Modeling, Behaviour Prediction, and Control – Tripartite Essentials of Contemporary Structural Engineering Education

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Abstract

Computers have today permeated every sphere of human endeavour and structural engineering is no exception. Multiply-hyperstatic structures which relatively recently took teams of structural engineers several months to analyse can now be elegantly analysed in a matter of seconds using computers equipped with readily available and user-friendly analysis/design software. This paper examined the contemporary structural engineer and the shift his education must accommodate in order for his relevance not to diminish in the face of the new realities posed by the computer. The results of the study showed that there should now be an orientation shift in the education of the structural engineer. More time should now be apportioned to learning the intricacies of structural modeling, understanding structural behaviour, and controlling same to achieve the desired design objective. On the other hand, although the classical analysis methods should still be taught, the time allocated to them should now be significantly reduced. Examples were given for quantitative illustration of the issues involved. In conclusion, the paper argued that the above three essentials should form the cornerstone of modern structural engineering education.

Keywords: Analysis, Design, Modeling, Structural Behaviour, Control, Approximate Methods.

Introduction

The structural engineer often designs structures made of materials (such as concrete) the structural properties of which he has an imperfect knowledge. He therefore makes some approximations of material behavior in his design. The geometry of an actual structure is also only approximately represented for the purpose of analysis. Thus a building frame with finite dimensions of width and depth of beams and columns is represented as a line drawing. Even the manner in which an actual structure is supported to the foundation and how its various members are connected and transmit internal forces among themselves at joints are usually only approximately represented for analysis. Finally, when the geometry and support conditions of the structure have been idealized, the designer discovers he has an incomplete understanding of how to obtain the characteristic values of certain types of loads on the structure. Thus, wind load, which is clearly a stochastic variable, is treated only semi-probabilistically in the current design procedures. This is because its probabilistic model as a stochastic process is very complex and still subject of active research (Ranganathan, 1999).

After all these approximations have been made, an analytical model emerges. This analytical model is simply an idealized representation of the actual structure. At this stage the engineer now imposes an "exact" analysis for the structure which in effect is a superposition of approximations. An exact analysis is any of those many classical equilibrium and compatibility methods of analysis the knowledge of which the engineer prides himself and which he spent most of his school days acquiring. From my student days to myinvolvement in engineering pedagogy at Ado-Ekiti, Akure and Bahir Dar, the emphasis in the engineering curricula (UNAD, 1999; FUTA, 2003; BDU, 2006) has remained the same, namely, sound grounding in the exact methods of analysis. This is understandable since a structure cannot be designed without sound knowledge of the analysis procedure. However, the level of emphasis and credit hours allotted to the study of exact methods scarcely leaves any time for educating the student on the other important aspects of the structure which are essential for the reliability of the results of an "exact" analysis.

One such important aspect is how to model the structure, material and load to obtain an analytical model that reasonably accurately reflects the actual structure and loading. This is because if the model greatly departs from reality, an “exact” analysis will simply yield results reflecting the “exact” departure from reality. Arguably, the time available within the school calendar for the quality all-round education of the civil/structural engineer (and I believe other engineers too) is such that if more time should be allotted to teaching the students essential details regarding the modeling of materials, loads, structural geometry, supports, prediction of deformation patterns under load, principle of transmission of actions among members connected to a joint and so forth, then less time should be allotted to teaching the “bag of tricks” embodied in the traditional exact analysis methods. Although such a suggestion would be inconceivable in the past, it is practicable today. This is because even the most complex structures can be efficiently analysed today by digital computation on the basis of the concise and systematic approach presented through the displacement method in its matrix formulation. Thus, whether it is a highly redundant rigidly jointed multistory skeletal structure which will normally take a team of engineers days to months for analysis, or an irregularly shaped continuum for which an analytical expression does not exist, as long as the skeletal structure can be sensibly modeled and the continuum correctly discretized, the computer in a matter of seconds will accurately analyse the structure based on the input model. Thus, the computer has released some extra time for the engineer. Although quality time should still be spent in teaching exact analysis methods, such time need not remain disproportionately high.

