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A Modified Procedure for Estimating the Population Mean in Two-occasion Successive Samplings

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Abstract. . This paper addresses the problem of estimating the current population mean in two occasion successive sampling. Utilizing the readily available information on two auxiliary variables on both occasions and the information on study variable from the previous occasion, some new estimation procedures have been developed. Properties of the proposed estimators have been studied and their respective optimum replacement policies are discussed. Relative comparison of efficiencies of the suggested estimators with the sample mean estimator when there is no matching from previous occasion and the optimum successive sampling estimator when no auxiliary information is used have been incorporated. Empirical study is carried out to judge the merits of the suggested estimators and suitable recommendations are given. We have also added a practical application in order to examine the performance of the proposed estimators.

Key words: Successive sampling, Study variable, Auxiliary variable, Mean squared error, Optimum Replacement Policy.
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Abstract in French - Résumé Ce papier traite du problème de l'estimation de la moyenne d'un paramètre d'une population dans le schéma d'un échantillonnage en deux étapes successives. De nouvellesprocédures d'estimation sont développées, basées sur l'information fournies par les deux variables auxiliaires lors dans les deux éetapes et celle reçue de l'estimation de la première étape.Les propriétés de ces estimateurs ont été étudiées et la stratéegie optimale de leur remplacement sont discutées. L'effcacitée relative des nouveaux estimateurs a été présentée, d'abord par rapport à la moyenne empirique lorsqu'il n'y a pas de correspondances avec la première étape, ensuite et par rapport à l'estimateur optimal de l'échantillonnage successif lorsque les variables auxiliaires ne sont pas utilisées. Une étude de simulation a été entreprise pour montrer les mérites de cette méthode, ayant aboutit à des recommandations. Enfin, nous avons ajouté une application pratique pour examiner la performance de nos estimateurs.

1. Introduction

Successive sampling is used widely in applied sciences, sociology, commerce, finance and economic researches .The sampling on two occasions is resorted to when the same variate is measured on two different occasions. In such situation, a subsample of units of the first occasion can be retained in the sample of the second occasion also (which is known as matched portion) and the estimators such ratio, regression, product and their ramification can be formed. These estimators are then combined with the estimators based on unmatched portion of the second occasion, for instance, see Cochran (1977), pp. 346-355. Sen (1971) considered the estimators for the population mean on the current occasion using information on two auxiliary variables available on previous occasion. Feng and (1997), and Biradar and Singh (2001) and Singh *et al.* (2011) used the auxiliary information on both the occasions for estimating the current population mean in two-occasion successive sampling. In some situations information on two or more than two auxiliary variables may be readily available or may be made available by diverting a small amount of fund available for the survey, for instance, see Singh and Vishwakarma (2007), Singh and Vishwakarma (2007) and Singh and Vishwakarma (2009), Singh and Pal (2015), Singh and Pal (2015), Singh and Pal (2015) and Singh and Pal (2015), Singh and Pal (2016), Singh and Pal (2016) and Singh and Pal (2016), and Singh *et al.* (2016).

The aim of the present work is to suggest a more efficient estimator of the population mean on current occasion using information on the two stable auxiliary variables on both the occasions. The behaviors of the suggested estimators are examined through empirical means and consequently suitable recommendations have been presented.

2. Formulation of Estimator

We assume that the population consists of N units, which is supposed to be remaining unchanged in size over two occasions. The character under study is designated (x, y) on the first (second) occasion respectively. It is assumed that the information on two stable auxiliary variables z_1 and z_2 whose population means are known and closely associated with x and y is available on first (second) occasion respectively. A simple random sample of size n is drawn on the first occasion. A random subsample of size $m = n\lambda$ is retained (matched) for its use on the second occasion; while a fresh (unmatched) simple random sample (without replacement) of size $u = (n - m) = n\mu$ is drawn on the second occasion from the entire population so that the sample size on the current (second) occasion remains n; λ and μ $(\lambda + \mu)$ are the fractions of the matched and fresh samples , respectively, on the current (second) occasion . The optimum values of λ and μ should be chosen for the purpose.

We have used the following notations throughout the paper.

$\overline{X}$, $\overline{Y}$: Population means of the study variables x and y respectively.

$\overline{Z}_1$, $\overline{Z}_2$: Population means of the auxiliary variables Z_2 and Z_1 respectively.

The sample means of the respective variables on the sample size shown in subscripts : $\overline{x}_n$ $\overline{x}_m$, $\overline{y}_u$, $\overline{y}_m$, $\overline{z}_{jn}$, $\overline{z}_{ju}$, $\overline{z}_{jm}$ (j=1,2).

The population variance of the variable x:

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{X})^2.$$

The population variances S_y^2, $S_{Z_1}^2$, $S_{Z_2}^2$ of the variables y, Z_1 and Z_2 respectively,

The population coefficients of variation C_y, C_x, C_{z_1}, C_{z_2} of the variables y, x, z_1 and z_2 respectively,

The sampling fraction $f = n/N$.

The population partial regression coefficient of y on z_1 :

$$\beta_{01.2} = \frac{\beta_{01} - \beta_{02}\beta_{21}}{1 - \beta_{12}\beta_{21}}$$

The population partial regression coefficient of y on z_2 :

$$\beta_{02.1} = \frac{\beta_{02} - \beta_{01}\beta_{12}}{1 - \beta_{12}\beta_{21}}.$$

The sample estimate of $\beta_{01.2}$ based on the sample of the size u

$$\hat{\beta}_{01.2}^{(u)} = \frac{\hat{\beta}_{01}^{(u)} - \hat{\beta}_{02}^{(u)}\hat{\beta}_{21}^{(u)}}{1 - \hat{\beta}_{12}^{(u)}\hat{\beta}_{21}^{(u)}}$$

The sample estimate of $\beta_{02.1}$ based on the sample of the size u,

$$\hat{\beta}_{02.1}^{(u)} = \frac{\hat{\beta}_{02}^{(u)} - \hat{\beta}_{01}^{(u)}\hat{\beta}_{12}^{(u)}}{1 - \hat{\beta}_{12}^{(u)}\hat{\beta}_{21}^{(u)}}.$$

