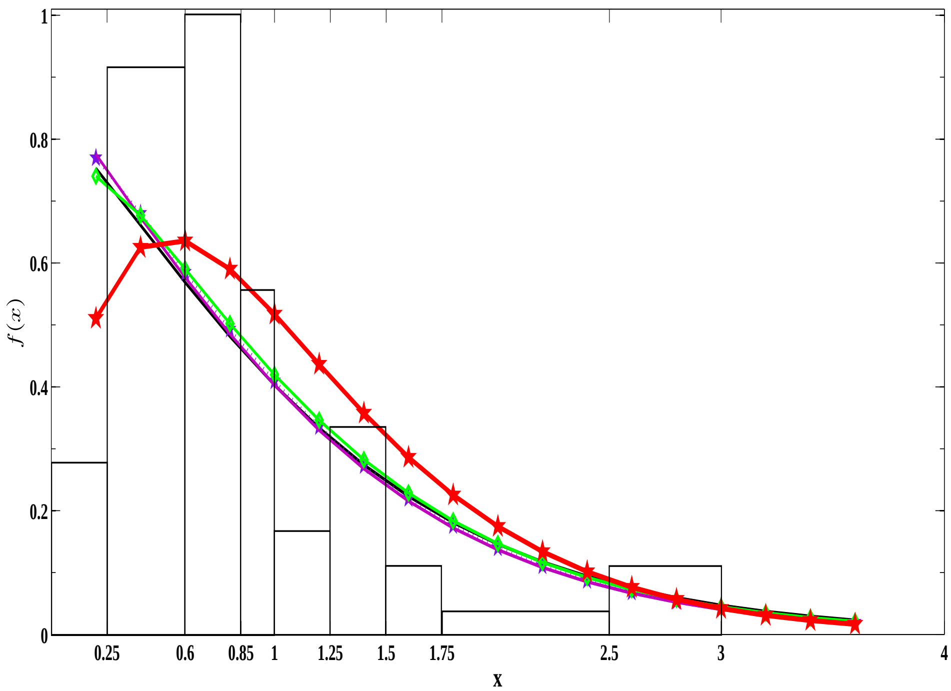

Fig. 2. Fitted probability density function for the real data set.

Table 1. Comparison of fit of $GSLMD$ using different methods of estimation of 72 guinea pigs under regimen 6.6

Count	*Observed*	Expected frequency by method of moments					Expected frequency by MLE				
		LD_1	LD_2	GLD_1	GLD_2	$GSLMD$	LD_1	LD_2	GLD_1	GLD_2	$GSLMD$
0-0.25	5	14	14	15	13	7	13	12	15	12	7
0.25-0.6	23	16	17	17	17	16	18	20	19	19	18
0.6-0.85	18	9	9	9	10	11	9	8	9	9	12
0.85-1.0	6	5	5	5	5	6	5	6	5	5	5
1.0-1.25	3	7	7	6	7	8	7	5	7	7	7
1.25-1.50	6	5	5	5	5	7	5	9	7	7	7
1.5-1.75	2	4	4	4	4	5	4	3	2	3	5
1.75-2.5	2	7	7	7	7	8	6	4	4	5	7
2.5-3	4	2	2	2	2	2	3	3	2	3	2
3-4	3	3	2	2	2	2	2	2	2	2	2
Total	72	72	72	72	72	72	72	72	72	72	72
df		6	4	3	4	2	6	4	3	4	2
Estimated values of parameters		$\widehat{\theta}$=1.42	$\widehat{\alpha}$= 1.21 $\widehat{\theta}$=1.51	$\widehat{\alpha}$=0.9 $\widehat{\gamma}$= 1.55 $\widehat{\theta}$=1.5	$\widehat{\alpha}$=2.1 $\widehat{\theta}$=1.51	$\widehat{\alpha}$= 2.6 $\widehat{\theta}$=2 $\widehat{\beta}$=1 $\widehat{\eta}$=1.5 $\widehat{\delta}$=0.6	$\widehat{\theta}$=1.58	$\widehat{\alpha}$=1.342 $\widehat{\theta}$=1.491	$\widehat{\alpha}$=1.231 $\widehat{\gamma}$=1.432 $\widehat{\theta}$=1.69	$\widehat{\alpha}$=2.314 $\widehat{\theta}$=1.74	$\widehat{\alpha}$=2.83 $\widehat{\theta}$=2.37 $\widehat{\beta}$=1.42 $\widehat{\eta}$=1.3 $\widehat{\delta}$=0.92
χ^2		25.788	20.656	20.751	17.193	13.906	22.398	18.917	19.237	16.862	9.753

9. Conclusions

A mixture of two Stacy gamma distributions called Generalized Stacy-Lindley mixture distribution ($GSLMD$) is proposed. Properties of $GSLMD$ including moments, conditional moments, vitality function, inequality measures and uncertainty measures are derived. Maximum likelihood estimation and moment estimation techniques are used to estimate the model parameters. In addition to this, the observed information matrix and asymptotic confidence interval are also included. Finally, the $GSLMD$ model is fitted to a real data set to illustrate the usefulness of the distribution and it is concluded that our model performs well compared to the existing Lindley models.çç

Acknowledgement. The authors are highly thankful for some of the helpful comments of the referee.

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