The aim of this investigation is to broadly examine the issues involved from the conception of a structure to its final design with a view to ascertaining those aspects where less-than-necessary attention is given at present and to which part of the time now freed by the computer should be transferred.

Review of Literature

Design Process

The engineering design process consists essentially of two stages, namely, a feasibility study (or conceptual design) which involves a comparison of the alternative forms of structure and selection of the most suitable type, and a detailed design of the chosen structure (Arya, 2003). According to this source, the success of the conceptual design stage relies largely on engineering judgment and instinct. Leet and Uang (2005) further decomposed the above two stages of the design process into the following six components: conceptual design, preliminary design, analysis of preliminary designs, redesign of the structures, evaluation of preliminary designs, and the final design and analysis phases. The first five components are in essence what Arya (2003) grouped together as feasibility study or conceptual design. In their own contribution, Olotuah and Arum (2007) offered that the complete structural design process consists of three main phases, namely, conceptual design, preliminary analysis and design, and finally, detailed analysis and design. Furthermore, they submitted, the first phase forms the basis for the remaining two phases.

Conceptual/Preliminary Designs

From the above conceptions, whether the design process is viewed as consisting of two, three or six stages, all are agreed that it has a preliminary and a final stage and very importantly, that the success of the final stage depends on the quality of the first. This is because at the conceptual design stage very important and far-reaching decisions concerning the future of any project are made, often within a very limited time span. At this stage, several possible structural schemes (layouts and structural systems) are drawn up and those layouts that lend themselves to efficient structural systems while satisfying the functional and aesthetic requirements of the project are established. Next, those structural systems that preliminarily appear most promising are selected and their major components are sized. Such preliminary sizing of the structural members demands from the designer an understanding of structural behavior (force and deformation characteristics under load) and a good knowledge of the loading conditions and their combinations that will most likely affect the design. Rough computations of structural response to load are required at this stage in order to make some estimates of the proportions of each member of the structure at critical sections. Such analysis is performed for all the promising structural systems earlier established. The approximate member proportions obtained at this stage are necessary for a preliminary structural analysis.

Since the analysis of indeterminate structures (and most real structures are indeterminate) requires foreknowledge of stiffness ratios of the interconnected members and such knowledge is still incomplete at this stage, embarking on exact analysis is usually unnecessary and the understanding of structural behavior by the engineer is especially important here. Such understanding will recommend to him an appropriate structural analysis method (usually an approximate method). Once the approximate magnitudes of the structural responses are obtained, they are used to redesign the structural members. After such redesign, all the alternative structural systems chosen earlier are evaluated and the variant best satisfying the promoter's requirements is chosen for the final analysis and design phase. It is usually at this stage when both the loads and the member proportions are already clearly established that it becomes necessary to employ an exact analysis. Fortunately, the computer makes this stage much easier today. Several software are available both for such analysis and the subsequent final design. However, the computer assumes that the model input for analysis is a true representation of the actual structure and loading and the correctness of the analysis depends on the correctness of the model.

From the above discussion, it is clear that most of the designer's work is now required at the conceptual/preliminary design stage rather than at the final analysis/design stage. The importance of the conceptual design stage in the entire design process was aptly captured by Mola (2007), who said that at present time conceptual design has become a well-defined way of approaching structural problems and many designers conceive their projects according to its basic principles.

Approximate Methods of Analysis

As evident from the preceding discussion on the various stages of the conceptual/preliminary design, approximate methods of analysis are widely used at that level. The choice of an appropriate approximate method at any stage on the other hand, is dependent on the designer's ability to predict structural behavior. Prediction of structural behavior helps to accurately sketch the deflection curves of the structure which in turn helps to locate zero moment positions for the reduction of the degree of indeterminacy of a structure. With the right choice of an approximate method, the designer can vary certain parameters of the structure to achieve the desired effect, i.e., control of structural behavior. In the words of Nilson, Darwin and Dolan (2004), in spite of the development of refined methods for the analysis of beams and frames, increasing attention is being paid to various approximate methods of analysis. According to them, prior to performing a complete analysis of an indeterminate structure, it is necessary to estimate the proportions of its members to determine their relative stiffness, upon which the analysis depends. On the other hand, the dimensions can be obtained on the basis of approximate analysis.