The estimate of population regression coefficient β_{01} of y on z_1 based on the sample of size u

$$\hat{\beta}_{01}^{(u)} = \frac{s_{yz_1(u)}}{s^2_{z_1(u)}},$$

The estimate of population regression coefficient β_{02} of y on z_2 based on the sample of size u,

$$\hat{\beta}_{02}^{(u)} = \frac{s_{yz_2(u)}}{s^2_{z_2(u)}},$$

The estimate of population regression coefficient β_{12} of z_1 on z_2 based on the sample of size u,

$$\hat{\beta}_{12}^{(u)} = \frac{s_{z_1z_2(u)}}{s^2_{z_2(u)}}$$

The estimate of population regression coefficient β_{21} of z_2 on z_1 based on the sample of size u,

$$\hat{\beta}_{21}^{(u)} = \frac{s_{z_1z_2(u)}}{s^2_{z_1(u)}}$$

where

$$s_{yz_1(u)} = \frac{1}{u-1}\sum_{i=1}^{u}(y_i - \overline{y}_u)(z_{1i} - \overline{z}_{1u}), \quad s_{yz_2(u)} = \frac{1}{u-1}\sum_{i=1}^{u}(y_i - \overline{y}_u)(z_{2i} - \overline{z}_{2u}),$$

$$s^2_{z_1(u)} = \frac{1}{u-1}\sum_{i=1}^{u}(z_{1i} - \overline{z}_{1u})^2, \quad s^2_{z_2(u)} = \frac{1}{u-1}\sum_{i=1}^{u}(z_{2i} - \overline{z}_{2u})^2$$

and

$$s_{z_1z_2(u)} = \frac{1}{u-1}\sum_{i=1}^{u}(z_{1i} - \overline{z}_{1u})(z_{2i} - \overline{z}_{2u}),$$

$\hat{\beta}_{01.2}^{(m)}$, $\hat{\beta}_{02.1}^{(m)}$, $\hat{\beta}_{01.2}^{(n)}$ and $\hat{\beta}_{02.1}^{(n)}$ are defined similarly.

We also have :

The sample estimate of population regression coefficient β_{0x} of y on x based on sample of size m

$$\hat{\beta}_{0x}^{(m)} = \frac{s_{0x(m)}}{s^2_{x(m)}}$$

The sample estimate of population regression coefficient β_{0x} of y on x based on sample of size n,

$$\hat{\beta}_{0x}^{(n)} = \frac{s_{0x(n)}}{s^2_{x(n)}}$$

$$s_{0x(m)} = \frac{1}{m-1}\sum_{i=1}^{m}(y_i - \overline{y}_m)(x_i - \overline{x}_m), \quad s^2_{x(m)} = \frac{1}{m-1}\sum_{i=1}^{m}(x_i - \overline{x}_m)^2$$

The sample estimate of the partial regression coefficient $\beta_{x1.2}$ of x on z_1 based on the sample of the size m

$$\hat{\beta}^{(m)}_{x1.2} = \frac{\hat{\beta}^{(m)}_{x1} - \hat{\beta}^{(m)}_{x2}\hat{\beta}^{(m)}_{12}}{1 - \hat{\beta}^{(m)}_{12}\hat{\beta}^{(m)}_{21}},$$

The sample estimate of the partial regression coefficient $\beta_{x2.1}$ of x on z_2 based on the sample of the size m

$$\hat{\beta}^{(m)}_{x2.1} = \frac{\hat{\beta}^{(m)}_{x2} - \hat{\beta}^{(m)}_{x1}\hat{\beta}^{(m)}_{21}}{1 - \hat{\beta}^{(m)}_{12}\hat{\beta}^{(m)}_{21}},$$

$$\hat{\beta}^{(m)}_{x1} = \frac{s_{xz_1(m)}}{s^2_{z_1(m)}}, \hat{\beta}^{(m)}_{x2} = \frac{s_{xz_2(m)}}{s^2_{z_2(m)}}$$

$$s^2_{z_1(m)} = \frac{1}{m-1}\sum_{i=1}^{m}(z_{1i} - \overline{z})^2, \quad s^2_{z_2(m)} = \frac{1}{m-1}\sum_{i=1}^{m}(z_{2i} - \overline{z})^2,$$

$\beta^{(n)}_{x1.2}$ and $\beta^{(n)}_{x2.1}$ are similarly defined.

ρ_{0x}, ρ_{01}, ρ_{02}, ρ_{x1}, ρ_{x2}, and ρ_{12} are the correlation between $(y\&x)$, $(y\&z_1)$, $(y\&z_2)$, $(x\&z_1)$, $(x\&z_2)$ and $(z_1\&z_2)$ respectively. Define also,

$$\rho_{01.2} = \frac{\rho_{01} - \rho_{02}\rho_{12}}{1-\rho^2_{12}}, \ \rho_{02.1} = \frac{\rho_{02} - \rho_{01}\rho_{12}}{1-\rho^2_{12}}, \ \rho^2_{0.12} = \frac{\rho^2_{01} + \rho^2_{02} - 2\rho_{01}\rho_{02}\rho_{12}}{1-\rho^2_{12}}$$

$$\rho_{x1.2} = \frac{\rho_{x1} - \rho_{x2}\rho_{12}}{1-\rho^2_{12}}, \ \rho_{x2.1} = \frac{\rho_{x2} - \rho_{x1}\rho_{12}}{1-\rho^2_{12}}, \ \rho^2_{x.12} = \frac{\rho^2_{x1} + \rho^2_{x2} - 2\rho_{x1}\rho_{x2}\rho_{12}}{1-\rho^2_{12}}$$

Now to estimate the population mean $\overline{Y}$ on the current (second) occasion in two occasion some estimators are suggested. One is based on the fresh sample of size $u = n\mu$ drawn on the second occasion and structured as

$$t_u = \overline{y}_u + \hat{\beta}^{(u)}_{01.2}(\overline{Z}_1 - \overline{z}_{1u}) + \hat{\beta}^{(u)}_{02.1}(\overline{Z}_2 - \overline{z}_{2u}). \quad (1)$$

Three modified regression type estimators based on the sample of size $m(= n\lambda)$ common to both the occasions are defined by

$$t_{m1} = \overline{y}_m + \hat{\beta}^{(m)}_{0x}(\overline{x}^*_n - \overline{x}^*_m) + \hat{\beta}^{(m)}_{01.2}(\overline{Z}_1 - \overline{z}_{1m}) + \hat{\beta}^{(m)}_{02.1}(\overline{Z}_2 - \overline{z}_{2m}). \quad (2)$$

$$t_{m2} = \overline{y}_m + \hat{\beta}^{(m)}_{0x}(\overline{x}^*_n - \overline{x}^*_m) + \hat{\beta}^{(m)}_{01.2}(\overline{Z}_1 - \overline{z}_{1m}) + \hat{\beta}^{(m)}_{02.1}(\overline{Z}_2 - \overline{z}_{2m}). \quad (3)$$

$$t_{m3} = \overline{y}_m + \hat{\beta}^{(m)}_{0x}(\overline{x}^{***}_n - \overline{x}^{***}_m) + \hat{\beta}^{(m)}_{01.2}(\overline{Z}_1 - \overline{z}_{1m}) + \hat{\beta}^{(m)}_{02.1}(\overline{Z}_2 - \overline{z}_{2m}). \quad (4)$$

where

$$\overline{x}_n^* = \left(\overline{x}_n + \hat{\beta}_{x1}^{(n)}(\overline{Z}_1 - \overline{z}_{1n})\right)\frac{\overline{Z}_2}{\overline{z}_{2n}}, \tag{5}$$

