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Kant's 'Transcendental Exposition' of Space and Time in the 'Transcendental Aesthetic': A Critique

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Abstract

Immanuel Kant's purpose in setting forth the system of the critical philosophy is to explain how scientific knowledge (which deals with the ideas of physical objects in space and time) is possible while denying the claims of metaphysics. His views on space and time are borne out of the historical background of the Newtonian 'absolutist' and the Leibnizian 'relationalist' debate in the early modern period. Kant held to the Newtonian view that space and time are absolute and not a system of properties, determinations or relations dependent on the existence of physical objects as proposed by Leibniz. Yet, in contrast to Newton, he denies that space and time are independently existing substances. In critiquing Kant's transcendental exposition of space and time, we explain the essential details of the metaphysical exposition of these concepts before giving the transcendental arguments. By tying the axioms of geometry to space and time and by showing that the latter are the universal, necessary structures of the way we experience nature, Kant preserves the a priori nature of geometry while arguing that it does give us objective knowledge about the world of experience that science studies. The work considers relevant statements in the *Critique of Pure Reason* in our interpretation of Kant's theory. We criticize his transcendental arguments on the

grounds that if we can show that they are problematic, then we are justified in claiming that his system is not adequate to do what it claims or does.

Introduction

In the first main section of the 'Transcendental Doctrine of Elements' of his magnum opus, the *Critique of Pure Reason* (1781) Kant writes:

> The science of all principles of a priori sensibility I call transcendental aesthetic. There must be such a science forming the first part of the transcendental doctrine of elements in distinction from the part which deals with the principles of pure thought,….which is called transcendental logic (66).

The use of the word 'Transcendental' here according to Kant signifies an inquiry into the possibility of a priori knowledge while the term 'Aesthetic' (from the Greek 'Aesthesis') is concerned with the capacity for sensibility and its forms. Thus, the 'Aesthetic' is 'Transcendental' because it undertakes a systematic investigation into space and time which for Kant are the "two pure forms of sensible intuition, serving as the principles of a priori knowledge…" (67). As a follow up, he ask: How are space and time known? Are they known empirically or a priori? What is the nature of space and time? In the principles of the 'Transcendental Aesthetic', Kant considers possible answers to these questions. As we shall see, it was the difficulties on the ontological status of space and time that were apparent with Newton and Leibniz that provided the springboard for the Kantian alternative.

Background to Kant's Thought

For Newton and his supporters referred to by Kant as the "mathematicians of nature" (A39-40), space and time consist of absolute entities that exist independently of physical objects. This means that even if there were no objects, there would still be empty space and empty time, while Leibniz and others as the "metaphysicians of nature" (B 56-57) see space and time as essentially relational and dependent on the existence of physical objects. For them, it makes no sense to conceive of space and time in isolation from physical objects. This also implies that if there were no objects there would be no space and no time. In the last pre-critical essay 'Concerning the Ultimate Foundation for the Differentiation of Region in Space' (1768), Kant broke away from the pre-critical period under the influence of the Leibnizian analysis of sense perception which explains the latter's relationalist account of space and time as merely appearances or properties/accidents (determinations) or relations of substances or things in themselves. On the contrary, he was convinced that "the Newtonian theory of absolute space which has its own reality independently of the existence of matter…" (*Inaugural Dissertation* 301) is more preferable than the Leibnizian relationalist account.

Two years later, Kant beginning with the critical system in his Inaugural Dissertation 'On the Forms and Principles of the Sensible and Intelligible World' (1770) articulated what was to be his final position in the *Critique*. In the *Metaphysical Foundations of Natural Science* (1780) and the *Prolegomena to Any Future Metaphysics* (1783), Kant expresses the transcendental ideality of space and time by saying that they are in us' merely as subjective forms of organizing the sensations of the mind and yet they are necessary conditions for the existence of phenomena. In his further analysis of the principles of 'Transcendental Aesthetic' of the first Critique, Kant goes on to provide the metaphysical and transcendental expositions of space and time. By 'exposition', he means "the clear though not necessarily exhaustive representation of that which belongs to a concept…" (*Critique* 68). He argues that "the exposition is metaphysical when it contains that which exhibits the concepts as given a priori" (68). In this, Kant presents four arguments to show the nature of space and by the same parallel arguments with respect to time as follows:

(i) That "Space is not an empirical concept which has been derived from outer experiences" (68) but is a priori and is presupposed by our experiences of outer objects.

