Rwandan grade 6 mathematics teachers' knowledge

[1]Jean Francois Maniraho, [2]Iben Maj Christiansen
[1]University Rwanda, College of Education, [2] School of Education, University of KwaZulu-Natal, South Africa

Abstract
Teachers' knowledge has been an interesting research topic, especially after the development of Shulman's teachers' knowledge categories in 1986. In this paper, we report on analysis of the knowledge of Rwandan grade 6 mathematics' teachers, sampled in 19 schools from 7 different districts through random stratified sampling. A test and a questionnaire were used as data collection tools including questions on Content Knowledge (CK) and Pedagogical Content Knowledge (PCK).The results reveal that the teachers' CK is best in numbers and measurements whereas their PCK is much better in unpacking mathematics compared to the other three PCK sub-categories considered in this study. As this is possible the first study of this kind done in Rwanda, further research to interrogate the above results is encouraged.

Key words: *Teacher knowledge, test scores, Rwanda, grade 6, content knowledge, pedagogical content knowledge, mathematics.*

Résumé
La connaissance des enseignants a été un sujet intéressant dans la recherche surtout après l'avènement des catégories de connaissance inventées par Shulman en 1986. Dans cet article, nous nous sommes fixé comme objectif d'analyser la connaissance des enseignants des Mathématiques de la sixième année primaire. Ces enseignants ont été sélectionnés dans 19 écoles de sept districts par échantillonnage stratifié. Les tests mathématiques et des questionnaires ont été utilisés comme instruments de collecte des données, comprenant des questions sur la connaissance de la matière et la connaissance pédagogique de la matière. Les résultats de la recherche indiquent que la connaissance de la matière des enseignants des mathématiques de la sixième année primaire est mieux avancée dans les systèmes d'unités de mesure alors que leur connaissance pédagogique de la matière est mieux dans la résolution des problèmes mathématiques comparativement aux autres trois catégories de la connaissance pédagogique de la matière qui ont été considérées dans cette étude. Comme cette recherche peut être la première de la sorte au Rwanda, d'autres recherches sont encouragées pour examiner plus profondément les résultats de cette recherché.

Introduction

In Rwanda, as argued by Uworwabayeho (2009), little is known about teachers' knowledge in general and teacher's mathematics knowledge in particular. Yet in the Kinyarwanda language[7], there is a saying that *"Ntawe utanga icyo adafite*''. This means that ''none can give or offer something which s/he does not have''. The fact that Rwandan learners generally perform inadequately in mathematics compared to other subjects (MINEDUC, 2006) is one of the factors which gave us the idea of investigating the knowledge of mathematics teachers so that we could understand the type of mathematics knowledge which they possess and are able to offer their learners, as well as the teaching approaches they use. Thus, a study was undertaken and data were collected on both learners and teachers, including video recordings of teaching. In this paper, however, we limit ourselves to engaging how we attempted to measure two types of teachers' knowledge, i.e. Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) of Rwandan grade six mathematics teachers.

[7] Kinyarwanda is a language spoken in Rwanda, one of the countries situated in East Africa. We would like to notify that it is the mother tongue of the first author of this paper.

Literature Review

In education, different scholars have investigated the kinds of knowledge that teachers are supposed to be equipped with, to perform their teaching job effectively[8] (Grossman, 1990; Krauss, Neubrand, Blum, & Baumert, 2006; Ma, 1999; Marais, Van der Westhuizen, & Tillema, 2013; Markic & Ingo, 2010; Mewborn, 2001; Shulman, 1986; Sorto, Marshall, Luschei, & Carnoy, 2009; Zhou, Peverly, & Xin, 2006). Results from the above studies show differences mainly due to the context in which each study was conducted. Thus, some studies find that variations in teacher knowledge, specialization or qualification make a substantial difference to learner performance (Akpo & Jita, 2013), while others find these less significant than socio-economic background, etc. For instance, the SAMEQ III study found that "a 100 points increase in teacher scores was associated with an average change of 4.8 points" in learner performance in Mathematics (Van der Berg et al., 2011, p. 5).

