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# Progressive Censored Burr Type-XII Distribution Under Random Removal Scheme: Some Inferences

**Gyan Prakash***

Department of Community Medicine, M. L. N. Medical College, Allahabad, U. P., India

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**Abstract.** When some sample values at either or both the extremes might have been assorted, a censoring scheme is much useful. Nevertheless, in some reliability experiments, the number of items fell out the experiment cannot be prefixed random in some situations. For such situations, a random removal scheme with censoring scheme may offer a good result. Here, a random removal scheme with the Progressive censoring plan is assumed for statistical inference, when fill out items of the experiments cannot be prefixed. The analysis of the present discussion has carried out with the help of a real data set for the Burr Type-XII distribution.

**Résumé.** Lorsque certaines valeurs d'échantillon à l'une ou l'autre des deux extrêmes ont pu être assorties, un schéma de censure est très utile. Néanmoins, dans certaines expériences de fiabilité, le nombre d'articles tombés en panne ne peut pas être préfixé. Dans de telles situations, un système de suppression aléatoire avec un système de censure peut offrir un bon résultat. Ici, un système de suppression aléatoire avec un plan de censure progressif est supposé pour l'inférence statistique, lorsque le nombre de pannes ne peut être préfixé. L'analyse de la présente discussion a été effectuée à l'aide d'un ensemble de données réelles issues de la distribution Burr Type-XII.

**Key words:** Random Removal Scheme, Progressive Type-II Censoring Plan, ML Estimation, Bayes Estimation, and Bayes Prediction Bound Length.
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* E-mail: ggyanji@yahoo.com

## 1. Introduction

In much life testing experiments, the experimenter may not be observing the lifetimes of all inspected units in life test. This may be because of time limitation and/or cost or material resources for data collection. One of the most common censored tests is Type-II censoring. It is noted that one can use Type-II censoring for saving time and money. However, when the lifetimes of products are very high, the experimental time of a Type-II censoring life test can be still too long. A generalization of this censoring is named as Progressive Type-II censoring, which is useful when the loss of life test units at the point other than the termination point is unavoidable. Recently, the Progressively Type-II censoring scheme has received much interest among the statisticians.

The cumulative density and probability density function of Burr Type-XII distribution are given as

$$F\left(x;\theta,\sigma\right) = 1-\left(1+x^{\sigma}\right)^{-\theta};\ \ \theta>0,\sigma>0,x\geq 0 \tag{1}$$

and

$$f\left(x;\theta,\sigma\right) = \sigma\theta x^{\sigma-1}\left(1+x^{\sigma}\right)^{-\theta-1};\ \ \theta>0,\sigma>0,x\geq 0 \tag{2}$$

The two-parameter Burr Type-XII distribution has unimodal or decreasing failure rate function given as

$$\rho\left(x\right) = \sigma\theta x^{\sigma-1}\left(1+x^{\sigma}\right)^{-1};\ \ \theta>0,\sigma>0,x\geq 0.$$

It is clear that the parameter $\theta$ does not affect the shape of the failure rate function $\rho\left(x\right)$ and $\sigma$ is the shape parameter. Also, $\rho\left(x\right)$ has a unimodal curve when $\sigma>1$ and it has decreased failure rate function when $\sigma\leq 1$. It has been applied in areas of quality control, reliability studies, duration and failure time modeling. Other areas of application include the analysis of business failure data, the efficacy of analgesics in clinical trials, and the times to failure of electronic components. Zimmer *et al.* (1998) discussed the statistical and probabilistic properties of the Burr Type-XII distribution and its relationship to other distributions used in reliability analyses.

Rodriguez (1977) presented complete guides about the Burr Type-XII distributions. Lots of work has done on underlying distribution, a little few of them are discussed here. Nigm (1988) presents some prediction bounds for the Burr Model. Al-Huesaini & Jaheen (1995) stated about the Bayes prediction bounds for the Burr Type-XII failure model. Ali-Mousa & Jaheen (1995) extend the results of the underlying model. Wu *et al.* (2010) obtained maximum likelihood (ML) estimates, exact confidence intervals and exact confidence regions for the parameters of the Gompertz and Burr Type-XII distributions based on failure-censored sampling, respectively.

Lee *et al.* (2009) obtained the Bayes and empirical Bayes estimators of reliability performances of this model under progressively Type-II censored samples. Soliman *et al.* (2012)

obtained some Bayes estimation from Burr Type-XII distribution by using progressive first-failure censored data.