$$\overline{x}_m^* = \left(\overline{x}_m + \hat{\beta}_{x1}^{(m)}(\overline{Z}_1 - \overline{z}_{1m})\right)\frac{\overline{Z}_2}{\overline{z}_{2m}},$$

$$\overline{x}_n^{**} = \left(\overline{x}_n + \hat{\beta}_{x2}^{(n)}(\overline{Z}_2 - \overline{z}_{2n})\right)\frac{\overline{Z}_1}{\overline{z}_{1n}}, \tag{6}$$

$$\overline{x}_m^* = \left(\overline{x}_m + \hat{\beta}_{x2}^{(m)}(\overline{Z}_2 - \overline{z}_{2m})\right)\frac{\overline{Z}_1}{\overline{z}_{1m}},$$

$$\bar{x}_n^{***} = [x_n + \widehat{\beta}_{x1.2}^{(n)}(\bar{Z}_1 - \bar{z}_{1n}) + \widehat{\beta}_{x2.1}^{(n)}(\bar{Z}_2 - \bar{z}_{2n})], \tag{7}$$

$$\bar{x}_m^{***} = [\bar{x}_m + \widehat{\beta}_{x1.2}^{(m)}(\bar{Z}_1 - \bar{z}_{1m}) + \widehat{\beta}_{x2.1}^{(m)}(\bar{Z}_2 - \bar{z}_{2m})]. \tag{8}$$

Combining the estimators t_u and t_{mi} $(i = 1, 2, 3)$, we have three estimators for the population mean $\bar{Y}$ as

$$t_1 = \eta_1 t_u + (1 - \eta_1)t_{m1}, \tag{9}$$

$$t_2 = \eta_2 t_u + (1 - \eta_2)t_{m2}, \tag{10}$$

and

$$t_3 = \eta_3 t_u + (1 - \eta_3)t_{m3}, \tag{11}$$

where the η_i's, $(i = 1, 2, 3)$ are unknown scalars to be determined under certain criterions.

3. Mean Squared Errors of the Estimators t_1, t_2 and t_3

In the following theorem we give the mean squared errors of the estimators t_u, t_m1, t_m2 and t_m3.

Theorem 1. *The mean squared errors (MSEs) of the estimators t_u, t_m1, t_m2 and t_m3, to the terms up to second order moments (or alternatively up to the terms of order), are respectively given by*

$$MSE(t_u) = \left(\frac{1}{u} - \frac{1}{N}\right) S_y^2 A_3, \tag{12}$$

$$MSE(t_{m1}) = S_y^2\left[\frac{1}{m}A_1 + \frac{1}{n}A_2 - \frac{1}{N}A_3\right], \tag{13}$$

$$MSE(t_{m2}) = S_y^2\left[\frac{1}{m}B_1 + \frac{1}{n}B_2 - \frac{1}{N}A_3\right], \tag{14}$$

$$MSE(t_{m3}) = S_y^2 \left[\frac{1}{m} D_1 + \frac{1}{n} D_2 - \frac{1}{N} A_3 \right], \tag{15}$$

where

$$A_1 = (A_3 - A_2),$$

$$\begin{aligned} A_2 &= 2\rho_{0x}\rho_{12}\rho_{01.2}\left(\frac{C_{z_2}}{C_x}\right) + 2\rho_{0x}\rho_{02.1}\left\{1 + \left(\frac{C_x}{C_{z_2}}\right)(\rho_{x1}\rho_{12} - \rho_{x2})\right\}\left(\frac{C_{z_2}}{C_x}\right) \\ &- 2\rho_{0x}\left\{\rho_{02} + \rho_{01}\rho_{x1}\left(\frac{C_x}{C_{z_2}}\right)\right\}\left(\frac{C_{z_2}}{C_x}\right) - \rho_{0x}^2\left(\frac{C_{z_2}^2}{C_x}\right)\left\{1 - \left(\frac{C_x^2}{C_{z_2}^2}\right)(1 + \rho_{x1}^2) + 2(\rho_{x1}\rho_{12} - \rho_{x2})\left(\frac{C_x}{C_{z_2}}\right)\right\}, \end{aligned}$$

$$A_3 = (1 - \rho_{01.2}^2),$$

$$B_1 = (A_3 - B_2),$$

$$\begin{aligned} B_2 &= 2\rho_{0x}\rho_{12}\rho_{02.1}\left(\frac{C_{z_1}}{C_x}\right) + 2\rho_{0x}\rho_{01.2}\left\{1 + \left(\frac{C_x}{C_{z_1}}\right)(\rho_{x2}\rho_{12} - \rho_{x1})\right\}\left(\frac{C_{z_1}}{C_x}\right) \\ &- 2\rho_{0x}\left\{\rho_{01} + \rho_{02}\rho_{x2}\left(\frac{C_x}{C_{z_1}}\right)\right\}\left(\frac{C_{z_1}}{C_x}\right) - \rho_{0x}^2\left(\frac{C_{z_1}^2}{C_x^2}\right)\left\{1 - \left(\frac{C_x^2}{C_{z_1}^2}\right)(1 + \rho_{x2}^2) + 2(\rho_{x2}\rho_{12} - \rho_{x1})\left(\frac{C_x}{C_{z_1}}\right)\right\}, \end{aligned}$$

$$D_1 = (A_3 - D_2),$$

$$D_2 = [\rho_{0x}^2(1 + \rho_{x.12}^2) - 2\rho_{0x}(\rho_{01}\rho_{x1.2} + \rho_{02}\rho_{x2.1})].$$

Proof. It is simple, and so, omitted.

The covariance between the estimator t_u and t_{mj}, $(j = 1, 2, 3)$ to the first degree of approximation, is given by .

$$\text{Cov}(t_u, t_{mj}) = -\frac{S_y^2}{N}(1 - \rho_{0.12}^2) = -\frac{S_y^2}{N} A_3 \tag{16}$$

The mean squared error of the combined estimator t_u and t_{mj}, $(j = 1, 2, 3)$ to the first degree of approximation is obtained as

$$\begin{aligned} \text{MSE}(t_j) &= E(t_j - \bar{Y})^2 \\ &= E\left[\eta_j t_u + (1 - \eta_j)t_{mj} - \bar{Y}\right]^2 \\ &= E\left[\eta_j(t_u - \bar{Y}) + (1 - \eta_j)(t_{mj} - \bar{Y})\right]^2 \\ &= \left[\eta_j^2 E(t_u - \bar{Y})^2 + (1 - \eta_j)^2 E(t_{mj} - \bar{Y})^2 + 2\eta_j(1 - \eta_j)E(t_u - \bar{Y})(t_{mj} - \bar{Y})\right] \\ &= \left[\eta_j^2 \text{MSE}(t_u) + (1 - \eta_j)^2 MSE(t_{mj}) + 2\eta_j(1 - \eta_j)\text{Cov}(t_u, t_{mj})\right], \ (j = 1, 2, 3) \end{aligned} \tag{17}$$

Putting the values of MSE(t_u), and MSE(t_{mj}) (j=1, 2, 3) from (12), (13), (14), (15) and (16), we get to the first degree of approximation in Corollary 1. Further we give the MSEs of the estimators t_u and t_{mj}, $(j = 1, 2, 3)$, and under the following assumption.