While there has been much focus on making teaching more participatory or learner centered – in our view, a vague notion which does little to assist in developing an understanding of what makes learning happen – the research points to other factors as being more important. In particular, curriculum coverage and high cognitive demand appears to make a difference in mathematics reflected in a conceptual focus (Mewborn, 2001; Reeves, 2005; Spaull, 2011; Van der Berg et al., 2011).

Considering this in relation to teacher knowledge, it is useful to consider if there is a special type of knowledge which teachers draw on to teach effectively. Contemplating this, Shulman (1987) suggests seven categories of knowledge which teachers could be equipped with, namely: Pedagogical Content Knowledge, Content Knowledge, General Pedagogical Knowledge, Curriculum Knowledge, Knowledge of Learners and their Characteristics, Knowledge of Educational Context, and Knowledge of Educational Ends. However, in this study, we only consider two of them - specifically CK and PCK - though we draw on more recent research in our conceptualization of PCK. To Shulman, PCK depends on an understanding of a particular topic, and knowing how to present these factors so as to make sense to the students, whereas he considers CK as the knowledge teachers have on the subject matter they are teaching. Researchers like Deapepe, Verschaffel, & Kelchtermans, (2013) argue that CK has a positive influence on PCK.

In their studies on PCK, some authors focus on the role played by the content, even if their notion of PCK also includes pedagogy (Sorto et al., 2009). A different perception of PCK comes with the work of Hill, Ball, & Schilling (2008) in which they consider Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Curriculum (KC) as PCK sub-categories. They emphasize KCS as a subset of PCK, which itself is a subset of the larger construct of what they call Mathematical Knowledge for Teaching (MKT). Based on their analysis of the mathematical demands of teaching, Ball, Thames, & Phelps (2008) hypothesize that Shulman's categories of CK and PCK can be subdivided into Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK) on

[8] What is understood by 'effective' could itself be interrogated and indeed has been so, but for the purpose of this paper we work from the assumption that effective teaching is that which offers learners the best possibilities to develop mathematical proficiency, possibly but not solely or in its entirety reflected in learner performance in tests.

one hand, and Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) on the other hand. As a matter of fact, some other researchers in mathematics education claim that mathematics teachers need a special type of CK, which they refer to as 'Mathematics for Teaching' (Adler & Zain, 2006) and which is different to the mathematics that Mathematicians or Engineers need. In particular, Mathematics for Teaching involves an 'unpacking' of the mathematical concepts taught. In the area of Mathematics Education, different researchers argue that mathematics teachers need more mathematics abilities to teach mathematics (Baker & Chick, 2006; Ma, 1999; Shulman, 1987). These abilities are in line with what is considered as Mathematics for Teaching (Adler & Davis, 2006; Ball, Hill, & Bass, 2005; Hill, Ball, & Schilling, 2008) or as Special Content Knowledge by Deapepe et al. (2013).

The literature in the Rwandan context on Mathematics teachers' knowledge or practices seems to be limited. Thus, the present study was deemed a useful contribution to our understanding of Rwandan grade 6 mathematics teachers' knowledge, and their teaching, as will be discussed in future publications.

Methods

The teacher knowledge test

The test we used in our study was adopted in a previous study spanning Botswana and South Africa (Carnoy, Chisholm, & Chilisa, 2012; Christiansen & Aungamuthu, in progress). It was constructed to interrogate both CK and PCK of teachers, but it is not clear to us to what extent it had engaged the different sub-categories of PCK suggested by Ball and her colleagues (2008). The coverage of the mathematical topics varied greatly, but there were some questions on numbers, geometry, measurement, statistics/probability, and algebra/number patterns. We identified four aspects of PCK (Hill, Blunk, et al., 2008) in the test, reflecting the way teachers may consider learners' thinking, how they unpack methods, their use of representation, and their ability to analyze learner errors, respectively. However, there were a limited number of questions in some of these categories, making drawing conclusions unreliable, so in this paper, we only considered the sub-categories of error analysis and unpacking mathematics. We still opted to use this test so as to allow for comparisons to the teachers from South Africa.