Jang *et al.* (2014) discussed some estimation based on Bayesian setup for Burr Type-XII distribution under progressive censoring. Danish & Aslam (2014) deals with Bayesian estimation of unknown parameters of Burr Type-XII distribution under the Koziol-Green model of random censorship assuming both the informative and non-informative priors. Recently, Rao *et al.* (2015) discussed about the multi-component stress strength reliability by assuming Burr Type-XII distribution. The research methodology they adopted for estimation of the parameter is ML estimation.

There are several situations in the life testing and reliability experiments or the survival analysis, in which units are lost or removed from the experiments while they are still alive. Either the loss may occur out of control or it is preassigned. The out of control case can be happened when an individual under study drops out or so. Other cases may occur because of limitation of funds or to save the time and cost.

However, in some reliability experiment, the number of items dropped out the experiment cannot be prefixed and they are random. In such situations, a random removals scheme is suited best with some censoring scheme. In this paper, random removals criteria have been used under Progressive Type-II censoring plan for Burr Type-XII distribution. Some statistical inferences have obtained and their performances are illustrated by a real life example.

## 2. Progressive Censoring with Random Scheme

The progressive censoring appears to be a great importance in planned duration experiments in reliability studies. In many industrial experiments involving lifetimes of machines or units, experiments have to be terminated early and the number of failures must be limited for various reasons. Progressively Type-II censored sampling is an important method of obtaining data in such lifetime studies. Singh *et al.* (2013) estimates the parameters of exponentiated Pareto distribution under random removals scheme. Azimi *et al.* (2014) presents some statistical inference for the generalized Pareto distribution based on progressive Type-II censored data with random removals.

Let us suppose an experiment in which $n$ independent and identical units $x_1, x_2, \ldots, x_n$ are placed on a live test at beginning time and first $m; (1 \leq m \leq n)$ failure items are observed. At the time of each failure occurring prior to termination point, one or more surviving units were removed from the test. The experiment is terminated at the time of $m^{th}$ failure, and all remaining surviving units are removed from the test.

Let $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(m)}$ are the lifetimes of completely observed units to fail and $R_1, R_2, \ldots, R_m; (m \leq n)$ are the numbers of units withdrawn at these failure times (See Prakash (2015 a) for more details).

Following Prakash (2015 a), the joint probability density function under progressive

Type-II censoring scheme is defined as

$$f_{\left(x_{(1:m:n)},x_{(2:m:n)},...,x_{(m:m:n)}\right)}\left(\underline{x}|\theta\right) = C_p\prod_{i=1}^{m} f\left(x_{(i)};\theta,\sigma\right)\left(1-F\left(x_{(i)};\theta,\sigma\right)\right)^{r_i}$$

$$\Rightarrow f_{\left(x_{(1:m:n)},x_{(2:m:n)},...,x_{(m:m:n)}\right)}\left(\underline{x}|\theta\right) = C_p\sigma^m A_p^*\left(\underline{x},\sigma\right)\theta^m \times exp\left(-\theta\sum_{i=1}^{m}\left(1+r_i\right)log\left(1+x_{(i)}^{\sigma}\right)\right); \quad (3)$$

where $A_p^*\left(\underline{x},\sigma\right) = \prod_{i=1}^{m}\left(\frac{x_{(i)}^{\sigma-1}}{1+x_{(i)}^{\sigma}}\right)$ and the progressive normalizing constant $C_p$ is $n\left(n-r_1-1\right)\left(n-r_1-r_2-1\right)...\left(n+1-\sum_{j=1}^{m-1}r_j-m\right)$.

Suppose an individual unit being removed from the test at $i^{th}\,(=1,2,\ldots,m-1)$ failure, and is independent of others with probability $p$ i.e., $r_i$ units removed at the $i^{th}$ failure follows a Binomial distribution with parameters $n-m-\sum_{k=1}^{i-1}r_k$ and $p$. Hence,

$$P\left(R_i=r_i|R_{i-1}=r_{i-1},...,R_1=r_1\right) = \left({}^{\left(n-m-\sum_{k=1}^{i-1}r_k\right)}C_{r_i}\right) \times p^{r_i}\left(1-p\right)^{n-m-\sum_{k=1}^{i}r_k}\;;\;\forall\, i=1,2,...,m-1. \quad (4)$$