Assumption 3.1: Since and denote the same study variable over two occasions and (z_1, z_2) are two auxiliary variables correlated to x and y, therefore, as mentioned in Murthy (1967), p.325, Reddy (1978), and Singh and Ruiz-Espejo (2003), the coefficient of variation is stable over time and following Cochran (1977) and Feng and (1997), the coefficient of variation x, y, z_1 and z_2 are approximately assumed to be the same i.e. $(C_x \approxeq C_{z_1} \approxeq C_{z_2} \approxeq C_y)$.

Corollary 1. *: Under the Assumption (3.1), the MSEs of the estimators t_u and t_{mj}, $(j = 1, 2, 3)$, given by (12), (13), (14) and (15) respectively reduce to:*

$$MSE(t_u) = \left(\frac{1}{u} - \frac{1}{N}\right) S_y^2 A_3, \tag{18}$$

$$MSE(t_{m1}) = S_y^2 \left[\frac{1}{m} A_1^* + \frac{1}{n} A_2^* - \frac{1}{N} A_3\right], \tag{19}$$

$$MSE(t_{m2}) = S_y^2 \left[\frac{1}{m} B_1^* + \frac{1}{n} B_2^* - \frac{1}{N} A_3\right], \tag{20}$$

$$MSE(t_{m3}) = S_y^2 \left[\frac{1}{m} D_1 + \frac{1}{n} D_2 - \frac{1}{N} A_3\right], \tag{21}$$

where

$$A_1^* = (A_3 - A_2^*),$$

$$A_2^* = \left[2\rho_{0x}\rho_{12}\rho_{01.2} + 2\rho_{0x}\rho_{02.1}(1 - \rho_{x2} + \rho_{x1}\rho_{12}) - 2\rho_{0x}(\rho_{02} + \rho_{01}\rho_{x1}) - \rho_{0x}^2(2\rho_{x1}\rho_{12} - 2\rho_{x2} - \rho_{x1}^2)\right],$$

$$B_1^* = (A_3 - B_2^*),$$

$$B_2^* = \left[2\rho_{0x}\rho_{02.1}(1 - \rho_{x1} + \rho_{x2}\rho_{12}) + 2\rho_{0x}\rho_{12}\rho_{0.21} - 2\rho_{0x}(\rho_{01} + \rho_{x2}\rho_{02}) - \rho_{0x}^2(2\rho_{x2}\rho_{12} - 2\rho_{x1} - \rho_{x2}^2)\right],$$

and A_3, D_1 and D_2 are same as defined earlier.

The covariance between the estimators t_u and t_{mj}, $(j = 1, 2, 3)$, to the first degree of approximation under the Assumption (3.1) is same as given in 16. Putting the values of $MSE(t_u)$, $MSE(t_{mj})$ and $Cov(t_u, t_{mj})$ from 16 and 18 to 21 one can easily get the $MSE(t_j)$, $(j = 1, 2, 3)$ to the first degree of approximation under the Assumption (3.1).

4. Minimum Mean Squared Errors of the Estimator t_j, $(j = 1, 2, 3)$

Differentiating 17 with respect to η_j and equating to zero i.e.,

$$\frac{\partial \text{MSE}(t_j)}{\partial \eta_j} = 0,$$

we get the optimum values of the η_j's (j=1, 2, 3) as

$$\eta_{j_{opt}} = \frac{[\text{MSE}(t_{mj}) - \text{Cov}(t_u, t_{mj})]}{[\text{MSE}(t_u) + \text{MSE}(t_{mj}) - 2\text{Cov}(t_u, t_{mj})]}. \tag{22}$$

Now inserting the value of $\eta_{j_{opt}}$ from equation (22) in (17), we get the minimum MSE of the estimator t_j as

$$\min \text{MSE}(t_j) = \frac{[\text{MSE}(t_u)\text{MSE}(t_{mj}) - \{\text{Cov}(t_u, t_{mj})\}^2]}{[\text{MSE}(t_u) + \text{MSE}(t_{mj}) - \text{Cov}(t_u, t_{mj})]}, \; \jmath = 1, 2, 3. \tag{23}$$

Substituting the values of $MSE(t_u)$, $MSE(t_{mj})$ and $Cov(t_u, t_{mj})$ from equations (12), (13), (14), (15) and (16) in (23) we get the simplified values of $\eta_{j_{opt}}$ and $\min .MSE(t_j)_{opt}$, $(j = 1, 2, 3)$ as

$$\eta_{1_{opt}} = \frac{\mu(A_3 - \mu A_2)}{(A_3 - \mu^2 A_2)}. \tag{24}$$

$$\eta_{2_{opt}} = \frac{\mu^*(A_3 - \mu^* B_2)}{(A_3 - \mu^{*2} B_2)}, \tag{25}$$

$$\eta_{3_{opt}} = \frac{\mu^{**}(A_3 - \mu^{**} D_2)}{(A_3 - \mu^{**2} D_2)}, \tag{26}$$

$$\min .MSE(t_1) = \frac{A_3[A_3(1-f) - \mu A_2 + \mu^2 f A_2]S_y^2}{n(A_3 - \mu^2 A_2)}, \tag{27}$$

$$\min .MSE(t_2) = \frac{A_3[A_3(1-f) - \mu^* B_2 + \mu^{*2} f B_2]S_y^2}{n(A_3 - \mu^* B_2)}, \tag{28}$$

$$\min .MSE(t_3) = \frac{A_3[A_3(1-f) - \mu^{**} C_2 + \mu^{**2} f D_2]S_y^2}{n(A_3 - \mu^{**} D_2)}, \tag{29}$$

where μ, μ^*, μ^{**} are the fractions of fresh samples drawn at the current (second) occasion. Under the Assumption (3.1), the expressions (24) to (29) respectively reduce to:

$$\eta_{1_{opt}}^{(1)} = \frac{\mu_{(1)}(A_3 - \mu_{(1)} A_2^*)}{(A_3 - \mu_{(1)}^2 A_2^*)}, \tag{30}$$

$$\eta_{2_{opt}}^{(1)} = \frac{\mu_{(1)}^*(A_3 - \mu_{(1)}^* B_2^*)}{(A_3 - \mu_{(1)}^{*2} B_2^*)}, \tag{31}$$

$$\eta_{3_{opt}}^{(1)} = \frac{\mu_{(1)}^{**}(A_3 - \mu_{(1)}^{**} D_2)}{(A_3 - \mu_{(1)}^{**2} D_2)}, \tag{32}$$

$$\min.MSE(t_1)_1 = \frac{A_3[A_3(1-f) - \mu_{(1)}A_2^* + \mu_{(1)}^2 f A_2^*]S_y^2}{n(A_3 - \mu_{(1)}^2 A_2^*)}, \tag{33}$$

$$\min.MSE(t_2)_1 = \frac{A_3[A_3(1-f) - \mu_{(1)}^* B_2^* + \mu_{(1)}^* B_2^* f]S_y^2}{n(A_3 - \mu_{(1)}^* B_2^*)}, \tag{34}$$

$$\min.MSE(t_3)_1 = \frac{A_3[A_3(1-f) - \mu_{(1)}^{**} C_2 + \mu_{(1)}^{**} f D_2]S_y^2}{n(A_3 - \mu_{(1)}^{***} D_2)}, \tag{35}$$

where μ, $\mu_{(1)}^*$, $\mu_{(1)}^{***}$are the fractions of fresh samples drawn at the current (second) occasion.