As the test is not in the public domain, we cannot include the actual questions from the test in this paper, but we have provided three sample questions below, adjusted from the originals in such a way that the main idea remains. Figure 1 shows an example of a CK question, while Figure 2 shows an example of a PCK question, classified as 'engaging learner thinking'.

How many decimal numbers are there between 0.80 and 0.90?

(Mark ONE answer.)

A. 9

B. 10

C. 99

D. *An infinite number*

Figure 1. An example of a CK test item. Correct answer in italics. 8 of the 19 teachers answered this question correctly.

In the CK question (Figure 1), the preferred incorrect answer was A, which 10 of the 19 teachers i.e. 52.6% chose. This reflects a common understanding of real numbers as having successors, the same way the natural numbers do. If the teachers hold this perception, they will not be in a position to engage the nature of the reals with their learners. The fact that so many of the teachers chose 9 as the answer, even though the boundaries were given with two decimals, also indicates a limited notion of the reals.

As Mr. Jantjes was assessing his learners' work from the day's lesson on multiplication, he noticed that Dylan had invented an algorithm that was different from the one taught in class. Dylan's work looked like this:

```
  547
   x3
 ----
  121
 1520
 ----
 1641
```

What is Dylan doing here?

(Mark ONE answer.)

A. Dylan is regrouping ("carrying") tens and ones, but this work does not record the regrouping.

B. Dylan is using the traditional multiplication algorithm but working from left to right.

C. *Dylan has developed a method for keeping track of place value in the answer that is different from the usual algorithm.*

D. Dylan is not doing anything systematic. He just got lucky, what he has done here will not work in most cases.

Figure 2. A PCK test item on engaging learner thinking. Correct answer in italics. 2 of the 19 teachers answered this item correctly.

On the task in Figure 2, the preferred incorrect answer was D, which 11 of 19 the teachers chose. This reflects a lack of openness to alternatives to the standard algorithms. The question requires teachers to not only recognize the answer as correct, but also to understand multiplication well enough to recognize the systematic way in which the learner has worked through the task. It thus combines an orientation towards learner responses as meaningful, as well as mathematical understanding. This differs from the 'unpacking mathematics' questions which were more about dealing with mathematical argumentation itself, as exemplified in Figure 3. An incorrect response to this question indicates a difficulty recognizing that the reasoning is not addressing ratio.

Noxolo and Musa are making porridge for their families. Noxolo uses 3 cups of oats and 5 cups of water. Musa uses 4 cups of oats and 7 cups of water. Who made the thickest porridge?

Learners agreed that Noxolo's porridge is the thickest, yet differed in their thinking about why.

Indicate whether each of the following offers correct reasoning about this problem.

A: The difference between the number of cups of oats and water is smaller for Noxolo (5 - 3 = 2) than for Musa (7 - 4 = 3).

Figure 3. An example of a PCK test item on unpacking mathematics. 9 of the 19 teachers correctly indicated that this is not correct reasoning about the situation.

Sampling

Through random sampling, the first author selected seven of the 30 districts in Rwanda, and then used stratified sampling to identify primary schools. The final sample consisted of 20 schools. To reduce travel costs, the districts were chosen from three of Rwanda's five provinces. Of these five provinces, one of them is a city. We included this one in the sample to ensure the presence of well-resourced schools in the sample. Using simple random sampling, the other two provinces were chosen from the four remaining. Within the three provinces, stratified sampling was used to identify districts.

As the strata had to be mutually exclusive, in this case, the school resources were taken into consideration in order to get a sense of school categories; those in which learners had basic learning materials like a book for each learner, and geometry kits for both learners and teachers were categorized as resourced schools, whereas those without facilities mentioned previously, were taken as non-resourced schools. This information was obtained from the Rwandan Ministry of Education, under the National Schools Inspectorate Division. There were insufficient official records to provide the information with complete accuracy, so the accuracy of information relies on the informant who had more than 15 years of experience working in the above mentioned division. The information matched with the situation in the actual schools visited.