Thus, $P\left(R=r\right)$ is now defined and obtained as

$$P\left(R=r\right) = P\left(R_1=r_1\right)P\left(R_2=r_2|R_1=r_1\right)... P\left(R_{m-1}=r_{m-1}|R_{m-2}=r_{m-2},...,R_1=r_1\right)$$

$$\Rightarrow P(R=r) = \Omega p^{\left(\sum_{i=1}^{m-1}r_i\right)}\left(1-p\right)^{\left((m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)}\;; \quad (5)$$

where $\Omega = \frac{(n-m)!}{\left(n-m-\sum_{i=1}^{m-1}r_i\right)!\prod_{i=1}^{m-1}r_i!}$

## 3. The Point and Interval Estimation

In the present section, maximum likelihood research methodology is applied for point and interval estimation. Hence, the likelihood function is defined and obtained as

$$L\left(\theta,p\right) = L\left(\theta;\underline{x}|R=r\right)\cdot P\left(R=r\right)$$

$$= C_p\sigma^m A_p^*\left(\underline{x},\sigma\right)\theta^m\, exp\left(-\theta\sum_{i=1}^{m}\left(1+r_i\right)\,log\left(1+x_{(i)}^{\sigma}\right)\right) \times\Omega\, p^{\left(\sum_{i=1}^{m-1}r_i\right)}\left(1-p\right)^{\left((m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)}$$

$$\Rightarrow L(\theta,p) = C^{*}\left\{\theta^{m}\, exp\left(-\theta\sum_{i=1}^{m}(1+r_i)\ log\left(1+x_{(i)}^{\sigma}\right)\right)\right\} \times \left\{p^{\left(\sum_{i=1}^{m-1} r_i\right)}(1-p)^{\left((m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)}\right\}. \quad (6)$$

The maximum likelihood (ML) estimators for the parameters $\theta$ and $p$ are obtained respectively as

$$\frac{\partial}{\partial\theta} logL(\theta,p) = 0 \Rightarrow \hat{\theta}_{ML} = \frac{m}{\sum_{i=1}^{m}(1+r_i)\ log\left(1+x_{(i)}^{\sigma}\right)} \quad (7)$$

and

$$\frac{\partial}{\partial p} logL(\theta,p) = 0 \Rightarrow \hat{p}_{ML} = \frac{\sum_{i=1}^{m-1} r_i}{\sum_{i=1}^{m-1} r_i + (m-1)(n-m) - \sum_{i=1}^{m-1}(m-i)r_i}. \quad (8)$$

Hence, the observed information matrix is now defined and obtained as

$$I = \begin{bmatrix} -\frac{\partial^2}{\partial\theta^2} logL(\theta,p) & -\frac{\partial^2}{\partial\theta\partial p} logL(\theta,p) \\ -\frac{\partial^2}{\partial p\partial\theta} logL(\theta,p) & -\frac{\partial^2}{\partial p^2} logL(\theta,p) \end{bmatrix}$$

$$\Rightarrow I = \begin{bmatrix} \frac{m}{\theta^2} & 0 \\ 0 & \frac{\sum_{i=1}^{m-1} r_i}{p^2} + \frac{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}{(1-p)^2} \end{bmatrix}.$$

Thus, the variance covariance matrix is now approximated as

$$\Rightarrow I = \begin{bmatrix} \frac{m}{\theta^2} & 0 \\ 0 & \frac{\sum_{i=1}^{m-1} r_i}{p^2} + \frac{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}{(1-p)^2} \end{bmatrix}^{-1}. \quad (9)$$

Now, the asymptotic distribution of $ML$ estimator $\left(\hat{\theta}_{ML}, \hat{p}_{ML}\right)$ is given as

$$\begin{pmatrix} \hat{\theta}_{ML} \\ \hat{p}_{ML} \end{pmatrix} \sim N\left[\binom{\theta}{p},\ V\ \right].$$

The expression, $V$ involves two unknown parameters $\theta$ and $p$. Hence, an estimate of $V\left(=\hat{V}\right)$ (say) is obtained by replacing the parameters $\theta$ and $p$ by its ML estimators $\hat{\theta}_{ML}$ and $\hat{p}_{ML}$ respectively as

$$\hat{V} = \begin{bmatrix} \frac{m}{\hat{\theta}_{ML}^2} & 0 \\ 0 & \frac{\sum_{i=1}^{m-1} r_i}{\hat{p}_{ML}^2} + \frac{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}{(1-\hat{p}_{ML})^2} \end{bmatrix}^{-1}. \quad (10)$$