4.1. Optimum Replacement Strategies of the Estimators $t_j (j = 1, 2, 3)$

To get the optimum values of the fractions μ, μ^* and μ^{**} of samples (to be drawn afresh sample at the second occasion) so that the population mean $\overline{Y}$ may estimated with maximum precision and minimum cost, it is desired to minimize $\min.MSE(t_1)$, $min.MSE(t_2)$ and $min.MSE(t_3)$ respectively with μ, μ^* and μ^{**}, which results in quadratic equations in μ, μ^* and μ^{**}, say $\widehat{\mu}$, $\widehat{\mu}^*$ and $\widehat{\mu}^{**}$, respectively

$$\mu^2 A_2 - 2\mu A_3 + A_3 = 0, \tag{36}$$

$$\widehat{\mu} = \frac{A_3 \pm \sqrt{A_1 A_3}}{A_2}, \tag{37}$$

$$\mu^{*2} B_2 - 2\mu^* A_3 + A_3 = 0, \tag{38}$$

$$\widehat{\mu}^* = \frac{A_3 \pm \sqrt{B_1 A_3}}{B_2}, \tag{39}$$

$$\mu^{**2} C_2 - 2\mu^{**} A_3 + A_3 = 0, \tag{40}$$

$$\widehat{\mu}^{**} = \frac{A_3 \pm \sqrt{D_1 A_3}}{D_2}. \tag{41}$$

It is obvious from equations (37), (39) and (41) that the real values of $\widehat{\mu}$, $\widehat{\mu}^*$ and $\widehat{\mu}^{**}$ exist, iff the quantities under square root are greater than or equal to zero . For any combination of correlations, which satisfy the condition of real situations, two real values of $\widehat{\mu}$, $\widehat{\mu}^*$ and $\widehat{\mu}^{**}$, it should be noted that $0 \leq \widehat{\mu} \leq 1$, $0 \leq \widehat{\mu}^* \leq 1$, and $0 \leq \widehat{\mu}^{**} \leq 1$, and all the other values of $\widehat{\mu}$, $\widehat{\mu}^*$ and $\widehat{\mu}^{**}$ are said to be inadmissible . If both the values of $\widehat{\mu}$, $\widehat{\mu}^*$ and $\widehat{\mu}^{**}$ are admissible, lowest one will be the best choice as it reduces the cost of the surveys.

Inserting the admissible values of $\widehat{\mu}$, $\widehat{\mu}^*$ and $\widehat{\mu}^{**}$ say μ_0, μ_0^* and μ_0^{**} respectively in (27), (28) and (29), we get the optimum values of minimum mean squared errors of the estimators t_1, t_2 and t_3, as:

$$min.MSE(t_1)_{opt} = \frac{A_3[A_3(1-f) - \mu_0 A_2 + \mu_0^2 f A_2]S_y^2}{n(A_3 - \mu_0^2 A_2)}, \tag{42}$$

$$min.MSE(t_2)_{opt} = \frac{A_3[A_3(1-f) - \mu_0^* B_2 + \mu_0^{*2} f B_2]S_y^2}{n(A_3 - \mu_0^{*2} B_2^*)}, \tag{43}$$

$$min.MSE(t_3)_{opt} = \frac{A_3[A_3(1-f) - \mu_0^{**} D_2 + \mu_0^{**2} f D_2]S_y^2}{n(A_3 - \mu_0^{**2} D_2)}, \tag{44}$$

Under Assumption (3.1), the expressions in (37), (39), (41), (42), (43) and (44) respectively reduce to:

$$\widehat{\mu}_{(1)} = \frac{A_3 \pm \sqrt{A_1^* A_3}}{A_2^*}, \tag{45}$$

$$\widehat{\mu}_{(1)}^* = \frac{A_3 \pm \sqrt{B_1^* A_3}}{B_2^*}, \tag{46}$$

$$\hat{\mu}_{(1)}^{**} = \frac{A_3 \pm \sqrt{D_1 A_3}}{D_2}, \tag{47}$$

$$min.MSE(t_1)_{1opt} = \frac{A_3\left[A_3(1-f) - \mu_{(1)0} A_2^* + \mu_{(1)0}^2 f A_2^*\right] S_y^2}{n\left(A_3 - \mu_{(1)0}^2 A_2^*\right)}, \tag{48}$$

$$minMSE(t_2)_{1opt} = \frac{A_3\left[A_3(1-f) - \mu_{(1)0}^* B_2^* + \mu_{(1)0}^{*2} f B_2^*\right] S_y^2}{n\left(A_3 - \mu_{(1)0}^{*2} B_2^*\right)}, \tag{49}$$

$$min.MSE(t_3)_{1opt} = \frac{A_3\left[A_3(1-f) - \mu_{(1)0}^{**} D_2 + \mu_{(1)0}^{**2} f D_2\right] S_y^2}{n\left(A_3 - \mu_{(1)0}^{**2} D_2\right)}, \tag{50}$$

where $\mu_{(1)0}$, $\mu_{(1)0}^*$ and $\mu_{(1)0}^{**}$ are optimum values of the fractions of fresh samples obtained from the equations (45), (46) and (47) respectively.

5. Efficiency Comparison

The percent relative efficiencies (PREs) of the estimators t_1, t_2, and t_3 relative to (i) : sample mean estimator $\bar{y}_n$, when there is no matching and (ii) : usual successive sampling estimator $\hat{\bar{Y}} = \eta^* \bar{y}_u + (1-\eta^*)\bar{y}'_m$, when there is no auxiliary information that is used at any occasion, where $\bar{y}'_m = \bar{y}_m + \beta_{0x}(\bar{x}_n - \bar{x}_m)$, have been computed for various choices of correlations and demonstrated in Table 1.