Approximate methods are also used to detect gross errors. Even with the availability of computers, for structures of minor importance, it is often satisfactory to design on the basis of results obtained by rough calculation, using only the knowledge of approximate sketches of the elastic curve of the structure. In agreement with the above submissions, Leet and Uang (2005) wrote that if designers understand the behavior of a particular structure, they can often use an approximate analysis to estimate closely, with a few simple computations, the approximate magnitude of the forces at various points in the structure. They affirmed that designers use the result of an approximate analysis to size the main members of a structure during the preliminary design phase and to verify the accuracy of an exact analysis.

Various techniques are available for approximate analysis of continuous beams and rigid frames for gravity load. Notable among the techniques are those of guessing the location of points of inflection and estimating the values of the member-end moments. Approximate methods are especially necessary for the analysis of highly redundant multi-story multi-bay reinforced concrete rigid frames subjected to lateral forces. The two established classical approximate methods for lateral-load-swayed frames are the Portal and the Cantilever methods (Wang, 1983; Kong, et. al., 1983; Kassimali, 1993; Englekirk, 2003; Raju, 2005). Approximate analysis using these methods is usually performed by effectively reducing the degree of static indeterminacy by suitable moment releases. Recognition of the major mode of racking deformation of the frame makes possible realistic predictions of the resulting points of contra-flexure in both beams and columns. The degree of indeterminacy is reduced by the number of inflection points assumed. The Portal and Cantilever methods make use of the same assumption that points of contra-flexure occur at the mid-height positions of all columns and at the mid-span positions of all beams. For single-bay frames, this single assumption is sufficient to reduce the structure to a statically determinate system. For multi-bay frames however, additional assumptions are made. The additional assumptions account for the difference in the two techniques. These additional assumptions, as well as other important aspects of the methods, including their limitations were extensively discussed elsewhere (Arum and Aderinlewo, 2005, 2006). In these works the authors used quantitative parameters to show that in practical building frames of not more than 10 storeys, the analysis results using the Portal method are closer to the exact results than do the results obtained by the Cantilever method.

Another valuable if relatively less common approximate method for multi-story lateral-load-swayed frames is the D-value method (Muto, 1974). Unlike the other methods however, the D-value method assumes some knowledge of members' stiffness ratios. Fortunately, an idea of members' stiffness ratios is usually available at the second phase of the conceptual/preliminary design, or the phase Olotuah and Arum (2007) referred to as the preliminary analysis and design phase. The D-value method consists essentially in expressing frame rigidity and distribution of lateral forces in terms of a distribution coefficient termed "D-value", which is based mainly on beam-to-column stiffness ratio and on the column stiffness.

In this work, a sample analysis will be presented with the intention to show the relative merits of the various methods and at what stage of the design process each is preferable. It will also be used to demonstrate how structural behavior can be controlled using either the Portal method or the D-value method. Finally, the results will be related to the author's position that engineering pedagogy should henceforth focus more on the issues related to conceptual design than has hitherto been the case.

Methodology and Sample Analysis

In this investigation a five-story, fixed-feet asymmetrical reinforced concrete frame with uniform story height was analysed using the Portal, a modified Portal, the D-value and the computer Stiffness methods. The interior-to-exterior column second moment of area ratio is assumed to be unity. The material and cross-sectional properties used were as follows:

$$E = 24821128 kN/m^2$$

$$I = 2.023 \times 10^{-3} m^4$$

$$\rho = 23.563 kN/m^3$$

$$\nu = 0.2$$

The analytical model with the applied lateral loading is shown in Fig.1.