Thus approximate $100(1-\epsilon)\%$ confidence intervals for the parameters $\theta$ and $p$ are derived as

$$\left(\hat{\theta}_{ML} \pm Z_{\epsilon/2}\sqrt{VAR\left(\hat{\theta}\right)}\right)$$

and

$$\left(\hat{p}_{ML} \pm Z_{\epsilon/2}\sqrt{VAR\left(\hat{p}\right)}\right)$$

where $VAR\left(\hat{\theta}\right)$ and $VAR\left(\hat{p}\right)$ are determined respectively from equation (10).

## 4. Bayes Estimation

A Bayesian procedure is applied in the present section for estimating the parameters of the Burr Type-XII distribution. The Two-parameter Gamma distribution is selected here as conjugate prior for the unknown parameter $\theta$ (when shape parameter $\sigma$ is considered to be known), having a probability density function

$$\pi_{(\theta)} = \frac{\alpha^{\beta}}{\Gamma\beta}\theta^{\beta-1}e^{-\alpha\theta}\ ;\ \alpha>0,\beta>0,\theta\geq 0. \tag{11}$$

Similarly, the prior density for unknown parameter $p$ is considered here as the Beta distribution, having a probability density function

$$\pi_{(p)} = \frac{1}{B\left(\gamma,\lambda\right)}p^{\gamma-1}\left(1-p\right)^{\lambda-1}\ ;\ \lambda>0,\gamma>0,0\leq p\leq 1. \tag{12}$$

Since, the parameters $\theta$ and $p$ are independent (when the parameter $\sigma$ is known) random variables, therefore, the joint prior density is given as

$$\pi_{(\theta,p)} = \frac{\alpha^{\beta}}{B\left(\gamma,\lambda\right)\Gamma\left(\beta\right)}\theta^{\beta-1}e^{-\alpha\theta}p^{\gamma-1}\left(1-p\right)^{\lambda-1}. \tag{13}$$

Based on the Bayes theorem, the joint posterior and marginal posterior distributions are obtained respectively as

$$\pi^*_{(\theta,p)} = \frac{L\left(\theta,p\right)\times\pi_{(\theta,p)}}{\int_p\int_\theta L\left(\theta,p\right)\times\pi_{(\theta,p)}\,d\theta\,dp}$$

$$\pi^*_{(\theta,p)} = J\theta^{m+\beta-1}\,exp\left\{-\theta\left(\alpha+\sum_{i=1}^{m}\left(1+r_i\right)log\left(1+x^{\sigma}_{(i)}\right)\right)\right\}$$
$$\times\, P^{\left(\gamma+\sum_{i=1}^{m-1}r_i\right)-1}\left(1-p\right)^{\left(\lambda+(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)-1} \tag{14}$$

$$\pi^*_{(\theta)} = \frac{\left(\alpha+\sum_{i=1}^{m}\left(1+r_i\right)log\left(1+x^{\sigma}_{(i)}\right)\right)^{m+\beta}}{\Gamma(m+\beta)}\theta^{m+\beta-1}$$

$$\times\, exp\left\{-\theta\left(\alpha+\sum_{i=1}^{m}(1+r_i)\, log\left(1+x_{(i)}^{\sigma}\right)\right)\right\} \tag{15}$$

and

$$\pi_{(p)}^{*}=\frac{p^{\left(\gamma+\sum_{i=1}^{m-1}r_i\right)-1}\,(1-p)^{\left(\lambda+(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)-1}}{B\left(\left(\gamma+\sum_{i=1}^{m-1}r_i\right),\left(\lambda+(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)\right)}; \tag{16}$$

where $J = \left\{\frac{\Gamma(m+\beta)}{\left(\alpha+\sum_{i=1}^{m}(1+r_i)\, log\left(1+x_{(i)}^{\sigma}\right)\right)^{m+\beta}} B\left(\left(\gamma+\sum_{i=1}^{m-1}r_i\right),(\lambda+(m-1)(n-m)\right.\right.$
$\left.\left.-\sum_{i=1}^{m-1}(m-i)r_i\right)\right)\Big\}^{-1}$.