The variance of $\bar{y}_n$ and optimum variance of $\hat{\bar{Y}}$ are respectively given by

$$Var(\bar{y}_n) = \frac{(1-f)}{n} S_y^2, \tag{51}$$

and

$$Var(\hat{\hat{Y}})_{opt} = \frac{\left\{\left(1+\sqrt{(1-\rho_{0x}^2)}\right)-2f\right\}S_y^2}{2n}.$$

Under the Assumption (3.1), we note that since, the optimum values of min.MSE(t_1), min.MSE(t_2) and min.MSE(t_3) respectively given by (48), (49) and (50) contain six correlations ρ_{0x}, ρ_{0x}, ρ_{02}, ρ_{x1}, ρ_{x2} and ρ_{12}, therefore for simplifying the expressions and to show the empirical results in tabular form it is assumed that

$$\rho_{x1} = \rho_{x2} = \rho_{01} = \rho_{02} = \rho_0, \tag{52}$$

which is an intuitive assumption and also considered by Cochran (1977) and Feng and (1997), see, Singh *et al.* (2016).

Under the above assumption (52), the values of the quantities A_1^*, A_2^*, A_3^*, B_1^*, B_2^*, D_1 and D_2 respectively reduce to:

$$A_1^* = (A_3^* - A_2^*),\ A_2^* = \left[\left(\frac{2\rho_{0x}\rho_0}{1+\rho_{12}}\right)(1-\rho_0+\rho_0\rho_{12}+\rho_{12}) - 2\rho_{0x}\rho_0(1+\rho_0) + \rho_{0x}^2\rho_0(2+\rho_0-2\rho_{12})\right],$$

$$A_3^* = \left(1-\frac{2\rho_0^2}{1+\rho_{12}}\right),\ B_1^* = (A_3^*-B_2^*),\ B_2^* = A_2^*,\ D_1^* = (1-\rho_{0x})\left[1+\rho_{0x}-\frac{2\rho_0^2(1+\rho_{0x})}{1+\rho_{12}}\right],$$

and

$$D_2^* = \rho_{0x}\left[\rho_{0x}-\frac{2\rho_0^2(2-\rho_{0x})}{1+\rho_{12}}\right].$$

Thus under the assumption (52), we have

$$min.MSE(t_1)_{1opt} = min.MSE(t_2)_{1opt}. \tag{53}$$

For the various choices of ρ_{0x}, ρ_0 and ρ_{12}, for a fixed value of the sampling fraction $f = n/N$, Table 1 shows the optimum values of $\mu_{(1)}$, $\mu_{(1)}^*$ and $\mu_{(1)}^{**}$ and percent relative efficiencies $(E_1^{(1)}, E_1^{(2)})$, $(E_2^{(1)}, E_2^{(2)})$ and $(E_3^{(1)}, E_3^{(2)})$ of t_1, t_2 and t_3 with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ respectively, where

$$E_1^{(1)} = \frac{Var(\bar{y}_n)}{\min MSE(t_1)_{opt}} \times 100,$$

$$E_1^{(2)} = \frac{Var(\hat{\hat{Y}})}{\min MSE(t_1)_{opt}} \times 100,$$

$$E_2^{(1)} = \frac{Var(\bar{y}_n)}{\min MSE(t_2)_{opt}} \times 100,$$

$$E_2^{(2)} = \frac{Var(\hat{\hat{Y}})_{opt}}{\min MSE(t_2)_{lopt}} \times 100,$$

$$E_3^{(1)} = \frac{Var(\bar{y}_n)}{\min MSE(t_3)_{lopt}} \times 100,$$

$$E_3^{(2)} = \frac{Var\left(\hat{\hat{Y}}\right)_{opt}}{\min MSE\left(t_3\right)_{opt}} \times 100$$

We note that $\mu_{(1)0} = \mu^*_{(1)0}$, $E_1^{(1)} = E_2^{(1)}$and $E_1^{(2)} = E_2^{(2)}$.

Findings are given in Table 2.

Table 1 : The optimum values $\mu_{(1)0}$, $\mu^*_{(1)0}$ and $\mu^{**}_{(1)0}$ and $PREs$ $\left(E_1^{(1)}, E_1^{(2)}\right)$ $\left(E_2^{(1)}, E_2^{(2)}\right)$ and $\left(E_3^{(1)}, E_3^{(2)}\right)$ with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for $f = 0.1$.

Table 1.

ρ_0	ρ_{12}	ρ_{0x}	$\mu_{(1)0} = \mu^*_{(1)0}$	$E_1^{(1)} = E_2^{(1)}$	$E_1^{(2)} = E_2^{(2)}$	$\mu^{**}_{(1)0}$	$E_3^{(1)}$	$E_3^{(2)}$
0.40	0.50	0.30	0.4882	123.8060	120.6379	0.4970	126.2853	123.0537
		0.40	0.4877	123.6655	117.9298	0.5038	128.1889	122.2434
		0.50	0.4889	123.9798	114.7519	0.5152	131.4220	121.6402
		0.60	0.4916	124.7621	110.8997	0.5326	136.3940	121.2391
	0.70	0.30	0.4887	120.0980	117.0248	0.4991	122.9365	119.7906
		0.40	0.4873	119.7307	114.1775	0.5062	124.8989	119.1061
		0.50	0.4871	119.6684	110.7615	0.5180	128.1314	118.5946
		0.60	0.4880	119.9093	106.5861	0.5357	133.0346	118.2530
	0.90	0.30	0.4886	117.2133	114.2139	0.5006	120.4185	117.3371
		0.40	0.4863	116.6037	111.1955	0.5081	122.4229	116.7448
		0.50	0.4847	116.1746	107.5277	0.5201	125.6534	116.3010
		0.60	0.4837	115.9188	103.0389	0.5380	130.5041	116.0036
0.50	0.50	0.30	0.4774	142.4897	138.8434	0.4858	145.2940	141.5761
		0.40	0.4752	141.7668	135.1916	0.4904	146.8005	139.9918
		0.50	0.4753	141.8089	131.2541	*	*	*
		0.60	0.4777	142.6186	126.7721	0.5160	155.3435	138.0832
	0.70	0.30	0.4793	135.1910	131.7315	0.4898	138.4640	134.9208
		0.40	0.4763	134.2534	128.0266	0.4951	140.1250	133.6259
		0.50	0.4750	133.8313	123.8701	0.5053	143.3434	132.6743
		0.60	0.4752	133.9068	119.0283	0.5218	148.5568	132.0505
	0.90	0.30	0.4802	129.7753	126.4544	0.4927	133.5213	130.1046
		0.40	0.4762	128.5802	122.6166	0.4986	135.2860	129.0114
		0.50	0.4733	127.7090	118.2036	0.5093	138.5135	128.2039
		0.60	0.4714	127.1384	113.0120	0.5261	143.6305	127.6715