Portal Method

In this method, the degree of statical indeterminacy of the example frame was suitably reduced through moment releases at mid-heights of columns and at mid-spans of beams. In addition, at each floor level the shear borne by the interior column was assumed to be twice the shear resisted by the exterior columns. The foregoing was in accordance with the strict theoretical format of the classical Portal method. With these assumptions, only the equilibrium equations of statics were required for the complete analysis of the frame.

Modified Portal Method

In this, format engineering judgment was used to make some modifications to the classical Portal theory as applicable to the example frame. The said engineering judgment derives from an understanding of the theoretical basis of the classical Portal method which consists in the following.

As mentioned earlier, one of the two assumptions that form the basis of the Portal method is that points of contra-flexure occur at the mid-height positions of all columns and at the mid-span positions of all beams. Usually beams are designed to be stiffer than the columns because of the relatively high magnitude of gravity loading they must resist. It is also known from frame behaviour that for laterally loaded portal frames, when beam-to-column stiffness is very high, neglecting axial deformation, the beam deflects horizontally as a rigid body and, for compatibility, the columns bend in double curvature and the points of contra-flexure in both columns and beams tend to be near their mid-lengths. In practice however, although the beams are usually stiffer than the columns, they are not infinitely rigid and errors naturally arise in the analysis results due to this fact. In addition the column end conditions are usually not the same throughout a building. The stiffness of the upper beams for instance may be different from that of the lower beams. In addition, the ground floor columns are bounded at the lower end by the foundation or foundation beams and therefore the stiffness at the lower boundary depends on whether the frame is assumed fixed or pinned at the bottom. A column fixed at the base will have its inflection point substantially moved upwards from its mid-length especially if the upper beam-to-column stiffness ratio is not great. On the other hand, if the column is pinned at the base, inflection point will not occur at all within the length of the column since a pin cannot transmit moment. Furthermore, it is well-established from studies of frame behaviour (Muto, 1974; Wang, 1983; Arum and Aderinlewo, 2005) that at the topmost story of any multi-story building, the effect of the stiffnesses of the top and bottom bounding beams is to move the inflection point of the column downwards from its mid-length. Since the assumption of inflection points at column and beam mid-lengths ignores the influence of the various aforementioned factors, inaccuracies arise in the analysis results, which are usually significant for the topmost and bottommost floors.

The second assumption of the Portal method is that the shear force borne by an interior column is twice that of an exterior column. This assumption is based on the fact that gravity load constitutes the primary loading in frames and since the tributary load area for an interior column is often about twice that for the exterior columns, the interior column usually has greater cross-sectional dimensions. In recognition of the shearing panel actions across the panels as the dominant behaviour mode of the frame and since the shear distributed to the columns supporting a particular floor is approximately proportional to their flexural stiffness (EI/h), the Portal method therefore assumes the shear resisted by the interior column to be twice that by the external column. In practice however, although the cross-sectional dimensions of the interior column is usually greater than those of the exterior column, the stiffness is rarely as much as twice. Buildings for which the exterior walls are constructed from heavy masonry and the tributary floor areas on columns are not great (small slab panel areas) often have about the same cross-sectional dimensions and therefore about the same magnitude of bending stiffness for the interior and the exterior columns. In other buildings the ratio of the interior-to-exterior column stiffness may be about 1.5. For such cases, errors (which can be significant) are introduced in the analysis based on the assumption that the stiffness ratio is equal to two (2). Such errors are even heightened if at a particular floor level, the bays are of different widths while the adjacent beams are of the same cross-section. In such a case the shorter beam is stiffer and will hold down the exterior column to its side more rigidly than the longer less rigid beam will do for the exterior column to its own side. Consequently, even if the interior column cross-sectional dimensions are twice those of the exterior columns, the column to the side of the stiffer beam will still resist greater shear than the one to the side of the more flexible beam. In such a case the proportion of the shear resisted by the interior column will tend to decrease with respect to the shear resisted by the more rigidly held of the exterior columns whereas it will increase with respect to the shear borne by the less rigidly bounded of the exterior columns.