Wide varieties of loss functions have been discussed in the literature to describe various types of loss structures. The symmetric squared-error (SE) loss is one of the most popular loss functions. It is widely employed in inference, but its application is motivated by its good mathematical properties, not by its applicability to representing a true loss structure.

A loss functions should represent the consequences of different errors. There are situations where overestimation and underestimation can lead to different consequences. For example, when we estimate the average reliable working life of the components of a spaceship or an aircraft, overestimation is usually more serious than underestimation. Being symmetric, the SE loss equally penalizes overestimation and underestimation of the same magnitude. A useful asymmetric loss known as the LINEX loss function (See Prakash (2015 b) for more details), rises approximately exponentially on one side of zero, and approximately linearly on the other side. The LINEX loss function LLF is defined as

$$L(\partial)=e^{a\partial}-a\partial-1;\ \partial=\hat{\theta}-\theta.$$

Here $'a'$ is the shape parameter and $\hat{\theta}$ is any estimate of unknown parameter $\theta$.

Following Prakash (2015 b), Bayes estimator corresponding to unknown parameter $\theta$ is denoted by $\hat{\theta}_B$, and obtained as

$$\hat{\theta}_B=-\frac{1}{a}\,ln\,E\left\{e^{-a\theta}\right\}=\frac{m+\beta}{a}ln\left\{1+\frac{a}{\alpha+\sum_{i=1}^{m}(1+r_i)\,log\left(1+x_{(i)}^{\sigma}\right)}\right\}. \tag{17}$$

Similarly, the Bayes estimator corresponding to the parameter $p$ say $\hat{p}_B$ is obtained by simplifying following equality

$$\hat{p}_B=-\frac{1}{a}ln\int_p\frac{e^{-ap}p^{\left(\gamma+\sum_{i=1}^{m-1}r_i\right)-1}\,(1-p)^{\left(\lambda+(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)-1}}{B\left(\left(\gamma+\sum_{i=1}^{m-1}r_i\right),\left(\lambda+(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i\right)\right)}dp. \tag{18}$$

The close form of $\hat{p}_B$ and the associated minimum posterior risk of Bayes estimators $\hat{\theta}_B$ and $\hat{p}_B$ do not exist. A numerical technique is applied herewith for drawing inferences.

### 5. Prediction of the Future Records

Since $\underline{x} = \left(x_{(1)}, x_{(2)}, ..., x_{(m)}\right)$ be first $m$ observed failure items from a sample of size $n$ under considered censoring plan for model (1). If we assume that $Y = \left(y_{(1)}, y_{(2)}, ..., y_{(s)}\right)$ be another independent random sample of observations from the same model. Then the Bayes predictive density of future observation $Y$ is denoted by $h\left(Y|\underline{x}\right)$ and obtained by simplifying the following relation

$$h\left(Y|\underline{x}\right) = \int_{\theta} f\left(y;\theta\right)\pi^{*}_{\theta} d\theta$$

$$\Rightarrow h\left(Y|\underline{x}\right) = \frac{\left(\alpha + \sum_{i=1}^{m}(1+r_i)log\left(1+x_{(i)}^{\sigma}\right)\right)^{m+\beta}(m+\beta)\,\sigma y^{\sigma-1}}{(1+y^{\sigma})\left(\alpha + \sum_{i=1}^{m}(1+r_i)log\left(1+x_{(i)}^{\sigma}\right) + log\left(1+y^{\sigma}\right)\right)^{m+\beta+1}}. \quad (19)$$

Let the lower and upper Bayes prediction limits are denoted by $l_1$ and $l_2$ for random variable $Y$ and $(1-\epsilon)$ is called the confidence prediction coefficient. Then one-sided Bayes prediction lower and upper limits are obtained by solving following equality

$$Pr\left(Y \le l_1\right) = \frac{\epsilon}{2} = Pr\left(Y \ge l_2\right). \quad (20)$$

Using equation (19) in (20) the one sided Bayes prediction lower and upper limits of $Y$ are obtained as

$$l_1 = \left\{exp\left\{\left(\alpha + \sum_{i=1}^{m}(1+r_i)log\left(1+x_{(i)}^{\sigma}\right)\right)(\epsilon^{*}-1)\right\} - 1\right\}^{1/\sigma}$$

and

$$l_2 = \left\{exp\left\{\left(\alpha + \sum_{i=1}^{m}(1+r_i)log\left(1+x_{(i)}^{\sigma}\right)\right)(\epsilon^{**}-1)\right\} - 1\right\}^{1/\sigma};$$

where $\epsilon^{*} = \left(1-\frac{\epsilon}{2}\right)^{-1/(m+\beta)}$ and $\epsilon^{**} = \left(\frac{\epsilon}{2}\right)^{-1/(m+\beta)}$.