(ii) That "Space is a necessary a priori representation which underlies all outer intuitions" (68). Here, Kant's argument is that space is logically prior to the object that exists in it. This means that space exist before object exist. "We can never represent to ourselves the absence of space, although we can quite well think it as empty of objects" (68).

(iii) That "Space is not a discursive or as we say, general concept of relations of things in general but a pure intuition" (69). This argument implies that space is not merely a way of thinking about the world; rather it is the way in which the world is. Kant points out that we can represent to ourselves only one space that is space is essentially a unitary whole. Different spaces are simply limitation or "part of one and the same unique space" (69). Therefore, different spaces do not stand in relation to space in general in the way that instance of a concept stand in relation to the concept itself. In this, Kant argues that space is not a concept but an intuition.

(iv) That "Space is represented as an infinite given magnitude" (69) and therefore must be known by an a priori intuition and not by concept.

The summary of Kant's 'metaphysical exposition' of space and time is that the first two arguments are to show that they are a priori while the second two are to show that they are known by intuitions and not by concepts. Having given an outline of Kant's

metaphysical exposition, we shall now proceed to state his transcendental arguments before drawing his final conclusions.

Kant on the 'Transcendental Expositions' of Space and Time

By a 'transcendental exposition', Kant means "the explanation of a concept as a principle from which the possibility of other a priori synthetic knowledge can be understood" (70). In his view, such knowledge is possible only on "the assumption of a given mode of explaining the concept" (70). Kant argues that "geometry is a science which determines the properties of (physical) space synthetically and yet a priori" (70). He tells us that in explaining the possibility of synthetic a priori geometrical knowledge, our knowledge of space must be in the form of an intuition. "For from a mere concept no propositions can be obtained which go beyond the concept – as happens in geometry" (70). For example, the statement that "space has three dimensions cannot be shown if we examine merely the concepts. For the concept of the subject does not contain the concept of the predicate as an analytic statement does. However, it is a necessary statement since it is not possible for space not to have three dimensions. Thus, in Kant's view, space must be a priori since no necessary statements can be merely empirical.

Kant's only alternative is to consider our knowledge of space grounded on an a priori intuition. Kant goes on from this analysis to show that this intuition must be in the subjects only, for it precedes the objects of experience (since it is known a priori and they are known empirically) and yet determines the concept of the objects. Furthermore, this intuition must be "merely the form of outer sense in general" (71), since it orders the appearances of these objects. In the transcendental exposition on time, Kant has analogous apodictic principles about time which can only be explained by thinking of time as a "subjective form of intuition", namely "time has only one dimension" and "different times are not coexistent but successive". Since Kant thinks these principles are synthetic a priori, he argues that his analysis of time, as the only possible explanation of these principles synthetic a prioricity, must be true. Besides the above principles about time which are explainable by his analysis, Kant also thought that it was necessary that time be known by a priori intuition to explain the possibility of motion and change. As he says, this is so because:

> …no concept… could render comprehensible the possibility of an alteration, that is, of a combination of contradictorily opposed predicates in one and the same object, for instance, the being and the not-being of one and the same thing in one and the same place. Only in time can two contradictorily opposed predicates meet in one and the same object, namely, one after the other. Thus, our concept of time explains the possibility of that body of a priori synthetic knowledge

> which is exhibited in the general doctrine of motion and which is by no means unfruitful (76).