ρ_0	ρ_{12}	ρ_{0x}	$\mu_{(1)0} = \mu^*_{(1)0}$	$E_1^{(1)} = E_2^{(1)}$	$E_1^{(2)} = E_2^{(2)}$	$\mu^{**}_{(1)0}$	$E_3^{(1)}$	$E_3^{(2)}$
0.6	0.50	0.30	0.4591	175.0016	170.5235	0.4675	178.5068	173.9390
		0.40	0.4543	172.9661	164.9439	0.4689	179.1010	170.7942
		0.50	0.4527	172.3066	159.4818	0.4760	182.1192	168.5640
		0.60	0.4543	172.9661	153.7477	0.4900	188.0545	167.1596
	0.70	0.30	0.4646	159.9152	155.8230	0.4753	164.0037	159.8070
		0.40	0.4591	157.8510	150.5298	0.4780	165.0268	157.3727
		0.50	0.4561	156.6972	145.0341	0.4861	168.1406	155.6258
		0.60	0.4552	156.3835	139.0075	0.5009	173.8248	154.5109
	0.90	0.30	0.4676	149.5006	145.6750	0.4808	154.1701	150.2249
		0.40	0.4612	147.2636	140.4334	0.4844	155.4577	148.2475
		0.50	0.4565	145.6174	134.7790	0.4933	158.6224	146.8160
		0.60	0.4533	144.4958	128.4407	0.5087	164.1301	145.8935
0.70	0.50	0.30	0.4256	241.5307	235.3501	0.4339	246.7322	240.4185
		0.40	0.4168	236.0730	225.1238	0.4309	244.8233	233.4683
		0.50	0.4126	233.5163	216.1356	0.4346	247.1483	228.7530
		0.60	0.4127	233.5411	207.5921	0.4459	254.1728	225.9314
	0.70	0.30	0.4397	204.9097	199.6662	0.4510	210.6842	205.2929
		0.40	0.4308	200.3747	191.0812	0.4500	210.1858	200.4373
		0.50	0.4254	197.6248	182.9155	0.4554	212.9196	197.0719
		0.60	0.4231	196.4176	174.5934	0.4679	219.3834	195.0074
	0.90	0.30	0.4477	182.7889	178.1115	0.4619	189.1733	184.3325
		0.40	0.4382	178.5331	170.2527	0.4624	189.4254	180.6398
		0.50	0.4312	175.4256	162.3686	0.4689	192.3595	178.0421
		0.60	0.4264	173.2796	154.0263	0.4824	198.4738	176.4211

Table 1 Continued.

ρ_0	ρ_{12}	ρ_{0x}	$\mu_{(1)0} = \mu^*_{(1)0}$	$E_1^{(1)} = E_2^{(1)}$	$E_1^{(2)} = E_2^{(2)}$	$\mu^{**}_{(1)0}$	$E_3^{(1)}$	$E_3^{(2)}$
0.80	0.50	0.30	0.3449	454.6663	443.0317	0.3532	466.4142	454.4790
		0.40	0.3310	435.0795	414.9003	0.3441	453.5424	432.5069
		0.50	0.3237	424.8189	393.1994	0.3433	452.3882	418.7168
		0.60	0.3217	421.9091	375.0303	0.3505	462.6396	411.2352
	0.70	0.30	0.3913	309.2647	301.3508	0.4032	319.5678	311.3903
		0.40	0.3778	297.7350	283.9259	0.3972	314.3759	299.7950
		0.50	0.3694	290.6000	268.9706	0.3987	315.6837	292.1873
		0.60	0.3651	286.9189	255.0390	0.4081	323.7918	287.8149
	0.90	0.30	0.4131	248.4087	242.0521	0.4286	258.5995	251.9821
		0.40	0.3995	239.5073	228.3989	0.4250	256.1909	244.3086
		0.50	0.3897	233.1649	215.8104	0.4283	258.3652	239.1350
		0.60	0.3830	228.8043	203.3816	0.4392	265.5696	236.0619
0.90	0.70	0.30	0.2352	944.2427	920.0802	0.2458	988.7842	963.4818
		0.40	0.2197	879.0969	838.3240	0.2353	944.5341	900.7262
		0.50	0.2104	840.2747	777.7328	0.2327	933.4444	863.9679
		0.60	0.2054	819.1575	728.1400	0.2370	951.8014	846.0457
	0.90	0.30	0.3371	441.5190	430.2208	0.3537	464.8516	452.9564
		0.40	0.3191	416.3095	397.0009	0.3446	452.0664	431.0993
		0.50	0.3068	399.1986	369.4861	0.3438	450.9401	417.3765
		0.60	0.2985	387.7046	344.6263	0.3511	461.1685	409.9276

Table 1 Continued.

Table 1 exhibits that the values of the percent relative efficiencies i.e $E_j^{(1)} = E_j^{(2)}; j = 1, 2, 3;$ greater than 100%. It follows the proposed estimator t_j's , $(j = 1, 2, 3)$ are better than the usual unbiased $\bar{y}_n$(when there is no matching) and the natural successive sampling estimator $\hat{\bar{Y}}$. It is interesting to note that the minimum value of μ_0 is 0.2054, which reflects that the fraction to be replaced at the current occasion is as low as about = 21 percent of the total sample size , which leads substantial reduction in cost of the survey. Thus the use of auxiliary variable in the development of the estimators is very much beneficial. It is also evident from Table 1 that if highly correlated auxiliary variables are used, relatively only a fewer fraction of sample on the current (second) is to be replaced by a fresh sample which reduce the cost of the survey. Thus the proposal of the estimators are justified and recommended for their use in practice.

5.1. A Practical Application

In this section we have examined the performance of the proposed estimator t_j's $(j = 1, 2, 3)$ over (i) sample mean $\bar{y}_n$ when there is no matching and, (ii) usual successive sampling estimator $\hat{\bar{Y}}$. defined in Section 6 through a natural population data earlier considered by Chaturvedi and Tripathi (1983), p.118.

Population: Source :[Chaturvedi and Tripathi (1983)].

The data under consideration were taken from Census 1961 and 1971, West Bengal, District Census Handbook, Malda District. The population consists of 278 villages under Gajole Police Stations with:

y=Number of agricultural labourers 1971.

x= Number of agricultural labourers 1961.

z_1 =Area of village in1961.

z_2=Area of village in1961.

$N = 278, n = 30, f = 0.1079, \bar{Y} = 39.068, \bar{X} = 25.111, \bar{Z}_1 = 173.74, \bar{Z}_2 = 339.95, S_y^2 = 3187.4117, S_x^2 = 1654.40, S_{Z_1}^2 = 25381.00, S_{Z_2}^2 = 123420.00, S_{yx} = 1696.4000,$
$S_{yZ_1} = 4653.4000, S_{yZ_2} = 15281.0000, S_{xZ_1} = 2596.6000, S_{xZ_2} = 11495.000, S_{Z_1Z_2} = 37556.0000, \rho_{x1} = 0.4007, \rho_{x2} = 0.8044, \rho_{12} = 0.6710, \rho_{0x} = 0.7387,$
$\rho_{01} = 0.5174, \rho_{02} = 0.7700.$