Recommendations which took the various factors mentioned above into account were given elsewhere (Arum, 2008) for rapid analysis of frames with number of stories not greater than 15 and with different stiffness ratios of members. The said recommendations were applied to the example frame in this work. Specifically, the shear resisted by the exterior column held by the stiffer beam, as a fraction of the interior column shear was taken at 0.82, 0.66 and 0.57 respectively for the first, intermediate and the topmost stories and for the exterior column held by the more flexible beam the corresponding shear fractions were taken at 0.73, 0.53 and 0.53. The column inflection point as a fraction of column height, measured from column lower end was taken for both exterior columns at 0.6, 0.5, 0.46, and 0.38 and for the interior column at 0.54, 0.5, 0.49 and 0.46, respectively for the first, second, intermediate and topmost stories.

D-Value Method

At a floor level in a lateral-load-swayed multistory building frame with columns $c_1, c_2, \ldots c_i, \ldots c_n$, the common deflection Δ of the column heads can be expressed as follows.

$$\Delta = \frac{h^2 Q_1}{12 EKk_{c1} a_1} = \frac{h^2 Q_2}{12 EKk_{c2} a_2} = \ldots = \frac{h^2 Q_i}{12 EKk_{ci} a_i} = \ldots = \frac{h^2 Q_n}{12 EKk_{cn} a_n}$$

where $h =$ height of the storey considered;

$Q_1, Q_2, Q_i, Q_n =$ Shear force borne by the 1st, 2nd, ith and nth columns, respectively;

$k_{c1}, k_{c2}, k_{ci}, k_{cn} =$ Stiffness ratios of the 1st, 2nd, ith and nth columns, respectively;

$a_1, a_2, a_i, a_n =$ Coefficient that depends on the beam-to-column relative stiffness ratio and on the boundary conditions of the column for the 1st, 2nd, ith and nth columns, respectively.

If the bounding beams are infinitely rigid, then $a_1 = a_2 = \ldots = a_i = \ldots = a_n = 1$ and Eqn (1) reduces to the expression for determining frame lateral deflection in the absence of joint rotation. In that case the lateral displacement would be exclusively a function of the flexural stiffness of the column. The coefficient a therefore accounts for joint rotation, which depends on the ratios of beam-to-column second moments of area. Thus, in its analysis procedure the D-value method recognizes the fact that the behavior of a subassembly subjected to a shear deformation is controlled by the stiffness of its two components, the beam and the column.

$K =$ Standard stiffness, given by $K = I_c / k_c h$.

From Eqn (1), we can write:

$$\frac{Q}{\Delta} = a_1 k_{c1} \left[\frac{12EK}{h^2}\right] = a_2 k_{c2} \left[\frac{12EK}{h^2}\right] = \ldots = a_i k_{ci} \left[\frac{12EK}{h^2}\right] = \ldots = a_n k_{cn} \left[\frac{12EK}{h^2}\right] \quad (2)$$

In general form, Eqn (2) can be written as: $\frac{Q}{\Delta} = ak_c\left[\frac{12EK}{h^2}\right]$ (3)

If $\Delta = 1$, then the right hand side of Eqn (3) becomes equal to the shear force. The quantity ak_c is known as D-value and is defined as the shear force corresponding to the unit relative deflection of the column head, having the unit of $12EK/h^2$. It is usually denoted by the letter D.

From Eqn (3), the D-value can be expressed as follows:

$$D = ak_c = \frac{Q}{\Delta}\left[\frac{h^2}{12EK}\right] \qquad (4)$$

By solving the slope-deflection equation for different frames and boundary conditions, and different ratios of beam-to-column second moments of area, the precise value for a can be obtained for each case. The values for some standard conditions were derived by Muto (1974).

The D-value method can also be used directly to determine the story lateral deflections for any multistory frame. Thus, from Eqn (4), we can write:

$$\Delta = \frac{Q}{D}\left[\frac{h^2}{12EK}\right]$$ (5), which is the story drift.

Stiffness Method

SAP 2000 software was employed for the exact analysis. All that was required for the analysis was correct input of the analysis model, including cross-sectional dimensions as well as loading.