Originally, 20 teachers were included in the sample, but the data from one were excluded from analysis because he was reluctant to complete the questionnaire and appeared to struggle to answer the questions due to language difficulties. All participation was voluntary. Teachers answered the questionnaire in their own time, but on the school grounds, they were not allowed to take it home.

The respondents

We then collected data from 19 teachers teaching in 19 different schools in Rwanda, including both private and public schools. Nine of the teachers were men whereas ten of them were women. Their average age was 41 years, while the youngest in the group was 22 years old and the oldest was 59 years old. Among them, the majority (73.7%) were senior six secondary diploma holders. Also, 15.7% held a D7 level, which is a level between senior six secondary and university studies. D7 level was offered in the earlier Rwandan school system but not offered nowadays. In addition to this, 10.6% of the teachers held bachelors' degrees, among which half (5.3%) had studied education and the other half (5.3%), accounting.

Amongst the sampled teachers, teaching experience was over two years each, and the majority, i.e. 64.7% had more than eleven years of teaching experience. Particularly as mathematics teachers, 58.8% had more than 11 years of experience, 35.2% ranged between two and five years, whereas 5.8% ranged between 6 and 10 years for the 16 teachers who answered this question. The mobility of Rwandan teachers between schools appears to be low as more than 64.6% had been teaching in the same school for a period of more than six years. Only one teacher in our sample was new in his/her school and in the process of completing his/her first year in that particular school. In-service teacher training seemed to be rare. In our sample, of the 8 teachers who responded to this item, 35.2% had not received any in-service mathematics training and 70.5% of those who had done only one or two courses in which they covered topics on numbers, geometry, data handling measurement and algebra, with no pedagogy or PCK content mentioned.

The test was written in English, although all of the teachers who wrote it had a French background in their studies as they all completed their studies in Rwanda and qualified as teachers before 2003 while the Rwandan school system was using French as the language of instruction (Gahigi, 2008), associated with the Rwandan colonial history.

Data analysis

To analyze the data we created an Excel spread sheet in which we entered each and every response given to each question by each teacher. The test questions were then categorized according to mathematical content domain and type of PCK, and the results analyzed across categories. We acknowledge that the sample is small and thus cannot be generalized, but it still has a reasonable likelihood of reflecting the current situation.

Results and Discussion

The tests analyzed in this study focused on numbers, algebra/patterns, geometry, measurement and statistics/probability, covering both CK and PCK questions (except for algebra where there were no PCK questions).

Due to a considerable age range of 37 years for our respondents, which is the same as the one for SACMEQ teachers (Hungi et al., 2010), we interrogated teachers' performance by age. Even though we were not making comparisons, we noted that the teaching experience for teachers was around 12 years in SACMEQ, whereas it was 11 years for teachers in our sample, as earlier mentioned. It was not surprising for us to find a correlation of -0.23 between teachers' performances and their ages as they all had almost similar education system backgrounds. In other words, there is no indication that older teachers do better on the test; there is a slight tendency for younger teachers to perform better.

We were expecting higher scores from teachers with higher levels of education on the given task. However, our results showed that there was no correlation (-0.08) between levels of education and performance in the test. Besides, we measured if teachers with more experience, particularly as mathematics teachers, did better in the test compared to other teachers in the sample. The correlation was 0.15, which was too low to claim any link.

The overall results on the test are shown in Figure 4 below according to topics. On one hand, teachers' content knowledge was best in the topic areas of 'numbers' with a mean score of 72.1% and 'measurement' with

78.9% as the mean score, with disappointing scores between 43% and 49% for the remaining three content areas i.e. algebra, geometry, probability/statistics. On the other hand, the teachers scored best on PCK questions for statistics/probability with a general average of 72.9%.