We have computed the optimum values of μ_0, μ_0^* and μ_0^{**} , and respectively by using the formulae 37 , 39 and 41 and the percent relative efficiencies (PREs) the proposed estimators t_j , $(j = 1, 2, 3)$ with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$ using the formulae:

$$PRE\left(t_1, \bar{y}_n\right) = \frac{(1-f)\left[A_3 - \mu_0^2 A_2\right]}{A_3\left[A_3(1-f) - \mu_0 A_2 + \mu_0^2 f A_2\right]} \times 100,$$

$$PRE\left(t_2, \bar{y}_n\right) = \frac{(1-f)\left[A_3 - \mu_0^{*2} B_2\right]}{A_3\left[A_3(1-f) - \mu_0^* B_2 + \mu_0^{*2} f B_2\right]} \times 100,$$

$$PRE\left(t_3, \bar{y}_n\right) = \frac{(1-f)\left[A_3 - \mu_0^{**2} D_2\right]}{A_3\left[A_3(1-f) - \mu_0^{**} D_2 + \mu_0^{**2} f D_2\right]} \times 100,$$

$$PRE\left(t_1,\ \hat{\bar{Y}}\right) = \frac{\left\{\left(1+\sqrt{1-\rho_{0x}^2}\right) - 2f\right\}\left[A_3 - \mu_0^2 A_2\right]}{2A_3\left[A_3(1-f) - \mu_0 A_2 + \mu_0^2 f A_2\right]} \times 100,$$

$$PRE\left(t_2,\ \hat{\bar{Y}}\right) = \frac{\left\{\left(1+\sqrt{1-\rho_{0x}^2}\right) - 2f\right\}\left[A_3 - \mu_0^{*2} B_2\right]}{2A_3\left[A_3(1-f) - \mu_0^* B_2 + \mu_0^{*2} f B_2\right]} \times 100,$$

$$PRE\left(t_3,\ \hat{\bar{Y}}\right) = \frac{\left\{\left(1+\sqrt{1-\rho_{0x}^2}\right) - 2f\right\}\left[A_3 - \mu_0^{**2} D_2\right]}{2A_3\left[A_3(1-f) - \mu_0^{**} D_2 + \mu_0^{**2} f D_2\right]} \times 100.$$

Findings are given in the Table 2.

Table 2: Optimum values $(\mu_0, \mu_0^*, \mu_0^{**})$ and the PREs the proposed estimators t_j , $(j = 1, 2, 3)$ (under the optimum conditions) with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$.

$\hat{\mu} = \mu_0 = 0.3942$	$PRE(t_1, \bar{y}_n) = 307.2732$	$PRE(t_1, \hat{\hat{Y}}) = 307.2732$
$\hat{\mu}^* = \mu_0^* = 0.4835$	$PRE(t_2, \bar{y}_n) = 475.2829$	$PRE(t_1, \hat{\hat{Y}}) = 373.7393$
$\hat{\mu}^{**} = \mu_0^{**} = 0.4979$	$PRE(t_3, \bar{y}_n) = 487.4067$	$PRE(t_3, \hat{\hat{Y}}) = 398.3596$

It is observed from Table 2 that the proposed estimators t_j , $(j = 1, 2, 3)$ are more efficient than the usual unbiased estimator $\bar{y}_n$ (when there is no matching) and natural successive sampling estimator $\hat{\hat{Y}}$ (when there is no auxiliary information is used at any occasion) with substantial gain in efficiency. Largest gain in efficiency is obtained by using the proposed estimator over both estimators $(\bar{y}_n, \hat{\hat{Y}})$ followed by the suggested estimator t_2.

Thus all the three suggested estimators t_j , $(j = 1, 2, 3)$ are to be preferred over the estimators $(\bar{y}_n, \hat{\hat{Y}})$ in practice.

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References

Biradar, R. S. and Singh, H.P. (2001). Successive sampling using auxiliary information on both the occasions. Calcutta Statist. Assoc. Bull., 51, 243-251.

Chaturvedi, D. K. and Tripathi, T.P. (1983). Estimation of population ratio on two occasions using multivariate auxiliary information. Jour. Ind. Statist. Assoc., 21, 113-120.

Cochran, W.G. (1977). Sampling Techniques. Third Edition. Johan Wiley, New York.

Feng, S. and Zou, G. (1997). Sampling rotation method with auxiliary variable. Commun. Statist. Theo. Meth., 26(6), 1497-1509.

Murthy, M.N. (1967). Sampling: Theory and Methods. Statistical Publishing Society, Calcutta India.

Reddy, V. N. (1978). A study on the use of prior knowledge on certain population parameters in estimation. Sankhya, C, 40, 29-37.

Sen A.R.(1971) Successive sampling with two auxiliary variables. *Sankhya*, C, 33, 371-376

Singh, G.N., Karna, J.P. and Prasad, S.(2011) On the Use of Multiple Auxiliary Variables in Estimation of Current Population Mean in Two-Occasion Successive (Rotation) Sampling. Sri Lankan Journ. Appli. Statist., 12, 101-116

Singh, H. P. and Pal, S. K. (2015). On the estimation of population mean in successive sampling. Int. Jour. Math. Sci. Applica., 5(1), 179-185.

Singh, H. P. and Pal, S. K. (2015). On the estimation of population mean in rotation sampling. Jour. Statist. Appl. Pro. Lett., 2(2), 131-136.

Singh, H. P. and Pal, S. K. (2015). On the estimation of population mean in current occasions in two-occasion rotation patterns. Jour. Statist. Appl. Prob., 4(2), 305-313.

Singh, H. P. and Pal, S. K. (2015). Improved estimation of current population mean over two-occasions. Sri Lankan Jour. Appl. Statist., 16(1), 1-19.

Singh, H. P. and Pal, S. K. (2016). An efficient effective rotation patterns in successive sampling over two occasions. Commun. Statist. Theo. Meth., 45(17), 5017-5027.

Singh, H. P. and Pal, S. K. (2016). Search of good rotation patterns through exponential type regression estimator in successive sampling over two occasions. Italian Jour. Pure. Appl. Math., 36, 567-582

Singh, H. P. and Pal, S. K. (2016). Use of several auxiliary variables in estimating the population mean in a two occasion successive sampling. Commun. Statist. Theo. Meth. 45(23), 6928-6942.

Singh, H. P. and Vishwakarma, G.K. (2007). Modified exponential ratio product estimators for finite population mean in double sampling. Austrian Jour. Statist., 36 (3), 217 – 225.

Singh, H. P., Kim, J. M. and Tarray, T. A. (2016). A family of estimators of population variance in two-occasion rotation patterns. Commun. Statist. Theo. Meth., 45(14), 4106-4116.

Singh, H. P. and Ruiz-Espejo, M. (2003). On linear regression and ratio-product estimation of a finite population mean. The Statistician 52(1), 59-67.

Singh, H. P. and Vishwakarma, G.K. (2007), A general class of estimators in successive sampling. Metron, 65(2), 201-227.

Singh, H. P. and Vishwakarma, G. K. (2009). A general procedure for estimating population mean in successive sampling. Commun. Statist. Theo. Meth., 38, 293-308.