Kant's conclusions from the above arguments with regards space (though the arguments about time are exactly parallel) are that: (a) "Space does not represent any property of things in themselves…" (71), when the subject viewing them is abstracted. This is so because space is known by an a priori intuition. However, nothing about objects can be intuited before they exist, which would be possible if we could know something about objects a priori. Thus, our a priori knowledge of space cannot be knowledge of anything about objects themselves. (b) "Space is nothing but the form of all appearances of outer sense. "It is the subjective condition of sensibility under which alone outer intuition is possible for us" (71). This conclusion, Kant says explains how space or time can be a priori since the form of all appearances can be given prior to experience in the receptivity of the subject. In so far as space or time determines all objects of experience "it can contain prior to all experience principles which determine the relations of these objects (for example the principles of geometry and for time arithmetic and mechanics)" (71). Kant's summarizing paragraph in the transcendental exposition concisely contains the metaphysical and epistemological view point about space and time which the arguments in the 'Transcendental Aesthetic' lead to:

> It is, therefore, solely from the human standpoint that we can speak of space, of extended things, etc. If we depart from the subjective condition under which alone we can have outer intuition; namely, liability to be affected by objects, the representation of space stands for nothing whatsoever (71).

Critical Remarks

Kant's transcendental arguments begin from the synthetic a priori nature of statements about space and Euclidean geometry to the conclusion that space must be known by an a priori intuition to explain the statements being synthetic a priori. These arguments are severely shaken by the formulation of possible non-Euclidean geometries that major scientific theories use as theories about space (e.g. Einstein's theory). Axioms of Euclidean geometry which state "A triangle must have 180^0 as the total of its internal angles" cannot be known a priori if non Euclidean geometries which assert that "triangles may have more than 180^0 as the sum of their internal angles" are possible or even thinkable. If both systems are imaginable and logically possible, then there is no reason to say that one is a priori true, for the only way to discover which of them actually applies to existing triangles is by some empirical method; thus, by experience. The problem of axioms in mathematical theories is indeed a difficult one, for they do, in some sense, seem to be necessary for the system in question. One way which mathematicians have solved the question is to assert that the axioms are analytic that is, that they define, for example what a triangle is to be, by the axioms that are

assumed about it. Thus, it might be considered a defining characteristic of "triangle" as used in Euclidean geometry, that have not more than 180^0 in the sum of its internal angles; whereas non-Euclidean geometry has a different definition of "triangle".

Statements about arithmetic are also considered by many analytic, so that the example "7 + 5" does equal by definition, "12", although Kant denied this. Some mathematicians have held that all geometries, though deducible from their premises, rest on synthetic axioms. Still others have claimed a solution to the problem of the status of axioms by holding them neither analytic nor synthetic, since they are not statements that can be true or false, but procedural rules or presuppositions which are pragmatically used to establish the theory. By this analysis, they are more like rules of inference than propositions. In any case, the way we take mathematical statements also depends upon whether we are considering pure or applied mathematics - a distinction which Kant never made. No matter how we try to solve the problem of conflicting geometries and the status of axioms, one need no longer hold, as Kant thought one must, that they are synthetic a priori. We disagree that an a priori intuition is necessary to connect concepts of space, geometry, and arithmetic which do not follow analytically from each other and yet whose connections are known a priori.

A similar argument can be applied to certain statements about time that Kant thought were synthetic a priori, namely (1) "different times are not coexistent" and (2) "time has only one dimension". A good case can be made out for calling these statements analytic; that is, expressing defining characteristics of our concept of time. Let us consider (1). A temporal instant is an infinitesimal interval or point which marks off from a finite length of time a previous time and a following time. However, if there were no length between time's intervals (t_1, t_2, etc.) then there would be no measure of duration (finite time) between t_1, t_2. In this case t_1 and t_2 would be at the same temporal place, or coexistent. Since we mean by "times" in the sense of statement 1, infinitesimal points which measure or mark off finite durations, t_1 and t_2 would not be coexistent times, but just two names for the same one time, which they do not in the event of their being at the same temporal point (coexistent). A temporal length must have two successive times marking off a temporal distance, just as a spatial distance must be marked by two points which are not in the very same location, or spot, The only possible exception that might be raised is if one interpreted "different times" in the statement to mean the times of specific changes or events. In this case, it could be true that two events which took the same length of time went on simultaneously.