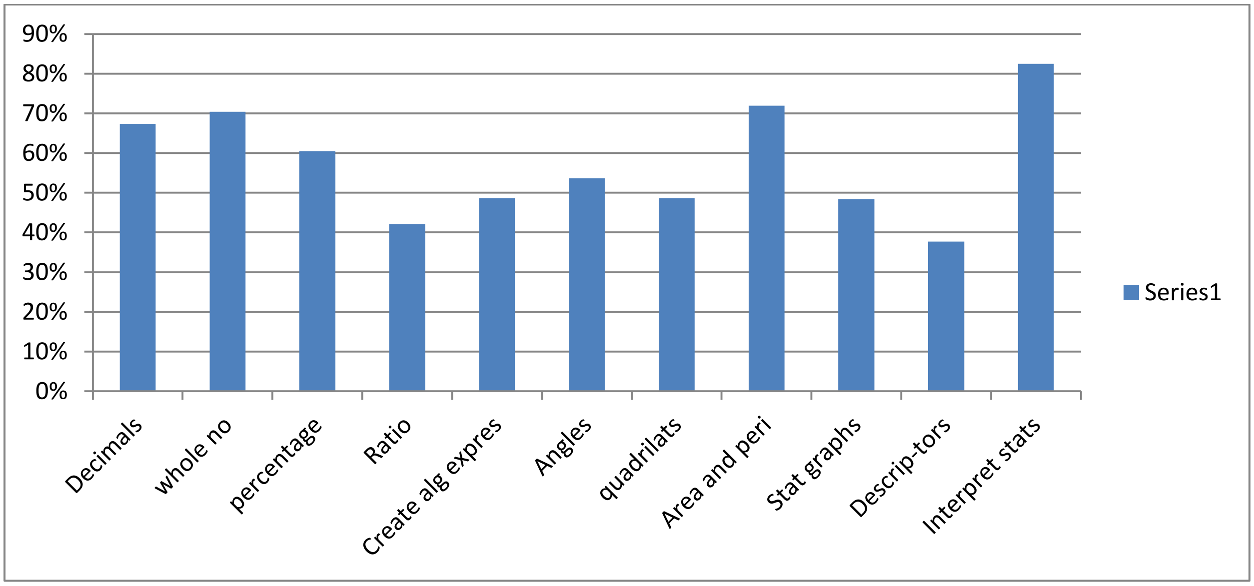

Figure 4: Teacher scores across topics, but encompassing both CK and PCK questions. Correct answers are given as a percentage of 19, thus *de fact* reflecting unanswered questions as incorrect

Although teachers generally performed better on certain questions like the ones related to numbers (Figure 4), there is a substantial spread in performances for some mathematics content domains. This is the case for scores which teachers got on questions related to decimals, percentages, creating algebra expressions, angle recognition and on statistical descriptors as representations with standard deviations of 0.26; 0.27; 0.33; 0.31 and 0.28 respectively. They performed well on CK questions in relation to numbers and measurement, and for the most part on PCK questions, especially in probability/statistics. However, for PCK, only five of the teachers correctly answered one of the two questions on the use of representations, and only two correctly answered both questions on learner thinking.

The means for the CK categories were 72.1%; 48.7%; 44.4%; 78.9% and 43.1% in numbers; function/algebra; geometry; measurement and statistics/probability respectively. On the PCK side, the mean scores were 52.2%; 53.7%; 64.9% and 72.9% in numbers; geometry; measurement and statistics/probability respectively. There were no PCK questions on function/algebra category in our test.

Some researchers (Beswick, Callingham, & Watson, 2012) argue that teachers with good PCK knew how to present mathematics concepts while teaching as well as respond to learners more constructively, so that their learners may assimilate the topic under study in a way which is more meaningful, with less misconceptions in their learning. However, Krauss et al.(2006) argue that PCK levels can at best explain learners' gain in a non-trivial way. In present day Rwanda, primary school teachers are trained in special schools known as Teacher Training Centers

(TTC) whereas secondary school teachers are mainly trained in a specific college of education of the University of Rwanda. It would be relevant to consider our findings in the light of the teaching at the Teacher Training Centers, and we hope to do so in future work. For now, we considered the correlations between the two types of knowledge.