Hence, the one-sided Bayes prediction bound length is obtained as

$$L = l_2 - l_1.$$

### 6. Numerical Analysis

The performance of the proposed procedures is studied by a numerical illustration based on a real data set for a clinical trial describe a relief time (in hours) for 30 arthritic patients considered here form data provided by Wingo (1993) and used recently by Wu *et al.* (2010). The data are given in the Table (1).

We carry out this analysis by considering the censored data of size $m\,(=5, 10, 15)$ with selected progressive censoring scheme (presented in Table (2). The maximum likelihood estimates for both parameters are presented in Table (3) for $\sigma = 1.00$. It is observed

**Table 1.** Relief Time (in hours) for 30 Arthritic Patients

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.70 | 0.58 | 0.54 | 0.59 | 0.71 | 0.55 | 0.63 | 0.84 | 0.49 | 0.87 |
| 0.73 | 0.72 | 0.62 | 0.82 | 0.84 | 0.29 | 0.51 | 0.61 | 0.57 | 0.29 |
| 0.36 | 0.46 | 0.68 | 0.34 | 0.44 | 0.75 | 0.39 | 0.41 | 0.46 | 0.66 |

that the values of ML estimator are increasing as censored sample size increases.

The risk corresponding to Bayes estimator $\hat{\theta}$ under LLF is obtained and presented in the Table (4). The selected values of shape parameter $'a'$ are 0.50 and 1.00. While the values of prior parameters are selected as $(\beta, \alpha) = (0.50, 0.70), (1.00, 1.00), (2.50, 1.58), (5, 2.30), (10, 3.16)$ and $(0, 0)$. Here, the criterion behind the selection of these prior parameter values is that the prior variance should be unity. Also, $\beta = \alpha = 0$ reflect the study under non-informative (Jeffrey's) prior. Hence, all the results should be valid for both informative and non-informative priors.

**Table 2.** Different Progressive Censoring Scheme

| Case | $m$ | $R_i; i = 1, 2, ..., m$ |
|---|---|---|
| 1 | 5 | 1 2 1 0 1 |
| 2 | 10 | 1 0 0 3 0 0 1 0 0 1 |
| 3 | 15 | 1 0 2 0 0 1 0 2 0 0 0 1 0 0 1 |

**Table 3.** Maximum Likelihood Estimates

| $\sigma = 1.00$ | $m \downarrow$ | | |
|---|---|---|---|
| $n = 30$ | 5 | 10 | 15 |
| $\hat{\theta}_{ML}$ | 0.7567 | 0.7619 | 0.7815 |
| $\hat{p}_{ML}$ | 0.8108 | 0.8211 | 0.8394 |

It is noted that the risk increases as the censored sample size $m$ increases. Similar trend also has seen when the values of shape parameter increases. Further, an opposite trend has seen when the set of prior parameter value increases. However, the magnitudes of risks are nominal.

Similarly, the risk corresponding to Bayes estimator $\hat{p}$ under LLF is obtained and presented in Table (5). However, the selection of hyper parametric values does not provide unity variance for prior defined in equation (12).

Similar properties have seen for risks of the Bayes estimator $\hat{p}$ under LLF as discussed earlier for the case of Bayes estimator $\hat{\theta}$. It is further noted that the risk's magnitude is larger for Bayes estimator $\hat{p}$ as compared to $\hat{\theta}$.

The Bayes prediction bound length and confidence interval are presented in Table (6), for $\epsilon = 99\%, 95\%, 90\%$. It is noted that when confidence level $\epsilon$ decreases the length of intervals tends to be closer. The bound's length tends to be closer also as the set of prior parameters increases when other parametric values are considered fixed. A decreasing trend has been seen in length when censored sample size increases. It is noted further that the bound length of based on ML procedure is closer than compared to Bayes procedure.