What the above arguments amount to saying is that they took place at one time, in the relevant sense of time. If one uses the word "time" unequivocally, it is a manifest contradiction to say that "different times can exist at one and the same time". The contradiction seems parallel to the contradiction of saying that "two places can exist at one place". But if the negation of the statement is a contradiction, then the original statement is analytic. Statement (2) is vague, for the case of "dimension" is modeled

on a spatial analogy. Einsteinian theory claims that there might be two different measures of time for systems traveling at near light velocities. This does not show that time can have two dimensions, but only that the measures of time in one system differ from those of the other. To say that the measures of time in each system are each dimensions of time would imply a super time of which the times of both systems were part. In the case of the super time, however, the intervals of the super time would measure the changes in both systems, and would be able to determine which equivalent change were taking place slower in which system. Thus, there would be no way of giving meaning, by this example, to the assertion that time could have two dimensions; (although, as we have noted, the use of the word "dimension" is only a metaphor). In any sense in which it can be claimed that 'time has dimension it can be further claimed that it is analytic that time has one dimension.

The main reason that may have led Kant to consider all the statements in the transcendental arguments synthetic a priori could be that his notion of the requirements for an analytic statement were too strict. Kant calls an analytic any statement in which the predicate is contained in the subject, or whose negation would be a contradiction in terms. However, this dependence on analytic statement as being of the subject-predicate form shows Kant's uncritical acceptance of Aristotelian logic and causes him to ignore statements that seem as analytic as these. For instance, "when it rains, it rains" seems as analytic as "All black cats are black". However, in the first example it is difficult to see what the subject is to determine whether the predicate is "contained" in it, or not. It is true that the negation of "when it rains, it rains" is a contradiction in terms (although Kant calls this one criterion for determining analytic statements), it seems that in practice he only applies the criterion of whether the subject is "contained" in the predicate. Not only is this criterion vague, but it is metaphorical and gives an unclear idea of just what examples of predicates fit the notion of being "contained" in their subjects.

In the statement "different times cannot be part of one and the same time" or "different times cannot be coexistent", how do we determine that the predicate is contained in the subject? If we take the predicate "not part of one and the same time", or the predicate "not coexistent", it is easy to see how Kant could deny that these are contained in the subject, "different times". All that the subject entails, he might hold, is that we may be speaking of two numerically distinct times, whereas the predicate commits us to saying that there cannot be two numerically distinct times in one temporal point, or simultaneous duration. Thus, the statement goes beyond a mere elaboration of what is merely in the subject, and must be synthetic. However, to hold that there are no synthetic a priori statements, one does not have much difficulty in holding that all a priori statements are analytic, and fitting Kant's examples to this analysis. Whenever Kant holds that a statement is synthetic a priori, it seems we can find that this position comes from a faulty analysis of the concept expressed by the

statement, either overlooking something that was entailed by the subject concept, or based on an equivocation of terms entailed by the subject and terms entailed by the predicate. Thus, if the notion of "time", is used univocally, it is manifestly contradictory (as has been discussed above) to say that "different times" are the "same time", which is what the negation of "different time are not coexistent" would assert. Therefore, there seems no reason not to hold this statement analytic.

In his transcendental argument about time, Kant thought that time would have to be known by an a priori intuition to justify synthetic a priori doctrines about motion. The notion of time, he thought, must be presupposed to avoid the otherwise contradictory predicates that motion and change would confer on objects (e.g. being A and –A, or being in place 1 and not in place 1 (in place 2). This point is obvious. However, it is unjustified to pass from this premise to the conclusion that time must be known by an a priori intuition to explain this fact. If Kant thinks that time must be pre-supposedly known by an a priori intuition to explain change, he must have had in mind some synthetic a priori statement about change and time that cannot be explained in any other way. This statement is "change (and motion) presupposes the existence of time". We must know this statement a priori or there would be no way to deny the possibility that a thing has contradictory properties. However, here again, it seems quite plausible to hold the statement analytic – a clarification of the notion of change, and to suppose that Kant's rather narrow view of what sentences could be analytic led him to call the statement synthetic. If all the statements that Kant considered synthetic a priori can be shown to be analytic, then we have divested the transcendental arguments of their importance. The transcendental arguments were needed to explain the possibility of synthetic a priori judgments. But if there are no synthetic a priori judgments which need explaining then there is no necessity to explain Kant's space and time as "a priori forms of the intuition".