The two types of knowledge measured were highly correlated, with a Pearson correlation coefficient of 0.773 significant at the 0.01 level (see Figure 5).

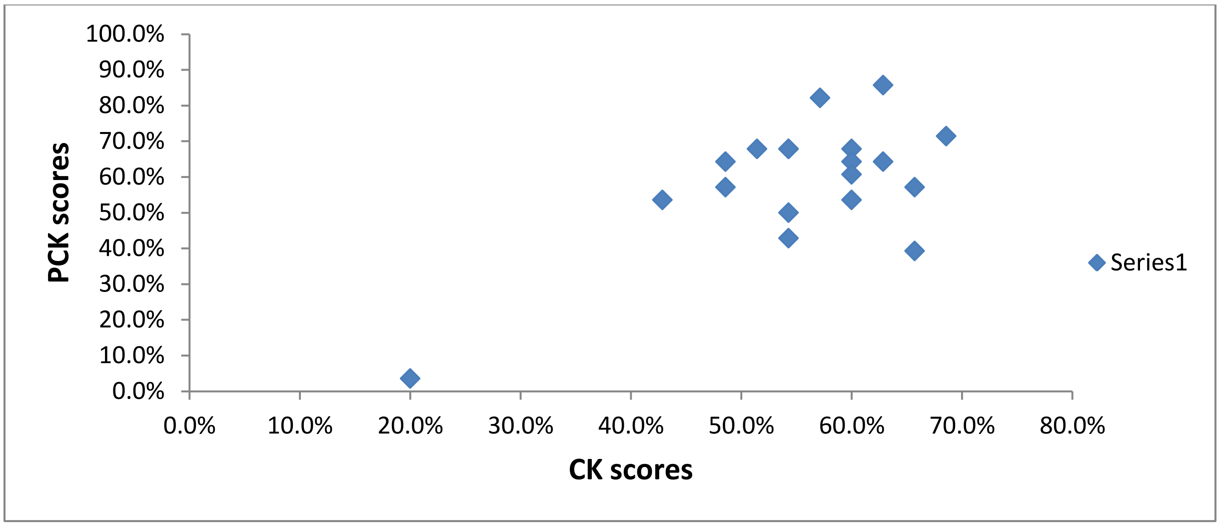

Figure 5: Overall correlation between CK scores and PCK scores

The positive correration between CK and PCK was also found in Germany by (Krauss et al., 2006) and in the grade 6 study in KwaZulu-Natal. The study furthermore suggests that CK is the basis of PCK, while an Australian study claims that there are aspects of the same knowledge domain (Beswick et al., 2012). Ball's study in the USA indicates that common and specialised knowledge are linked but not equivalent as there is a possibility of someone developing CK yet lacking the specific types of knowledge required to teach (Hill, Schilling, & Ball, 2004, p. 15).

We further investigated the correlation between scores in CK and PCK across the different domains. The correlation between CK scores and PCK scores on the content area 'number' was very strong, i.e. 0.948 (Figure 6).