**Table 4.** Risk Corresponding to Bayes Estimator $\hat{\theta}$

| $n = 30$ $\sigma = 1.00$ | | $\leftarrow (\beta, \alpha) \rightarrow$ | | | | | |
|---|---|---|---|---|---|---|---|
| $a \downarrow$ | $m \downarrow$ | 0.50, 0.70 | 1.00, 1.00 | 2.50, 1.58 | 5.00, 2.30 | 10, 3.16 | 0, 0 |
| 0.50 | 5 | 0.7211 | 0.7139 | 0.6686 | 0.6449 | 0.6048 | 0.6421 |
| | 10 | 0.8517 | 0.8333 | 0.8013 | 0.7825 | 0.7701 | 0.7385 |
| | 15 | 1.0206 | 1.0192 | 1.0101 | 0.9099 | 0.9016 | 1.0047 |
| 1.00 | 5 | 0.7392 | 0.7237 | 0.7032 | 0.6828 | 0.6583 | 0.6711 |
| | 10 | 0.8769 | 0.8626 | 0.8503 | 0.8328 | 0.8027 | 0.8277 |
| | 15 | 1.0741 | 1.0535 | 1.0317 | 1.0184 | 1.0088 | 1.0148 |

**Table 5.** Risk Corresponding to Bayes Estimator $\hat{p}$

| $n = 30$ $\sigma = 1.00$ | | $\leftarrow (\gamma, \lambda) \rightarrow$ | | | | | |
|---|---|---|---|---|---|---|---|
| $a \downarrow$ | $m \downarrow$ | 0.50, 0.70 | 1.00, 1.00 | 2.50, 1.58 | 5.00, 2.30 | 10, 3.16 | 0, 0 |
| 0.50 | 5 | 0.8447 | 0.8309 | 0.8101 | 0.7929 | 0.7651 | 0.7103 |
| | 10 | 0.9511 | 0.9252 | 0.8914 | 0.8596 | 0.8145 | 0.7417 |
| | 15 | 1.1193 | 1.1426 | 1.0651 | 1.0296 | 1.0148 | 1.0018 |
| 1.00 | 5 | 0.9884 | 0.9707 | 0.9562 | 0.9251 | 0.8922 | 0.9425 |
| | 10 | 1.0233 | 1.0174 | 1.0062 | 1.0004 | 0.9833 | 0.9875 |
| | 15 | 1.2341 | 1.2107 | 1.1864 | 1.1337 | 1.1263 | 1.1081 |

**Table 6.** Central Coverage Bayes Prediction Bound Lengths under Progressive Type-II Censoring Plans (Based on Simulation Data)

| | | Bayes Procedures | | | | | | ML Procedures | |
|---|---|---|---|---|---|---|---|---|---|
| $n = 30$ | | $\leftarrow (\beta, \alpha) \rightarrow$ | | | | | | | |
| $m \downarrow$ | $\epsilon$ | 0.50, 0.70 | 1.00, 1.00 | 2.50, 1.58 | 5.00, 2.30 | 10, 3.16 | 0, 0 | $\theta$ | $p$ |
| 5 | 99% | 3.1034 | 3.0855 | 3.0527 | 3.0492 | 3.0139 | 3.0672 | 2.4191 | 2.8916 |
| | 95% | 2.3542 | 2.3064 | 2.2243 | 2.1547 | 2.0191 | 2.2154 | 2.1147 | 2.8518 |
| | 90% | 2.0394 | 2.0118 | 1.9375 | 1.8983 | 1.6922 | 1.8112 | 1.5071 | 1.7215 |
| 10 | 99% | 3.0745 | 3.0616 | 3.0194 | 2.9547 | 2.8949 | 3.0531 | 1.9311 | 2.8788 |
| | 95% | 2.3124 | 2.2922 | 2.1774 | 2.1251 | 2.0049 | 2.1815 | 1.8316 | 2.7311 |
| | 90% | 1.9057 | 1.8216 | 1.7998 | 1.7675 | 1.6868 | 1.7801 | 1.4711 | 1.6014 |
| 15 | 99% | 2.8295 | 2.8058 | 2.7876 | 2.7566 | 2.6801 | 3.0102 | 1.8851 | 2.5801 |
| | 95% | 2.1553 | 2.1399 | 2.1003 | 1.0976 | 1.0921 | 2.1275 | 1.7303 | 2.5171 |
| | 90% | 1.7724 | 1.6402 | 1.5708 | 1.0558 | 1.0461 | 1.7109 | 1.4123 | 1.1404 |

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