Results and Discussion

The results of the sample frame analysis for member-end moments for different analysis methods are presented in Table1 for all the columns, while the story drifts are shown in Table2. The Portal method in its strict theoretical format was employed only for the case when the second moment of area of the columns is equal to that of the beams. This was because since the method is based on the conversion of redundant structures to determinate ones before analysis, it is unable to take account of variations in member stiffness ratios. On the other hand, using the data recommended from the results of a study of frame behavior (Arum, 2008), the modified Portal method took cognizance of members' stiffness ratios as well as frame asymmetry.

As the analysis results in Table1 indicate, although the classical Portal method can be employed to obtain sensible estimates of the moments in members of a sway building frame, some pronounced deviations of the results from those of the exact method exist for some member-ends. This is due to the fundamental assumption in the method that the shear force borne by an interior column is twice that of the exterior column. In the sample frame analysed in this work, this assumption is far from the fact. The results can be far more accurate if the theoretical basis for the method is well appreciated and applied in a judgement-based manner to suit particular situations, such as was done in this example, more so that ready-to-use tabular data are available for such application (Arum, 2008). Knowledge of the fundamental ideas on which a particular method is based can greatly improve the results of approximate methods.

Comparison of the values of member-end moments obtained using either the modified Portal or the D-value methods and the values obtained by the exact method shows that the difference in value is generally within 5% except for a few local deviations. This shows that approximate methods are very powerful analysis tools when judiciously applied. Judicious application on the other hand, is based on knowledge of frame behaviour. The results obtained by using the D-value and exact analysis are shown in Table2 for lateral deflection. Although the story drifts can also be obtained using the Portal method, the principle of virtual work needs to be applied. It can be seen from

Table 2 that as the beam-to-column stiffness ratio increases for a given material, the lateral deflection reduces. Thus, since story drift is an important design criterion for tall buildings, by varying the beam-to-column stiffness ratios, the desired effect on the frame can be achieved. This in effect is using design to control frame behaviour.

Conclusions

This study has shown that knowledge of modeling (since the analytical model is the major input to the computer), ability to predict the structural behavior as exhibited by the structure's force and deformation responses to load, as well as the understanding of the effect of cross-sectional characteristics of members on structural response, are all vital tools in the complete design process. A quantitative example was given to illustrate the role of approximate methods of analysis in achieving the above objective. The paper showed the need to allocate more time in the civil/structural engineering curricula to the teaching of various aspects of structural modeling and the conceptual and preliminary analysis techniques so as to ensure that this all-important stage on which the quality and cost of a project often depend, becomes less heuristic and more objective, measurable by quantitative parameters. It is recommended that while sufficient attention should be given to those classical exact methods which encourage good understanding of structural behaviour, and the computer-oriented matrix methods, less time should be alloted to the teaching of some of the hand-oriented exact methods which are gradually becoming anachronistic in view of the new realities posed by the computer.

Nomenclature

a coefficient which depends on the beam-to-column stiffness ratio and on column boundary conditions;

E modulus of elasticity;

h storey height;

I second moment of area;

I_b beam second moment of area;

I_c column second moment of area;

K standard stiffness;

k_c column stiffness ratio;

M_{ij} bending moment at end i of member ij;

Q shear force;

Δ storey lateral deflection (storey drift);

ρ unit weight of concrete;

ν Poisson's ratio

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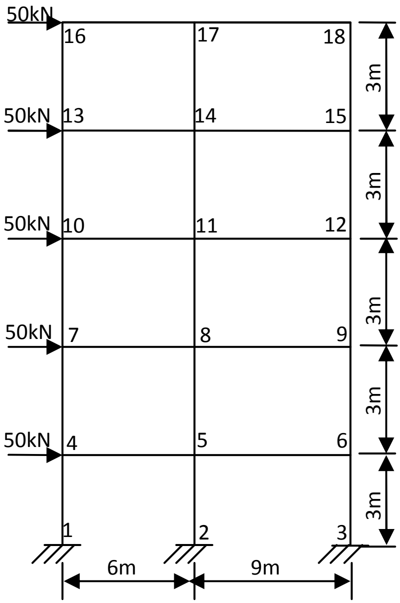