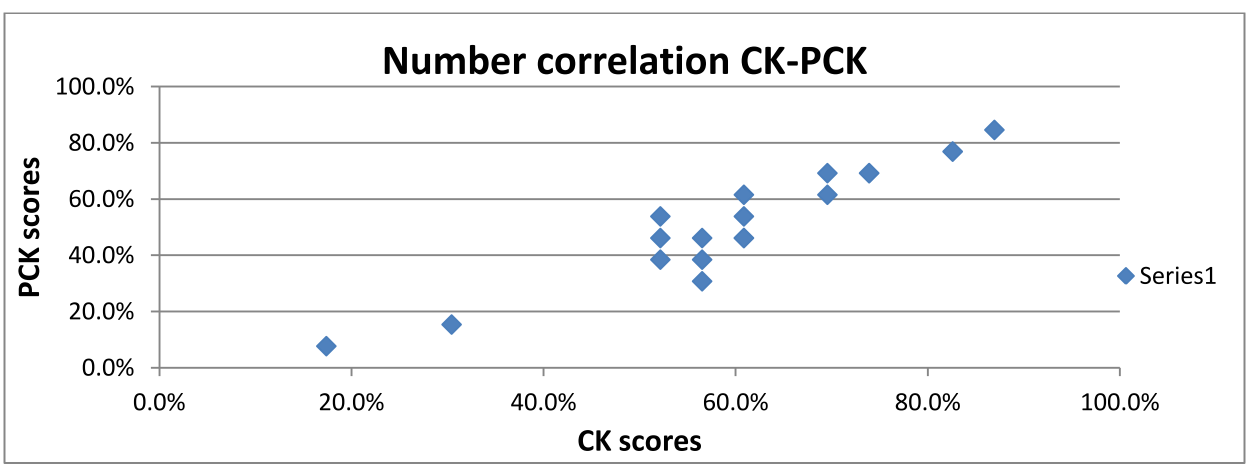

Figure6: Correlation between CK scores and PCK scores on 'number'

For geometry, there was no link between CK and PCK (correlation of -0.086) and there was a weak correlation for measurement (0.449) and a weaker one for statistics/probability (0.352). However, CK overall was strongly correlated with unpacking mathematics PCK (0.703).

As we previously mentioned, the results showed an overall correlation of 0.773 between teachers' CK and their PCK, as illustrated in Figure 5 above.

Generally, the teachers found it difficult to unpack learners' thinking in responding to a particular mathematics task. This was also the case in the study conducted by Beswick, Callingham, & Watson (2011). However, teachers were able to recognise if the learners' answers were correct or incorrect.

Conclusion

In this paper we have presented the main correlations measured between CK and PCK. However, due to the sample size we opted not to generalize the findings even if the method used could indicate the status of grade 6 mathematics teachers' knowledge. Our results may still help the education policy makers to think about which aspects to focus on during mathematics teacher training in order to shape teachers' CK and PCK. From what we have seen, we suggest this may particularly include addressing teachers' awareness of learners' thinking, but also to address content knowledge gaps in algebra, geometry, statistic and probability.

The next issue is, of course, to what extent the teachers' knowledge translate into teaching in the classroom. This will be interrogated in future papers, where we examine the extent to which classroom teaching supports or contradicts the impressions gained from the teacher tests alone.

References

Adler, J., & Davis, Z. (2006). Opening Another Black Box: Reseaching Mathematics for Teaching in Mathematics Teacher Education. *Journal for research in mathematics Education, 37(4)*, 270-296.

Akpo, S. E., & Jita, L. C. (2013). The influence of selected teacher inputs on students' academic achievement in the junior secondary school certificate mathematics in Namibia. *TD: The Journal for Transdisciplinary Research in Southern Africa: Special edition: Sustainable learning environments and social justice, 9*(3), 465-479.

Baker, M., & Chick, H. (2006). *Pedagogical content knowledge for teaching primary mathematics: A case study of two teachers.* Paper presented at the 29th Conference of the Mathematics Education Research Group of Australasia.

Ball, D.L., Thames, M.H., & Phelps, G.C. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59(5)*, 389-407.

Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *, 29*(1), 14-17,20-22,43-46.

Beswick, K., Callingham, R., & Watson, J. (2012). The nature and development of middle school mathematics teachers' knowledge. *Journal of mathematics Teacher Education, 15*(2), 131-157.

Carnoy, M., Chisholm, L., & Chilisa, B. (Eds.). (2012). *The low achievement trap*. Cape Town, South Africa: HSRC Press.

Christiansen, I. M., & Aungamuthu, Y. (in progress). The mathematical content knowledge of grade 6 teachers in KwaZulu-Natal.

Deapepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in wich the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12-25.