Since we have shown that there is, according to most philosophers, no serious problem of dissolving the apparently synthetic a priori nature of mathematical sciences into either synthetic a posteriori or analytic a priori propositions, we have also rendered implausible one of Kant's conclusions as to space and time. The second of Kant's conclusion states that space is a form of outer intuition. However, none of his previous arguments give a reason for this conclusion. All that has apparently been shown is that space (and time) is known by formal or pure (a priori) intuitions, not that the form of empirical outer intuition is space. The reason for this conclusion is indicated in a footnote where Kant hints that only by considering space as known a priori and yet also the form of sensuous intuitions can one explain how the synthetic a priori judgments of geometry can apply to objects of experience. Here, Kant is trying to solve the problem of applied geometries. If we grant the above argument that mathematical sciences are not synthetic a priori, then we can consider the problem of applied geometry as empirical problems of finding out which geometry "fits" the world. Setting

this problem has nothing to do with the status of pure mathematics, however. We may consider the axioms analytic of two conflicting geometries, if we like. Of course, we must insist that each is defining different entities if they contain axioms that would contradict otherwise. The problem of applied mathematics still remains: to determine which sorts of entities there are, as a matter of fact, in the world: the entities defined by geometry A or those of geometry B.

General Comments

From the preceding arguments of the transcendental exposition, we can advance some general comments on Kant's views on space and time as necessary conditions for making judgments in mathematics and other aspects of geometry. At a superficial glance, it can be seen that none of Kant's argument gives sufficiently strong support for his conclusion that the representation of space and time "stands for nothing" outside the form of our intuitions. From his arguments, all that follows at the most is that we cannot know whether things in themselves are in space and time or not outside of our experience. Nothing in his argument rules out the possibility that, as a matter of fact, things in themselves happen to be in a space and time which is constructed in exactly the same way as the forms of our sense experience would lead us to think. If Kant's views were true, it would be "problematic" for us to discuss whether space and time did exist for things in themselves, since we could never know. However, neither would we have any grounds for a positive denial that things in themselves exist in space and time. It is to establish more evidence for the positive denial that Kant spins out the problems of the antinomies of pure reason. He attempts to show throughout the 'dialectic' the illegitimate endeavour reason adopts if it attempts to draw conclusions about things in themselves, which are outside the realm of possible experience. According to Kant, objective/scientific knowledge is possible as long as we confine ourselves to our experience. Experience is the only sphere of knowledge where we can achieve objective results. On the contrary, as soon as we try to go beyond our experience, our reason falls into many unsolvable contradictions.

Kant holds that the "transcendental illusion" of pure reason rests on its dialectical inferences concerning things in themselves from the three sorts of syllogisms possible in traditional logic, the categorical, hypothetical and necessary. To these three systems of inference there correspond the three ideas of pure reason, soul, world and God; and Kant's three sets of difficulties that thinking about these ideas involves: the paralogisms, the antinomies and the arguments for the existence of God. He argues that whenever the metaphysician tries to discuss these transcendental ideas of pure thought as though they were objects of experience or objective conditions of the universe, all sorts of errors occur.

> When we try to expand our knowledge…outward, we come to complete impasse…when we try to move beyond the phenomenal

> world to the realm of things-in-themselves, we are again unable to proceed (Korner *Kant 201*).