Fig.1 Frame Analytical Model

Table 1. Member-end Moments (kNm) as Function of Members' I-ratios for Different Analysis Methods

Moment Notation	$I_b = I_c$			$I_b = 3I_c$				
	Exact	D-value	Portal	Exact	D-value	% Error	Modified Portal	% Error
M_{14}	180.27	182.93	93.75	145.18	147.67	1.7	144.70	
M_{41}	63.90	60.98	93.75	95.89	98.45	2.7	96.47	
M_{47}	87.48	94.26	75.00	87.35	97.71	11.9	90.41	
M_{74}	91.48	94.26	75.00	94.19	97.71	3.7	90.41	
$M_{7,10}$	53.73	56.56	56.25	62.32	69.63	11.7	62.39	
$M_{10,7}$	79.89	84.83	56.25	73.09	76.95	5.3	73.24	
$M_{10,13}$	28.55	32.99	37.50	38.64	41.53	7.5	41.59	
$M_{13,10}$	59.29	61.27	37.50	50.22	56.18	11.9	48.83	
$M_{13,16}$	7.45	11.78	18.75	15.56	18.33	17.8	15.47	
$M_{16,13}$	31.77	35.35	18.75	25.33	30.54	20.6	25.24	
M_{25}	194.20	187.14	187.50	162.25	158.15	2.5	158.82	
M_{52}	100.82	100.77	187.50	132.53	129.40	2.4	135.30	
M_{58}	140.27	138.48	150.00	137.52	126.42	8.1	136.98	
M_{85}	139.78	138.48	150.00	139.10	126.42	9.1	136.98	
$M_{8,11}$	96.33	93.47	112.50	99.38	94.82	4.6	100.68	
$M_{11,8}$	118.26	114.25	112.50	106.63	94.82	11.1	104.79	
$M_{11,14}$	59.50	55.39	75.00	65.11	56.89	12.6	67.12	
$M_{14,11}$	84.26	83.09	75.00	72.38	69.53	3.9	69.86	
$M_{14,17}$	27.83	21.95	37.50	31.90	26.86	15.8	32.86	
$M_{17,14}$	52.47	47.29	37.50	39.61	36.35	8.2	38.57	
M_{36}	167.66	170.87	93.75	135.99	140.61	3.4	128.83	
M_{63}	43.15	47.35	93.75	78.16	75.72	3.1	85.88	
M_{69}	68.42	67.26	75.00	66.13	75.84	14.7	72.60	
M_{96}	72.58	67.26	75.00	75.70	75.84	0.2	72.60	
$M_{9,12}$	36.41	40.36	56.25	47.86	51.19	7.0	50.09	
$M_{12,9}$	65.39	60.53	56.25	60.71	62.57	3.1	58.81	
$M_{12,15}$	17.54	17.96	37.50	30.02	30.34	1.1	33.40	
$M_{15,12}$	50.85	49.30	37.50	43.62	45.50	4.3	39.20	
$M_{15,18}$	2.27	5.62	18.75	12.78	13.27	3.8	14.39	
$M_{18,15}$	28.20	28.01	18.75	24.81	24.65	0.7	23.47	

Table 2. Frame Storey Drift (mm) as Function of Members' I-ratios for Different Analysis Methods

Storey Drift	$I_b = I_c$			$I_b = 3I_c$		
	Exact	D-value	% Error	Exact	D-value	% Error
Δ_6	11.1	9.1		6.0	6.4	
Δ_9	18.0			6.7		
Δ_{12}	27.1	23.2		13.0	12.8	
Δ_{15}	14.4			1.5		
Δ_{18}	40.5	33.8		18.5	17.9	
	16.5			3.2		
	49.9	40.8		22.2	21.3	
	18.2			4.1		
	55.4	44.3		24.2	23.0	
	20.0			5.0		