Gahigi. (2008). Rwanda: English language teaching kicks off. *All Africa. November, 4*.

Grossman, P.L (1990). The making of a teacher: Teacher knowledge and teacher education. *New York: Teacher College Press*.

Hill, H.C., Ball, D.L., & Schilling, S.G. (2008). Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teachers'Topic-Specific Knowledge of Students. *Journal for research in mathematics Education, 39(4)*, 372-400.

Hill, H.C., Blunk, M.L, Charalambous, C.Y, Lewis, J.M, Phelps, G.C., Sleep, L& Ball, D.L. (2008). Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study. *26*(4), 430-511. http://www.tandfonline.com/doi/pdf/10.1080/07370000802177235

Hill, H.C., Schilling, S.G., & Ball, D.L. (2004). Developing Measures of Teachers' Mathematics Knowledge for Teaching. *The Elementary School Journal, 105*(1), 11-30. doi: 10.1086/428763

Hungi, N., Makuwa, D., Ross, K., Saito, M., Dolata, S., van Cappelle, F., Paviot, L. & Vellien,J. (2010). *SACMEQ III Project Results: Achievement levels in reading and mathematics: Working Document Number 1*. 2011, from http://www.sacmeq.org/downloads/sacmeqIII/WD01_SACMEQ_III_Results_Pupil_Achievement.pdf

Krauss, S., Neubrand, M., Blum, W., & Baumert, J. (2006). The Professional Knowledge of German Secondary Mathematics Teachers: Investigations in the Context of the COACTIV Project.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*: Lawrence Erlbaum Associates Mahwah, NJ.

Marais, C., van der Westhuizen, G., & Tillema, H. (2013). Teacher knowledge of learners' help-seeking in mathematics problem solving. *African Journal of Research in Mathematics, Science and Technology Education, 17*(1-2), 63-73.

Markic, S., & Ingo, E. (2010). A mixed methods approach to characterize the beliefs on science teaching and learning of freshman science student teachers from different science teaching domains. In M.F. Taşar & G. Çakmakcı (Eds.). *Contemporary science education research: teaching and Teacher Education*, 21-28.

Mewborn, D. (2001). Teachers'content knowledge, teacher education and their effects to preparation of elementary teachers in the United State. *Mathematics Teacher Education and Development, 3*, 28-36.

MINEDUC, R. (2006). Rwanda Education Sector: Long-term strategy and financing framework 2006-2015 (LTSFF).

Reeves, C. (2005). *The effect of opportunity-to-learn and classroom pedagogy on mathematics achievement in schools serving low socio-economic status communities in the Cape Peninsula.* (PhD), University of Cape Town, Cape Town.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher, 15(2)*, 4-14.

Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Education Review, 57*, 1-22.

Sorto, M.A., Marshall, J.H., Luschei, T.F.,&Carnoy, M. (2009). Teacher knowledge and teaching in Panama and Costarica: A comparative study in primary and secondary education. *Revista Latinoamericana de Investigacion en matematica Education, 12(2)*, 251-290.

Spaull, N. (2011). A preliminary analysis of SACMEQ III. *Stellenbosch Economic Working Papers,* (11/11). www.ekon.sun.ac.za/wpapers/2011/wp112011/wp-11-2011.pdf

Uworwabayeho, A. (2009). Teachers' innovative change within countrywide reform: a case study in Rwanda. *J Math Teacher Educ, 12*, 315–324.

van der Berg, S., Burger, C., Burger, R., de Vos, M., du Rand, G., Gustafsson, M., Moses, E., Shepherd, D., Tayler, S., van Broekhuizen, H., von Fintel, D. (2011). *Low Quality Education as a Poverty Trap*. Stellenbosch, South Africa: Stellenbosch University.

Zhou, Z., Peverly, S. T., & Xin, T. (2006). Knowing and teaching fractions: A cross-cultural study of American and Chinese mathematics teachers. *Contemporary Educational Psychology, 31*(4), 438-457.