Kant claims that many of the problems philosophers have met are due to the fact that they have tried to apply reason to questions that are of a metaphysical nature (e.g. trying to prove the existence of God). When we apply our knowledge to matters beyond experience, we fall into the antinomies of reason that is proposition that make opposite claims but for which we can provide equal justification. For example, we can both say that the universe is finite and that it is infinite. Our reason can justify both statements. We can also say that the universe cannot be finite; because the idea of something finite implies the idea of a beginning and the idea of a beginning imply something that precedes that beginning. These antinomies, according to Kant are to be avoided if we realize that the cognitive faculties (i.e. space and time etc.) are only equipped to understand, shape and organize the data of experience. They cannot go beyond it.

Again, Kant's conclusion as to the status of space and time is that they are empirically real but transcendentally ideal. To assert that something is empirically real is to assert that it is an actual object which is given in experience, and which can be known to be true. Analogously, to assert that something is transcendentally ideal would be both to assert that it is an actual thing in itself or property of a thing in itself, and that it can be known to be true. To be consistent Kant must deny that there are things which are known to be transcendentally ideal. He does often imply that at least the existence of things in themselves can be known to be real as grounds for the existence of phenomena, although this is inconsistent with his epistemology. For Kant, all knowledge is restricted to objects which are empirically real, but transcendentally ideal, namely, phenomena, or space and time. In the case of space and time, however, Kant thinks he has proved by the Antimonies the impossibility of space and time being transcendentally ideal in the sense that they could be properties of things in themselves. From his conception of the antinomies we could know that space and time do not apply to things in themselves, but only to objects of experience.

Summary and Conclusion

We now come to the end of our analysis and critique of Kant's transcendental exposition of space and time. No doubt, the difficulty in understanding the principles of the 'Transcendental Aesthetic of the *Critique of Pure Reason* in which he gives the metaphysical and transcendental expositions of space and time has led commentators to place greater emphasis on the contextual background of his theory. For this reason, we began our analysis of Kant's views as part of a number of specific philosophical problems discussed by Newton and Leibniz in the early modern period. This aroused Kant's interest leading to the formulation of his final views in the critical philosophy. In our analysis and critique of Kant's transcendental exposition of space and time, we began by presenting his arguments of the metaphysical exposition and proceeded to

present the transcendental arguments using the postulates of mathematical knowledge. If considered in their pure intuition, mathematical entities Kant argued are conceived a priori but meaningful only if they are manifested in empirical objects. It was in fact, Kant's contention that the geometrician construction of mathematics is as example of an a priori concept that is possible empirically. In his view, "geometry is seen as a science which determines the properties of space synthetically yet a priori". Here, Kant's argument was that we can make synthetic a priori judgments about spatial concepts such as "a straight line is the shortest distance between two points". For him, the condition for making such judgment is that space and time are a priori forms of sensibility since they "must be found in us prior to any perception of an object and must therefore be pure not empirical intuition" (70).

With this exposition, Kant was convinced that this was the only explanation that not only made possible for geometry to be seen as a system of synthetic a priori knowledge but also provide a criterion with which other a priori synthetic knowledge can be justified with the greatest certainty. In this way, our analysis of Kant has shown that the employment of such a priori principles and their relation to physical objects seen as phenomena constitute the necessary condition for the validity of objective empirical science. By critiquing Kant's transcendental arguments on space and time, the inadequacy of his position has been shown. If Kant is consistent, then certain scientific theories such as Newton's cannot be justified from his system (although Kant wanted to prove them justified). In any case, no matter how we look at Kant's argument, we cannot but agreed that the metaphysical and scientific theories are justifiable systems of knowledge. That physical objects existing in space independently us are also possible objects of knowledge is not in doubt. Whatever inference that justifies Newton's theory will also justify physical space dependent on the existence of objects as advocated by Leibniz. The existence of conflicting geometries and relativistic theories of space (for example Einstein) are evidence against Kant's position that the properties of space must be known a priori to account for the synthetic a priori truths of Euclidean geometry. This evidence has indicated that applied geometries are merely synthetic and depend for their truths on observed measures of objects and their geometrical properties. Thus, to hold in the transcendental exposition that space and time are 'subjective forms of intuition of the mind known a priori' as Kant did have no positive reasons in its favour.

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